David A. King

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PREFACE

The papers reprinted in this volume would not have seen the light of day had it not been for my family, my teachers and various academic institutions. In paying due tribute, it may be appropriate to explain briefly how I came to an interest in the subject of medieval Islamic astronomy.

I imbibed the history of astronomy from an early age. When I was at high-school in England, my father, Henry C. King, wrote *The History of the Telescope* (1955) and *The Background to Astronomy* (1957). But my own interests were elsewhere, in other parts of this planet and other cultures. Little did I know at that time that I would one day choose to study the same Greek and Islamic astronomical traditions which my father had been researching for that second book!

Somehow I survived the rigours and absurdities of a higher education restricted to mathematics, perhaps only by reading in history and religion and travelling regularly to Southern Europe. At the age of twenty I visited the Middle East: the peoples, culture and history of that region captivated me. My first appointment was with the Sudan Government Ministry of Education (1964–67), teaching English and Mathematics to high-school students in the provinces.

From El-Fasher in Darfur I wandered to New Haven, Connecticut, there to immerse myself in Near Eastern Studies at Yale University. It seemed like a good idea at the time and turned out to be a move I have never regretted. Prof. Franz Rosenthal taught me how to read Arabic manuscripts instead of newspapers; Prof. Bernard Goldstein introduced me to Arabic scientific manuscripts and Ptolemaic planetary models; and Prof. Asger Aaboe taught me things not recorded in any book. At a seminar at Brown University I met Prof. E. S. Kennedy, the leading scholar in the history of Islamic astronomy, who at that time was on leave from the American University of Beirut.

Through these personal and academic encounters my own professional fate was decided. It was quite apparent to me that the history of science in the Islamic world was an exciting field in which to conduct research on primary sources. Men who had discovered the same in previous decades, such as the Sédillots, Woepcke, Suter, Nallino, Wiedemann, Krause, Schoy, Luckey, Renaud, Millas, Neugebauer, Hartner, Kennedy, Ullmann, Pingree, Toomer and Goldstein, were as easy to admire as they were difficult to emulate. Scholars such as Brockelmann, Storey and Sezgin, who had by their monumental labours documented the Arabic and Persian manuscript sources available for the study of Islamic civilization, likewise aroused my deepest admiration and respect.

For my doctoral dissertation, Bernie Goldstein suggested an investigation of the *Ḥākimī Zīj* by the Fatimid astronomer Ibn Yūnus. (A *zīj* is an astronomical handbook with tables and explanatory text.) I set about gathering microfilms of the relevant manuscripts, and in Beirut Ted Kennedy guided me through that *zīj* and others. George Saliba was also a student of Ted's at that time: he too was immersed in *zīj*es. Our colleague Dave Gordon in the English Department at A.U.B. coined the phrase "zijjing about" to describe our activities. Also in Beirut for that year was Prof. Owen Gingerich of the Harvard-Smithsonian Center for Astrophysics, who introduced both George and me to medieval astrolabes and modern computers.

Before I had finished the thesis (1972), I had stumbled upon the Cairo corpus of tables for timekeeping and the various tables of al-Khalīlī, and had recovered some of the "lost" material of Ibn Yūnus in later Egyptian and Yemeni sources. Bernie Goldstein wisely advised me to suppress all this at least until I had finished the thesis, and it was to take many years before this new material had been digested and evaluated.

Owen Gingerich saved a graduating doctoral student from the bleak prospect of unemployment by helping to organize a research project to catalogue the scientific manuscripts in Cairo. Initially proposed for two years, the project lasted seven (1972–79) and achieved rather more than its original aim. Based at the American Research Center in Egypt, it was funded by the Smithsonian Institution and National Science Foundation, with additional support from the American Philosophical Society and the Ford Foundation. The project afforded me the opportunity to visit most of the major manuscript libraries in Europe and the Near East. No less stimulating was my appointment at New York University (1979 – present), teaching classical Arabic, Islamic Studies, and History of Science. Research funding was provided by the National Science Foundation, the National Endowment for the Humanities, the Hagop Kevorkian Fund, and the University itself. During this period, my interests moved towards folk science and its practical applications in daily life, although I still enjoyed "zijjing about" and trying to crack medieval tables.

The papers collected here describe some of the materials located in previously untapped manuscript sources. Paper no. I offers some personal reflections on the present state of the field, which I hope will draw attention both to the importance of the Islamic astronomical tradition and to the vast amount of material awaiting scholars in the future.

Paper II describes the tables not discussed by Ted Kennedy in his survey of Islamic *zīj*es published in 1956. In a sense, both of our papers are mistitled, and mine is simply a supplement to his.

Part of my own research has been to survey each of these categories of tables. For example, Papers IX, X and XII are overviews of astronomical timekeeping in the three main areas where it was practised, and Papers XIV and XV describe the different kinds of tables used by medieval astronomers for multiplication and division.

Paper XIII concerns the determination of the qibla or sacred direction in Islam. A great deal of my more recent research has been on the non-mathematical procedures which were commonly used in practice and which account for the wide variation in orientations of medieval mosques. Papers on this aspect of the qibla problem, which fall into the category of folk science, are not reprinted in this volume.

The reviews of two popular works on Islamic science are included as Papers XVII and XVIII in the hope that at least readers of these reviews will see that the books concerned should not be taken as authoritative. Future authors of such surveys should familiarize themselves with at least some of the abundant secondary literature on Islamic science before attempting to describe the Muslim achievement in this area.

To each of the institutions and individuals mentioned above I owe a great deal. To my wife Patricia, who shared my involvement with Ibn Yūnus, al-Khalīlī, and various other medieval astronomers from Andalusia to Central Asia, I owe still more. This volume is dedicated to my parents, who made the whole venture possible and encouraged me at every step of the way.

DAVID A. KING

PUBLISHER'S NOTE

The articles in this volume, as in all others in the Collected Studies Series, have not been given a new, continuous pagination. In order to avoid confusion, and to facilitate their use where these same studies have been referred to elsewhere, the original pagination has been maintained wherever possible.

Each article has been given a Roman number in order of appearance, as listed in the Contents. This number is repeated on each page and quoted in the index entries.

Some Reflections on the History of Islamic Astronomy

(Adapted from an article in the *Bulletin of the Middle East Studies Association of North America*, 4 (1980), pp. 10-26)

Before the advent of Islam the Arabs of the peninsula had an intimate knowledge of the sun and moon and the changing night sky throughout the year, as well as the meteorological phenomena associated with the seasons. Since the sun, moon, stars and winds are specifically mentioned in the *Qur'ān*, there was considerable interest in the heavens, both in the early Islamic community of the Hejaz and, in later centuries, throughout the entire Islamic world from the Maghrib to Indonesia. The astronomical folklore of the Arabs was documented for the first time in the eighth and ninth centuries and was discussed in a wide variety of treatises thereafter. Applications of folk astronomy to certain aspects of religious life, such as the regulation of the lunar calendar, the determination of the times of prayer, and the fixing of the sacred direction (*qibla*), are discussed in independent compilations and also feature in books dealing with the sacred law.

During the millenium which followed the introduction of a far more "sophisticated" mathematical astronomy from Indian, Sasanian and Hellenistic sources to the vigorous cultural scene of Abbasid Iraq in the eighth and ninth centuries, Muslim scholars made substantial contributions to all aspects of astronomy. They compiled a remarkably rich and varied corpus of astronomical literature, dealing with planetary astronomy, spherical astronomy, timekeeping, instruments and astrology. In addition, these works treat of the regulation of the calendar, prayer-times and sacred direction by mathematical means.

The two distinct branches of astronomy practised by Muslim scholars until the nineteenth century possessed a pronounced Islamic flavour and went far beyond classical folk astronomy and mathematical astronomy in their interests and applications. Islamic astronomy should not be thought of merely as an intermediary between classical and early European astronomy, not least because the vast majority of Islamic astronomical works were unknown in the West and hence were without any influence there.

Islamic texts on folk astronomy and mathematical astronomy are preserved in the manuscript libraries of the Near East, India, Central Asia, Europe and the United States. Catalogues of varying quality exist for most collections of manuscripts written in Arabic, Persian and Turkish, but most were prepared by individuals who had little or no interest in scientific literature *per se*. The model catalogue is that of W. Ahlwardt for the Berlin collection, but most cataloguers are not so fortunate as Ahlwardt, who had the manuscripts shipped to him at his home. For some of those collections which were catalogued at an early date, such as the Bodleian Library in Oxford, there is no supplementary catalogue of more recent acquisitions. Likewise, the holdings of what is now the Egyptian National Library in Cairo were catalogued in 1890, and the new manuscripts acquired between 1930 and 1950 were catalogued in the 1950's. But vast numbers of manuscripts were acquired between 1890 and 1936 and these were catalogued only in the 1970's. Both in Europe and the Near East there are several important collections which are not yet catalogued at all. The largest and richest collection of Arabic manuscripts in general, and scientific manuscripts in particular, namely, that in the Süleymaniye Library in Istanbul, is without a published catalogue.

The first bio-bibliographical survey of the available medieval literature relating to astronomy and mathematics was published by H. Suter in 1900. This fundamental reference work was based mainly upon the information contained in catalogues of European collections. Additional information for Arabic and Persian sources was provided in the monumental surveys of Arabic and Persian literature by C. Brockelmann and by C.A. Storey. H.P.J. Renaud contributed useful additions, mainly for Maghribi scientists, and M. Krause identified many, but not all, of the early scientific works represented in the Süleymaniye.

The recent publication of F. Sezgin's volumes of his history of the Arabic literature up to *ca.* 1050 (in German) relating to mathematics, astronomy, folk astronomy and astrology in the early period of Islamic civilization, and the survey (in Russian) by G.P. Matvievskaya and B.A. Rosenfeld of the scientific literature up to the eighteenth century, open up numerous new avenues for research. For the later period the number of available manuscripts has been increased considerably through my recent cataloguing extravaganza in Cairo.

Anyone who leafs through the new volumes of Sezgin's survey will see the amount of scientific material available from the first four centuries of Islam alone, but the sources from later centuries are yet more abundant. Also – and this is a point to be stressed with those who underestimate the importance of the history of Islamic science – the physical size of Sezgin's volumes on alchemy and medicine, and astronomy and mathematics, amounts to almost the same as the total size of his volumes on traditional Islamic sciences, such as *Qur'ān* studies and commentaries, *ḥadīth*, theology, law, history and poetry. This observation is not intended to suggest that the amount of scientific source material currently available in any way approaches the amount of available religious, historical or literary material, but to F. Sezgin and his monumental work goes the credit for illustrating the importance of science in Islamic civilization.

Numerous Islamic astronomical instruments are preserved in museums around the world. A valuable survey of Muslim instrument makers was published in 1956 by L.A. Mayer. The forthcoming survey of Islamic astrolabists and their works by A. Brieux and F. Maddison promises to include a substantial amount of new information and to revive some interest in Islamic instrumentation.

The history of astronomy and mathematics in Islamic civilization has been documented by a series of scholars of diverse national backgrounds who have somehow acquired the necessary linguistic and technical skills and then investigated some of these manuscripts for themselves. Some of these scholars, with their diverse backgrounds and intellectual interests, are Arabists who have become interested in the Islamic scientific tradition; others started life as classicists or mathematicians. (There is no need to be an astronomer to undertake research in the history of Islamic astronomy.) Such is the extent and variety of the source material that all of these scholars have made original discoveries in whichever aspect of the subject attracted their attention.

The study of the history of Islamic astronomy and mathematics began in earnest with the labours of J. Sédillot and his son, L.A.P. Sédillot, and the German F. Woepcke in nineteenth-century Paris. At the beginning of the present century, the Italian Arabist C.A. Nallino and H. Suter, a mathematics teacher from Zurich, were the leading scholars in the field. Between World Wars I and II the most serious activity was conducted in Germany, with scholars such as J. Ruska, E. Wiedemann, K. Schoy, M. Krause and P. Luckey making outstanding contributions. In the early fifties, the American scholar E.S. Kennedy turned his attention to Islamic astronomical handbooks with tables (*zījes*) and initiated a series of studies still in progress. Based at the American University of Beirut, Kennedy was inspired by the example of O. Neugebauer, whose treatment of the sources for the history of ancient astronomy and mathematics stood as a model.

In the U.S.S.R., groups of scholars working mainly in Moscow and Tashkent and inspired by the example of such scholars as A.P. Youschkevitch and B.A. Rosenfeld, have also made valuable contributions to the field. In postwar Germany, W. Hartner produced a series of masterful monographs. In Spain, the eminent scholar J. Millás Vallicrosa initiated a series of studies himself and inspired several younger scholars to continue in the same tradition. In Turkey, A. Sayili conducted several studies of major importance, and found at least one student to continue. In the past two decades several prominent Arab scholars, working mainly in Europe or the U.S.A., have made substantial contributions.

Nowadays, individual scholars working mainly in the U.S.A. and the U.S.S.R., as well as Canada, France, (F.R.) Germany, Spain and Turkey, are publishing on Islamic astronomy and mathematics. Inspired by the example of Neugebauer and Kennedy, and aided by the bibliographical tools of Suter, Storey, Sezgin, and Matvievskaya and Rosenfeld, this is an exciting field for original research. The *Festschrift* for Prof. Kennedy, to be published in 1985, reveals the wide spectrum of backgrounds and interests of many of the scholars currently working in the field.

Most of the secondary literature is in German, French, English, Spanish, Russian, Italian and Turkish, and no one scholar has yet attempted to survey the whole field using the wealth of available material. Various books which have appeared in recent years purporting to present an overview of Islamic science have been quite unsuccessful, not least because their authors were familiar with neither the primary nor the secondary sources. The time is ripe for a survey, but it should be a collaborative effort of the various specialists in each field. R. Rashed in Paris is currently gathering contributions on different aspects of Islamic science for a general history of the subject.

As a branch of Islamic studies, the history of Islamic astronomy has not fared well. In J. Sauvaget's *Introduction to the History of the Middle East* (revised ed., 1965), the sciences in Islam were dismissed in a few lines. In the publication entitled *The Study of the Middle East: Research and Scholarship in the Humanities and the Social Sciences* (1976), sponsored by the Middle East Studies Association of North America, the sciences in Islam were completely ignored. Aware of this deficiency, Prof. Jere L. Bacharach of the University of Washington in Seattle encouraged me to write the article (now completely revised and substantially enlarged) from which these remarks are taken, as he did Prof. J. Len Berggren to write on recent research in Islamic mathematics.

As a branch of the history of astronomy in general, Islamic astronomy has not yet gained its rightful place. Historians of astronomy still tend to see Muslim astronomers as preservers and transmitters of classical astronomy to Europe. In fact, in the literature on the history of science (as distinct from Islamic studies), there has been no improvement as yet on the chapter "Oriental Astronomers" in J.L.E. Dreyer's history of astronomy first published about 1900. B.A. Rosenfeld and the present writer have contributed a survey of Islamic astronomy to the forthcoming *General History of Astronomy*.

The scientific endeavours of Muslim scholars in medieval times have yet to be fully recognized and duly appreciated in the West, but it will take still longer for them to be recognized in the Muslim world. Most popular accounts in contemporary Arabic are summarized from Western popular accounts or they are either simply incompetent or based on fantasy. It is to be hoped that the achievements of Muslim scientists in the medieval period will be presented to future generations in their proper historical perspective (which is not that presented in most Western accounts). Of particular interest to Muslim audiences should be the history of the different aspects of astronomy as applied to everyday religious practice, but there are still no accounts of this subject in Arabic.

The establishment of the Institute for the History of Arabic Science in Aleppo a few years ago, and the appearance there of a first-rate journal devoted entirely to the history of Islamic science (edited at first mainly by E.S. and M.-H. Kennedy) was thought to augur well for the future of our subject. At the present time, however, this Institute seems partially paralyzed by political developments in Syria, and a good share of hope for the future centres on the newly-founded Institut für Geschichte der Arabisch-Islamischen Wissenschaften in Frankfurt, directed by Prof. Sezgin, and the new journal to be published there starting in 1985.

There are so few scholars working on the vast corpus of available materials that it would be most useful to have at hand facsimile reproductions of manuscripts of particular importance. The new Institute in Frankfurt has undertaken a series of such editions. At present the historian of Islamic science has to rely mainly on microfilms or photographs of manuscripts, which some libraries are unable or simply refuse to supply. The more fortunate scholar with a travel grant may have access to the major libraries of Arabic scientific manuscripts; but it would be useful indeed to have a central repository of microfilms and the new Institute in Frankfurt has this potential.

Not only is the importance of the scientific tradition in medieval Islam unquestionable, but there is also no dearth of primary research material available for anyone who has the basic qualifications necessary to handle it. In all, only three out of about 200 Islamic astronomical handbooks (*zījes*) have been published. We have no published edition of the Arabic versions of the *Elements* or the *Almagest*, nor of any Arabic recensions thereof or commentaries thereon. But then only in 1974 did there appear the first modern commentary on the *Almagest* (by O. Pedersen), and only in 1984, the first scholarly English translation of this monumental work (by G. Toomer).

L.A. Sédillot, whose privilege it was to work in the rich collection of manuscripts in Paris, wrote in 1845: "Chaque jour amène quelque découverte nouvelle et vient démontrer l'extrême importance d'un examen approfondi des manuscrits de l'Orient." Given the vast amount of available material gathering dust in the libraries of Europe, the Near East, India and Central Asia, and the relatively small number of scholars currently working in this field, Sédillot's statement is no less true in our time than it was over 125 years ago.

II

ON THE ASTRONOMICAL TABLES OF THE ISLAMIC MIDDLE AGES

The traditional approach of historians of science to Islamic astronomy has been to stress the contribution which it made to European astronomy[1]. The works of the two most famous astronomers of Islam, al-Khwārizmī[2] and al-Battānī[3], are based respectively on the Indian and Hellenistic traditions, and judged by the achievements of these individuals Islamic astronomy ranks as little more than an intermediary, albeit an important one, between classical and medieval European astronomy. Two other Islamic works which were known in medieval Europe, namely the treatises of al-Farghānī[4] and al-Biṭrūjī[5], confirm this impression: the former is a summary of the non-mathematical parts of Ptolemy's *Almagest* and the latter a misguided attempt to adapt simplified Ptolemaic planetary models to an Aristotelian system.

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[1] The best summaries of Islamic astronomy are by J. L. E. Dreyer in *A History of Astronomy from Thales to Kepler*, second edition., New York, 1953, Chapter 11, and C. Nallino's article "Astronomy" in the *Encyclopaedia of Islam*, first edition, Leiden, 1913–34, (both long out of date), and D. Pingree's article "^{c}Ibn al-hay'a" in the *Encyclopaedia of Islam*, 2nd. ed., Leiden, 1960–. See also the penetrating study in D. Pingree, *The Greek Influence on Early Islamic Mathematical Astronomy*, "Journal of the American Oriental Society", 93, 1973, pp. 32–43.

[2] al-Khwārizmī's astronomical handbook, compiled ca. 840, is unfortunately lost, but we do possess a Latin version of a greatly modified recension of the original work by al-Majrīṭī (fl. ca. 1000), published in H. Suter, *Die astronomischen Tafeln des Muḥammed ibn Mūsā al-Khwārizmī in der Bearbeitung des Maslama ibn Aḥmed al-Madjrīṭī und der latein. Uebersetzung des Athelhard von Bath*, "Kgl. Danske Vidensk. Skrifter, 7. R., Hist. og filos. Afd." 3/1, 1914, and supplemented with an English translation and new commentary by O. Neugebauer, *The Astronomical Tables of al-Khwārizmī*, "Kgl. Danske Vidensk. hist.-fil. Skrifter", 4/2, 1962.

[3] On al-Battānī see the article by W. Hartner in the *Dictionary of Scientific Biography*, New York, 1970–. His astronomical handbook was edited with an annotated Latin translation by C. Nallino, *al-Battani sive Albatenii Opus Astronomicum*, 3 vols., Milan and Rome, 1899–1907.

Now the manuscript libraries of the Near East, Europe, and North America, contain thousands of Islamic astronomical manuscripts, the contents of which demand a complete reappraisal of the Muslim achievement in the exact sciences. These manuscripts are mainly of works which were not transmitted to Europe, and many of them represent the results of keen and scholarly activity in astronomy during the period from 750 to 1500. The high scientific level of this activity is especially apparent in the extensive tables which were compiled by the Muslim astronomers, and the purpose of this paper is to describe certain categories of Islamic tables which were not transmitted to Europe, as far as we know[6], and which have only recently come to light during research on manuscripts.

Very few of the numerous collections of Arabic and Persian manuscripts have been properly investigated yet. For certain major libraries there exist catalogues, although of widely divergent quality[7], and the availability of microfilms of manuscripts in certain collections lessens the need for the modern scholar to wander from one library to another like a medieval peripatetic.

The manuscripts which are our source material vary greatly in the quality of their contents, particularly since the uncritical copying of medieval astronomical manuscripts continued in the Near East until the 19th century. Many works from the early period of Islamic astronomy are unfortunately lost, and the majority of extant manuscripts are of later works. Nevertheless there are enough reliable manuscripts of works of consequence dating from the 8th to the 15th centuries for us to reconstruct a picture of the real contributions of the Muslim scholars.

The first attempt to make a bibliographical survey of the Muslim scientists and their works was made by H. Suter[8], who in 1900 published

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[4] al-Farghānī's epitome of the *Almagest* was published in 1669 by J. Golius. See also note 43 below.

[5] The Arabic text of his treatise and a medieval Hebrew translation, together with an English translation and a valuable commentary, have been published by B. Goldstein, *Al-Biṭrūjī: On the Principles of Astronomy*, 2 vols., New Haven and London, 1971.

[6] Further research on Hebrew and Latin astronomical manuscripts may reveal transmission of some of this material to medieval Europe.

[7] Catalogs are listed in A. J. W. Huisman, *Les Manuscrits Arabes dans le Monde: une Bibliographie des Catalogues*, Leiden, 1967.

[8] See H. Suter, *Die Mathematiker und Astronomen der Araber und ihre Werke*, "Abhandlungen zur Geschichte der mathematischen Wissenschaften", 10, 1900, and 14, 1902, pp. 157–185, and the additional information in H. J. P. Renaud, *Additions et Corrections à Suter "Die Mathematiker und Astronomen der Araber"*, "Isis", 18, 1932, pp. 166–183, and M. Krause, *Stambuler Handschriften islamischer Mathematiker*, "Quellen und Studien zur Geschichte der Mathematik, Astronomie, und Physik", Abt. B, 3/4, 1936, pp. 437–532.

a list of over 500 Muslim astronomers and mathematicians, noting the titles of all their known compilations and all of the extant manuscripts attributed to each author. Suter's valuable book was largely based on the manuscript catalogues available to him and on medieval Arabic biographies and bibliographies. Although very much out of date it is still the most useful tool of the student of the exact sciences in Islam. The information which it contains is now supplemented by the indispensable bibliographical works of C. Brockelmann and F. Sezgin. G. Sarton's voluminous writings on the Muslim scientists contain useful bibliographical information but otherwise are all highly derivative[9].

The first large-scale attempt to investigate Islamic astronomical tables was made by E. S. Kennedy. In 1956 he published a survey of the Islamic astronomical handbooks known as *zījes*[10], of which the *zījes* of al-Khwārizmī and al-Battānī are examples. In Arabic the word *zīj* could also be applied to the *Almagest* and the *Handy Tables*, and to the *De revolutionibus* and the *Prutenic Tables*. Kennedy's survey listed over 120 Islamic *zījes*, many of which are still extant, although until now only three have been published[11]. His research has also involved a systematic analysis of the parameters underlying the planetary tables in those *zījes* which are still available, in order to establish their relations with other sources, and the preparation of detailed studies on individual tables for planetary equations and latitudes, parallax, lunar and planetary visibility, and eclipse computations[12]. There are numerous *zījes* of great interest which deserve further investigation, but the number of workers in this field is few indeed.

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[9] Cf. C. Brockelmann, *Geschichte der arabischen Litteratur*, 2 vols., (2nd. ed.), Leiden, 1943–49, and *Supplementbände*, 3 vols., Leiden, 1937–42; F. Sezgin, *Geschichte des arabischen Schrifttums*, Leiden (volumes on mathematics and astronomy are currently in press); and G. Sarton, *Introduction to the History of Science*, 3 vols., Baltimore, 1927–48.

[10] E. S. Kennedy, *A Survey of Islamic Astronomical Tables*, "Transactions of the American Philosophical Society", N.S., 46/2, 1956, pp. 123–177. It is hoped to prepare a new edition of the survey, including several dozen *zījes* that have come to light in the past twenty-five years, and listing the basic parameters used in all extant *zījes* or noted in other sources.

[11] Namely, the *Zījes* of al-Khwārizmī (in a form rather different from the original) and al-Battānī (see notes 2 and 3 above), and al-Bīrūnī's *al-Qānūn al-Mas^{c}ūdī* (edited by M. Krause and published in Hyderabad, 1954–56). A summary of the latter by E. S. Kennedy is shortly to appear in "al-Abhāth". Mention should also be made of the so-called Almanac of al-Zarqāllu (ca. 1100), published by J. Millás Vallicrosa, *Estudios sobre Azarquiel*. Madrid-Granada, 1943–50, pp. 153–237, and analyzed by M. Boutelle, *The Almanac of Azarquiel*, "Centaurus", 12, 1967, pp. 12–20; the *Toledan Tables*, analysed by G. J. Toomer, *A Survey of the Toledan Tables*, "Osiris", 15, 1968, pp. 5–174; the introduction to the *Zīj* of Ibn al-Bannā', edited and translated by J. Vernet Ginés, *Contribución al Estudio de la Labor Astronómica de Ibn al-Bannā'*, Tetuan, 1952; the introduction to the *Zīj* of Ulugh Beg, edited and translated by L. A. Sédillot, *Prolégomènes des Tables Astronomiques*

The tables which I describe in the sequel are not generally contained in *zījes*. They have been located by searching through manuscript catalogues and inspecting large numbers of manuscripts. The computational accuracy of these tables, generally high, has been investigated using electronic computers[13].

Sexagesimal multiplication tables

We shall begin with arithmetical multiplication tables. All Islamic astronomical tables have entries written in Arabic alphabetical notation[14] and expressed sexagesimally, that is, to base 60. A number equivalent to

$$23\ 15\ 54 \text{ seconds,}$$

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d'Oloug Beg, 2 vols., Paris, 1847–1853; and al-Marrākushī's treatise on spherical astronomy, translated by J.-J. Sédillot, *Traité des Instruments Astronomiques des Arabes Composé au Treizième Siècle par Aboul Hhassan Ali de Maroc Intitulé Jāmi^{c} al-mabādi' wa-l-ghāyāt*, 2 vols., Paris, 1834–35.

[12] Examples of such studies are the papers by E. S. Kennedy, *Parallax Theory in Islamic Astronomy*, "Isis", 47, 1956, pp. 33–53; *A Set of Medieval Tables for Quick Calculation of Solar and Lunar Ephemerides*, "Oriens", 18–19, 1967, pp. 327–334; (with M. Agha) *Planetary Visibility Tables in Islamic Astronomy*, "Centaurus", 7, 1960, pp. 134–140; (with M.-L. Davidian) *Al-Qāyinī on the Duration of Dawn and Twilight*, "Journal of Near Eastern Studies", 20, 1961, pp. 145–153; (with N. Faris) *The Solar Eclipse Technique of Yaḥyā b. Abī Manṣūr*, "Journal for the History of Astronomy", 1, 1970, pp. 20–38; (with M. Janjanian) *The Crescent Visibility Table in al-Khwārizmī's Zīj*, "Centaurus", 11, 1965, pp. 73–78; (with H. Salam) *Solar and Lunar Tables in Early Islamic Astronomy*, "Journal of the American Oriental Society", 87, 1967, pp. 492–497; and (with W. Ukashah) *Al-Khwārizmī's Planetary Latitude Tables*, "Centaurus", 14, 1969, pp. 86–96. There are several classes of Islamic astrological tables on which there is as yet no published material, such as tables for the astrological houses and the projections of the rays, but Prof. Kennedy has also done extensive research on these.

[13] On the use of computers in the history of astronomy see, for example, E. S. Kennedy, *The Digital Computer and the History of the Exact Sciences*, "Centaurus", 12, 1967, pp. 107–113, and O. Gingerich, *Applications of High-Speed Computers to the History of Astronomy*, in A. Beer, ed., *Vistas in Astronomy*, 9, London, 1967, pp. 229–236. I have recomputed most of the examples known to me of the various categories of tables described in this paper.

[14] On this convention see R. A. K. Irani, *Arabic Numeral Forms*, "Centaurus", 4, 1955, pp. 1–12.

which in modern sexagesimal notation is

$$23;\ 15,\ 54,$$

means

$$23+15/60+54/3600,$$

i. e.

$$23.265.$$

In sexagesimal arithmetic, more so than in decimal arithmetic, it is useful to have a multiplication table at hand. Such tables are also useful for division. We now have numerous examples of Islamic sexagesimal multiplication tables [15] which give products

$$m \times n\,(m \text{ and } n = 1, 2, \ldots, 60)$$

such as

$$27 \times 51 = 22{,}57\ (= 1377).$$

One particular table gives products

$$m \times n\,(m = 0{,}1,\ 0{,}2,\ \ldots,\ 59{,}59,\ 60{,}0)$$
$$(n = 1{,}2,\ \ldots,\ 60)$$

such as

$$40{,}27 \times 37 = 24{,}56{,}39$$

The more common variety contains about 3,600 entries and the large table contains a grand total of 216,000 entries. Fig. 1 shows an extract from this table: there are 180 such pairs of facing pages in the entire set.

Trigonometric tables

Most *zījes* contain tables of the sine and tangent functions for each whole, or half, or quarter degree of arc[16]. Entries are generally given to three sexagesimal digits, corresponding roughly to five decimal digits. But certain Muslim scholars compiled more extensive sets of trigonometric tables which were not included in *zījes*. Already in the early 10th century al-Samarqandī prepared a set of tangent tables with entries to

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[15] See further D. A. King, *On Medieval Islamic Multiplication Tables*, "Historia Mathematica" I, 1974, pp. 317–323.

[16] See further Kennedy, *op. cit.* (note 10), pp. 139–140; and C. Schoy, *Beiträge zur arabischen Trigonometrie*, "Isis", 5, 1923, pp. 364–399, and *Über den Gnomonschatten und die Schattentafeln der arabischen Astronomie*, Hannover, 1923.

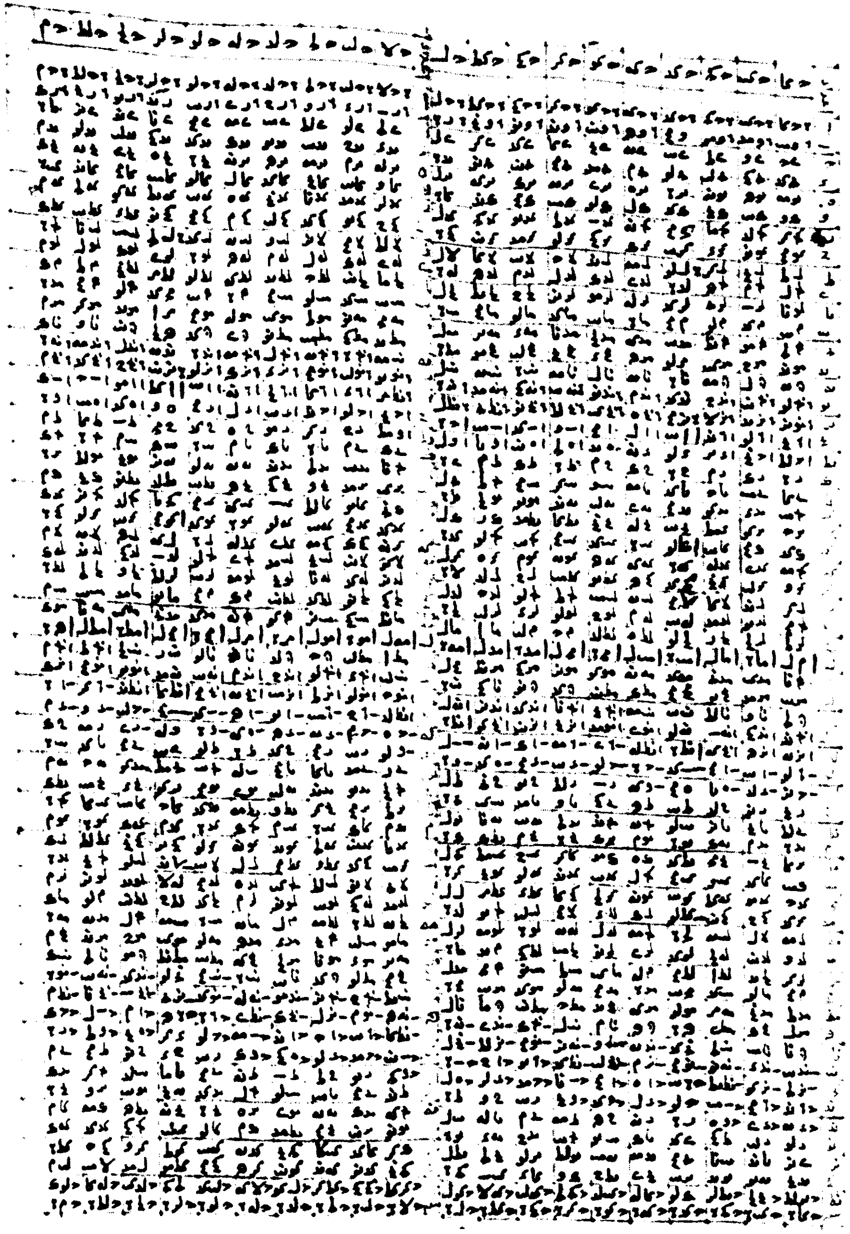

Fig. 1. Extract from the sexagesimal multiplication tables in MS Paris Bibliothèque Nationale ar. 2552 (fols. 11v – 12r). The horizontal arguments in this particular subtable are 3,21, 3,22, ..., 3,40, and the vertical arguments are 1, 2, ..., 60. Reproduced with kind permission of the Director of the Section Orientale de la Bibliothèque Nationale, Paris.

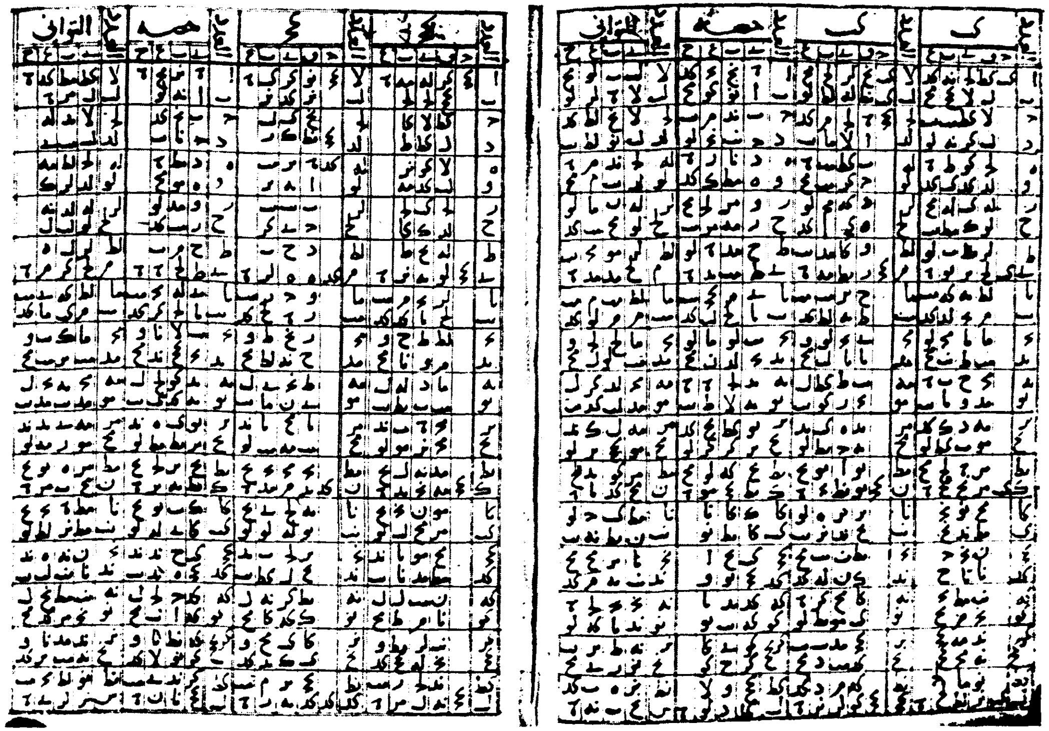

Fig. 2. Extract from Ibn Yūnus' sine tables in MS Berlin Ahlwardt 5752 (= Landberg 1038, fols. 13v – 14r). The right hand page serves arguments between 22;1° and 23;0° and the left hand page serves arguments between 23;1° and 24;0°. The two main columns on the right hand side of each page display the values of the sine function to five sexagesimal digits for each minute of arc, and the two remaining main columns on each page display the differences to be added for each second of arc. Reproduced with kind permission of the Director of the Orientalische Abteilung der Staatsbibliothek, Berlin.

three sexagesimal digits for each minute of arc[17]. Again, in the late 10th century the Egyptian astronomer Ibn Yūnus tabulated the sine function to five sexagesimal digits, or about nine decimal digits, for each minute of arc, also giving the differences for each second. He also tabulated the tangent function for each minute of arc, and the solar declination for each minute of solar longitude[18]. Fig. 2 shows two pages of entries from Ibn Yūnus' sine tables. Whilst these tables are not particularly accurate, they prove that for a mathematician his heart was in the right place. Over four centuries were to elapse before the compilation in Samarqand of the magnificent trigonometric tables contained in the *Zīj* of Ulugh Beg and later also copied separately. These give the values of the sine and tangent to five sexagesimal digits for each minute of argument, and are generally accurate in the last digit[19].

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[17] Suter, *op. cit.* (note 8), no. 501a. (al-Samarqandī is mentioned in the late 10th century *Hākimī Zīj* of Ibn Yūnus.)

[18] See further C. Schoy, *Beiträge zur arabischen Trigonometrie*, (see note 16), pp. 394 – 395, and D. A. King, *The Astronomical Works of Ibn Yūnus*, doctoral dissertation, Yale University, 1972, pp. 85 – 89, 96 – 99, and 109.

Spherical astronomical tables

Certain *zījes* contain extensive tables of spherical astronomical functions. Already in the late tenth century *Ḥākimī Zīj* of Ibn Yūnus, for example, the oblique ascensions are tabulated for each degree of terrestrial latitude and each degree of ecliptic longitude[20].

However, there is another class of such tables which exists in the manuscript sources, though not generally in *zījes*, concerned with what is called in Arabic *ᶜilm al-mīqāt*, timekeeping by the sun and stars[21]. Already about the year 950 ᶜAlī b. Amājūr of Baghdad compiled a table giving the time since sunrise as a functions of solar meridian altitude and instantaneous altitude, for each degree of both arguments[22]. Numerous similar tables for timekeeping by the sun and stars were compiled by later Muslim astronomers for different latitudes, as well as tables based on approximate formulae which work reasonably well for all latitudes. We also have several examples of tables giving the time as a function of solar altitude and the solar longitude. Each of these tables for timekeeping contains several thousand entries. One remarkable table compiled in Cairo about 1250 by Najm al-Dīn al-Miṣrī[23], which gives the time since rising of the sun or a star as a function of its altitude for all declinations and terrestrial latitudes, contains over a quarter of a million entries. In this table one feeds in three arguments, namely, the meridian altitude of the sun or star, the instantaneous altitude, and half the arc of visibility, and the appropriate entry is the time since rising. Fig. 3 shows an extract from this enormous table.

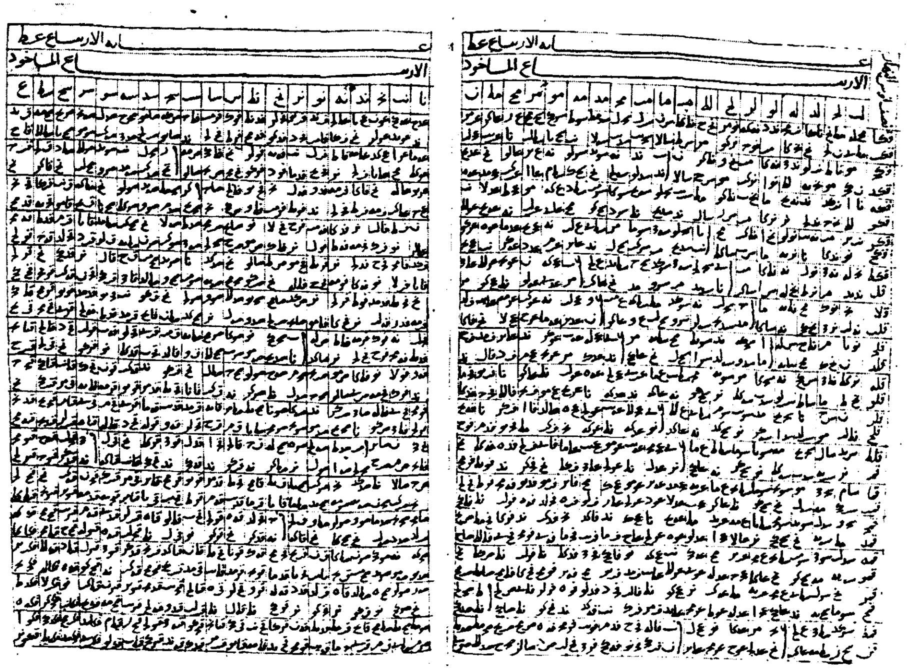

Fig. 3. Extract from Najm al-Dīn al-Miṣrī's universal table for timekeeping, taken from MS Oxford Bodleian Marsh 672 (fols. 33v – 34r). Both pages serve meridian altitude 79° and half arcs of visibility 121°, 122°, ..., 150°, entered vertically. The instantaneous solar or stellar altitude 32°, 33°, ..., 70° is entered horizontally, and the entry in the table is the time since rising of the sun or star, expressed in equatorial degrees and minutes. Reproduced with kind permission of the Keeper of Oriental Books, Bodleian Library, Oxford University.

Other Islamic tables give, for example, the solar azimuth as a function of the solar altitude and longitude for particular latitudes, or the longitude of the ascendant as a function of the altitude of the sun or fixed stars. Fig. 4 shows an extract from a table displaying the longitude of the ascendant as a function of solar altitude and longitude, computed for the latitude of Taiz by the Yemeni astronomer Abū l-ᶜUqūl about the year 1300.

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[19] On these tables see C. Schoy, *Die Trigonometrischen Lehren des persischen Astronomen Abū 'l-Raiḥān Muḥ. Ibn Aḥmad al-Bīrūnī dargestellt nach al-Qānūn al-Masᶜūdī*, Hannover, 1927, pp. 92 – 108. On the *Zīj* of Ulugh Beg see E. S. Kennedy, *op. cit.* (note 10), no. 12.

[20] See further D. A. King, *op. cit.* (note 18), pp. 139 – 141.

[21] I have prepared a survey of several dozen medieval Islamic tables for reckoning time by the sun and stars, which is currently being submitted for publication. The only previous analysis of such a table is contained in B. R. Goldstein, *A Medieval Table for Reckoning Time from Solar Altitude*, "Scripta Mathematica", 27, 1963, pp. 61 – 66.

[22] On Ibn Amājūr see Suter, *op. cit.* (note 8), no. 99.

[23] On Najm al-Dīn al-Miṣrī see Suter, *op. cit.* (note 8), no. 460. An incomplete Oxford manuscript of his main tables was investigated for the first time in July, 1973. The other half of the same manuscript was discovered in the Egyptian National Library on the day before I left Cairo for the Colloquia Copernicana in Toruń.

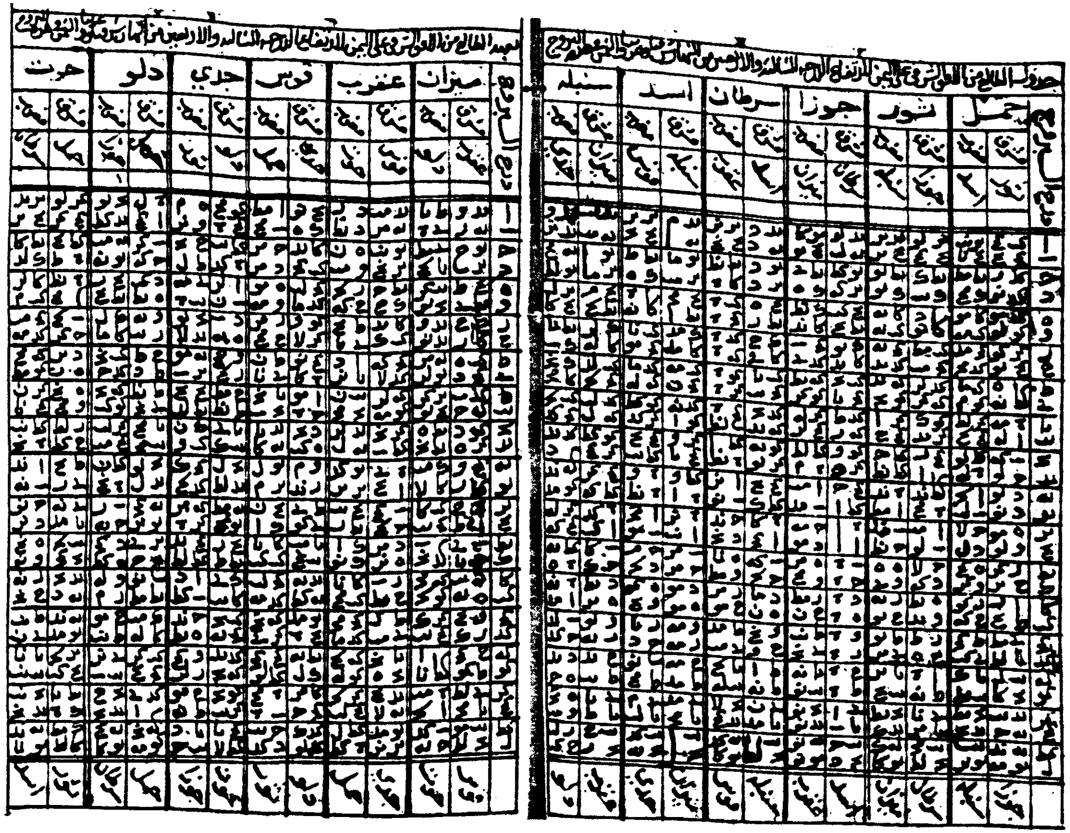

Fig. 4. Extract from the tables for timekeeping computed for the latitude of Taiz (taken as 13;40°) by Abū l-ᶜUqūl, contained in the unique source MS Berlin Ahlwardt 5720 (=Mq. 733, fols. 56v–57r). This particular table displays the longitude of the ascendant (horoscopus) as a function of solar longitude for solar altitude 43°. For each degree of solar longitude entries are given in signs, degrees, and minutes for both eastern and western altitudes. Reproduced with kind permission of the Director of the Orientalische Abteilung der Staatsbibliothek, Berlin.

Tables for the times of Muslim prayer

An important aspect of Islamic timekeeping was the regulation of the times of the five daily prayers[24]. These are defined in terms of the apparent daily passage of the sun across the heavens, and their determination was the concern of the *muwaqqits*, that is, the astronomers

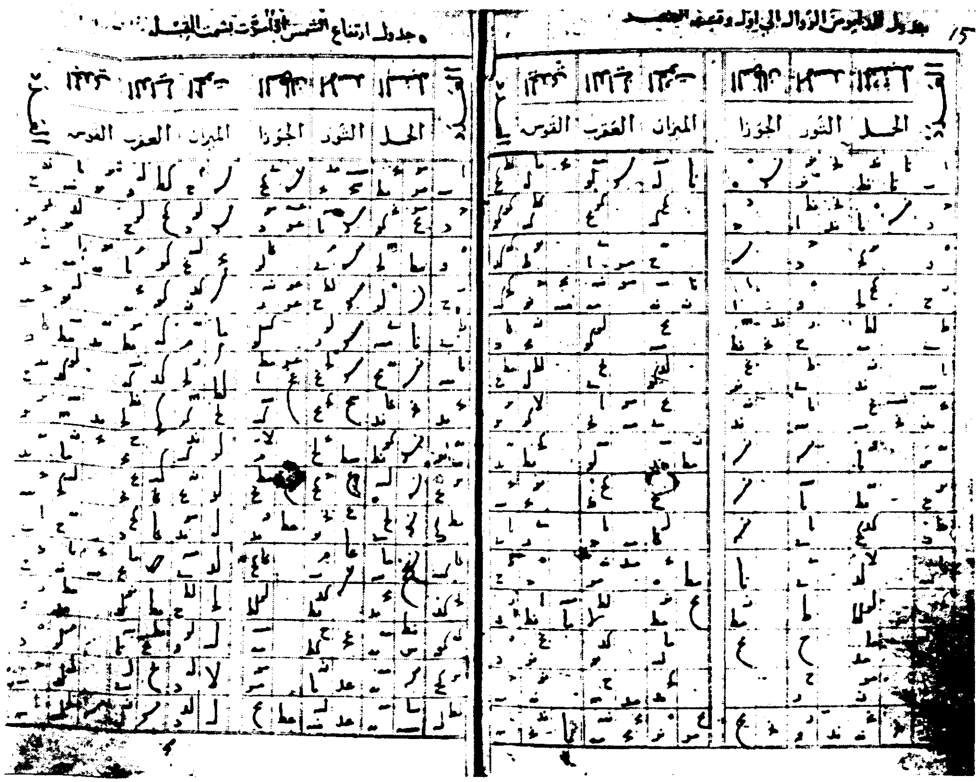

Fig. 5. Extract from the main Cairo corpus of tables for timekeeping, generally attributed to Ibn Yūnus, taken from MS Chester Beatty 3673 (fols. 15v–16r). The right hand page displays the time in equatorial degrees from midday to the beginning of the afternoon prayer, and the left hand page displays the solar altitude when the sun is in the azimuth of Mecca. Entries are given to two sexagesimal digits for each degree of solar longitude. The three right hand and left hand columns on each page serve northern and southern declinations, respectively, taking advantage of the symmetry of the tabulated functions about the solstices. Reproduced with kind permission of the Librarian of the Chester Beatty Library, Dublin.

associated with the principal mosques. The Muslim day begins at sunset, and the interval during which the first prayer is to be performed lasts from sunset to nightfall. The interval for the second prayer begins at nightfall and lasts until daybreak. The third prayer is performed in the interval between daybreak and sunrise. The permitted time for the fourth prayer begins when the sun has crossed the meridian and ends when the interval for the fifth prayer begins, namely, when the shadow of an object equals its meridian shadow increased by the length of the object. The interval for the fifth prayer may last until the shadow increases again by the length of the object or until sunset.

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[24] The standard work on the determination of the times of Muslim prayer is J. Frank and E. Wiedemann, *Die Gebetszeiten im Islam*, "Sitzungsberichte der phys.-med. Sozietät zu Erlangen", 58, 1926, pp. 1–32, reprinted in E. Wiedemann, *Aufsätze zur arabischen Wissenschaftsgeschichte*, Hildesheim, 1970, vol. 2, pp. 757–788. The origins of the definitions of the prayertimes are discussed in A. J. Wensinck's article "*Mīḳāt*" in the *Encyclopaedia of Islam*, first ed., Leiden, 1913–34. See also note 26 below.

The earliest known tables for regulating the times of prayer date from the tenth century. In the corpus of tables for the latitude of Cairo, some of which were computed by Ibn Yūnus, numerous functions such as the length of half daylight and night, the hour-angle at the beginning of the afternoon prayer, the duration of morning and evening twilight and total darkness, are tabulated for each degree of solar longitude[25]. Fig. 5 shows an extract from these tables. Various later sets of prayer-tables for different latitudes are preserved in the manuscript sources[26]. The twilight tables are based on the assumption that daybreak and nightfall occur when the sun is a certain angle of depression below the horizon, and in a few medieval prayer-tables corrections are given for the effect of refraction at the horizon[27].

In the Muslim world nowadays the muezzin's call to prayer is likewise regulated by tables, computed for each day of the year by modern methods and displayed in almanacs, pocket-diaries, and newspapers. The Egyptian Government Survey Department will prepare prayer-tables for any locality where these are requested by a Muslim community.

Tables giving the direction of Mecca

The Muslim is supposed to perform his five daily prayers facing Mecca[28]. The determination of the qibla or the direction of Mecca for a given locality is a non-trivial problem in spherical trigonometry, and the qibla is defined in terms of the geographical coordinates of both localities by a rather complicated formula. Exact solutions of this problem were devised already in the ninth century[29], and by the 13th century the extensive geographical tables found in *zījes* often had the qibla indicated for each city of consequence[30]. In the late 14th century however, al-Khalīlī, who was also the author of a very fine corpus of tables for timekeeping computed for the latitude of Damascus, compiled a set of tables displaying the azimuth of Mecca for each degree of terrestrial latitude and longitude. This table, which contains almost 3,000 entries computed with remarkable accuracy, is the most sophisticated trigonometric table from the entire medieval period.

Fig. 6 shows an extract from al-Khalīlī's qibla table. Several other Muslim astronomers compiled qibla tables based on a simple approximate formula[31].

Tables of auxiliary functions for computations in spherical astronomy

In the mid 9th century the Damascus astronomer Ḥabash al-Ḥāsib compiled a small set of tables of mathematical functions of such a nature that ordered applications of them would lead to the solution of the standard problems of spherical astronomy[32]. A similar set was compiled about the year 1000 by Abū Naṣr[33]. These tables are in fact rather limited in their application, and several centuries were to pass before the Syrian

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[25] These tables are analyzed in detail in D. A. King, *Ibn Yūnus' "Very Useful Tables" for Reckoning Time by the Sun*, "Archive for History of Exact Sciences", 10, 1973, pp. 342–394. See also note 26.

[26] I have prepared a survey of several dozen medieval Islamic tables for regulating the times of prayer, which is currently being submitted for publication. This survey also contains more information on the main Cairo corpus (see note 25).

[27] See, for example, D. A. King, *op. cit.* (note 25), pp. 373–376, and my study of Islamic prayer-tables (note 26).

[28] See further the article "*Ḳibla*" by A. J. Wensinck and C. Schoy in the *Encyclopaedia of Islam*, first ed., Leiden, 1913–34.

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[29] On the methods of Ḥabash (ca. 850) and al-Nayrīzī (ca. 900), for example, see Y. Id and E. S. Kennedy, *A Letter of al-Bīrūnī: Ḥabash al-Ḥāsib's Analemma for the Qibla*, "Historia Mathematica", I, 1974, pp. 3–11, and C. Schoy, *Abhandlung von al-Faḍl b. Ḥātim al-Nayrīzī über die Richtung der Qibla*, "Sitzungsberichte der math.-phys. Klasse der Bayerischen Akademie der Wissenschaften zu München", 1922, pp. 55–68. On the methods of Ibn Yūnus (ca. 990) and Ibn al-Haytham (ca. 1020) see C. Schoy, *Gnomonik der Araber*, in E. von Bassermann-Jordan, ed., *Die Geschichte der Zeitmessunq und der Uhren*, Band IF, Berlin–Leipzig, 1923, pp. 36–42, and *Abhandlung des al-Ḥasan ibn al-Ḥasan ibn al-Haiṭam (Alhazen) über die Bestimmung der Richtung der Qibla*, "Zeitschrift der Deutschen Morgenländischen Gesellschaft", 75, 1921, pp. 242–253, and also D. A. King, *op. cit.* (note 18), pp. 256–268 and note 31 below.

[30] For example, Schoy's article on al-Nayrīzī cited in note 29 contains (pp. 67–68) a list by the Damascus astronomer Ibn Zurayq, a successor of Ibn al-Shāṭir. The geographical coordinates in numerous *zījes* and related works have been data processed by F. Haddad and E. S. Kennedy at the American University of Beirut.

[31] A detailed analysis of al-Khalīlī's table and information on various approximate tables is contained in D. A. King, *al-Khalīlī's Qibla Table*, "Journal of Near Eastern Studies", 1975.

[32] See R. A. K. Irani, *The "Jadwal al-Taqwīm" of Ḥabash al-Ḥāsib*, Master's dissertation, American University of Beirut, 1956, for a detailed discussion of these.

[33] See further C. Jensen, *Abū Naṣr Manṣūr's Approach to Spherical Astronomy as Developed in His Treatise "The Table of Minutes"*, "Centaurus", 16, 1971, pp. 1–19.

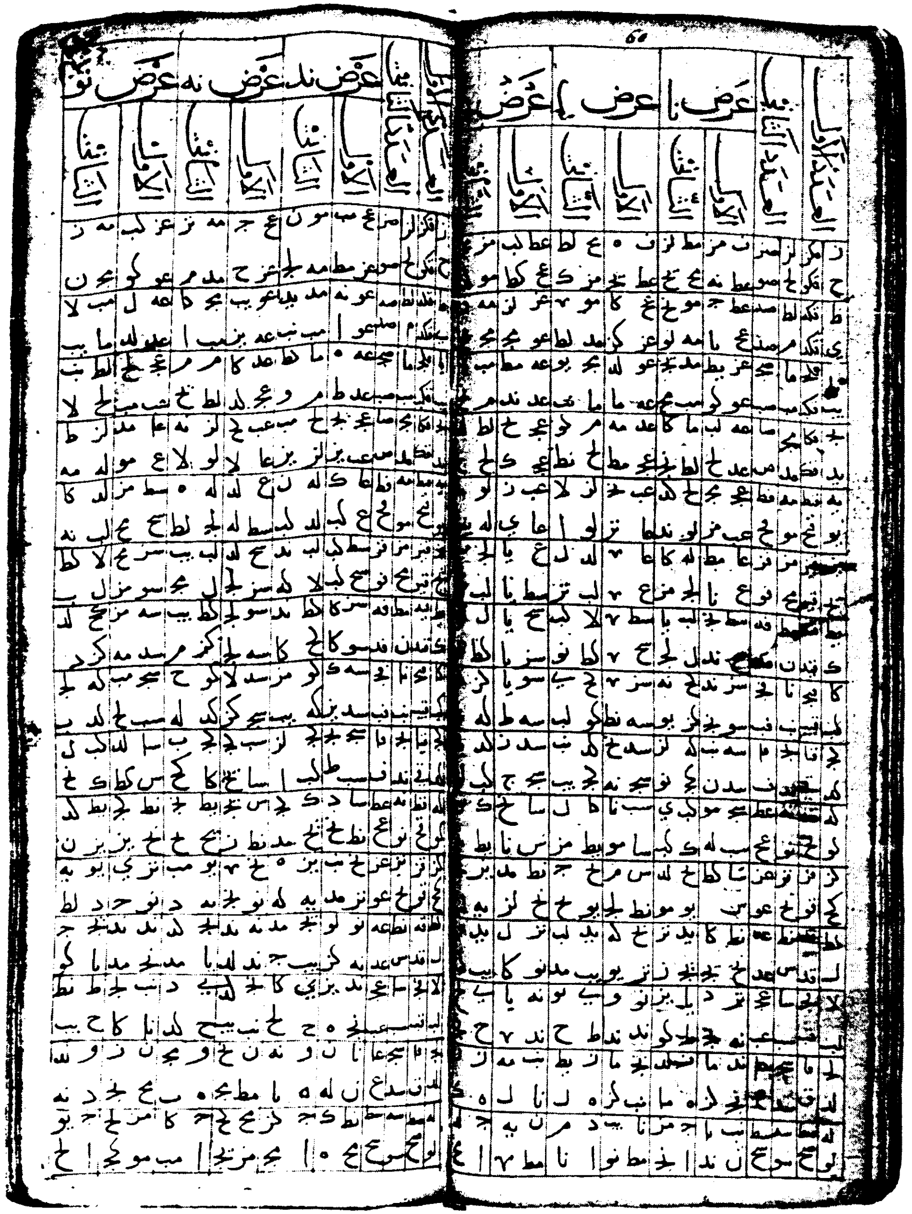

Fig. 6. Extract from al-Khalīlī's qibla table, taken from MS Paris Bibliothèque Nationale, ar. 2558 (fols. 60v – 61r). This particular section of the table serves latitudes 51°, 52°, ..., 56°, and displays the qibla as an angle to the meridian expressed in degrees and minutes for longitudes 1°, 2°, ..., 60° east or west of Mecca. The underlying formula is

$$q = \operatorname{arc\,cot}\left\{\frac{\sin\varphi\cos\Delta L - \cos\varphi\tan\varphi_M}{\sin\Delta L}\right\},$$

where q is the qibla, φ is the local latitude, φ_M is the latitude of Mecca (taken by al-Khalīlī as 21;30°), and ΔL is the difference in longitude from the meridian of Mecca. Most of the entries in al-Khalīlī's table, which number almost 3,000, are accurate or in error by ±1 or ±2 in the second sexagesimal digit. Reproduced with kind permission of the Director of the Section Orientale de la Bibliothèque Nationale, Paris.

astronomer al-Khalīlī compiled his *Universal Table* containing over 13,000 entries. al-Khalīlī tabulated three functions with the aid of which one can solve all of the standard problems of spherical astronomy for any latitude, with no computation beyond interpolation[34]. This fine set of tables marks the zenith of the Islamic achievement in spherical astronomy.

Tables for marking sundials

Islamic sundials have been generally neglected by historians of science, but the valuable researches of C. Schoy have laid the foundations for further study[35]. Recently L. Janin published the first description of the magnificent sundial of the 14th century Syrian astronomer Ibn al-Shāṭir which adorned the north minaret of the Umayyad Mosque in Damascus[36]: the complex markings on this superb instrument were probably drawn using tables especially compiled for the purpose.

Numerous Muslim astronomers prepared tables for marking the curves on sundials oriented in different planes. One set computed in the late 13th century by the Egyptian astronomer al-Maqsī gives pairs of orthogonal coordinates of the points on the solistitial shadow traces indicating the seasonal hours and the afternoon prayer, for sundials oriented in particular planes[37]. For several latitudes al-Maqsī tabulates these coordinates for horizontal sundials, and, specifically for Cairo, he tabulates them for vertical sundials erected at an angle to the meridian, giving a table for each degree of inclination. Fig. 7 shows an extract from

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[34] See further D. A. King, *al-Khalīlī's Auxiliary Tables for Solving Problems of Spherical Astronomy*, "Journal for the History of Astronomy", 4, 1973, pp. 99 – 110. I have analyzed several other sets of Islamic auxiliary tables in my survey of Islamic tables for timekeeping (see note 21).

[35] See C. Schoy, *Gnomonik der Araber* (cited in note 29 above) and *Sonnenuhren der spätarabischen Astronomie*, "Isis", 6, 1924, pp. 332 – 360.

[36] See further L. Janin, *Le Cadran Solaire de la Mosquée Umayyade de Damas*, "Centaurus", 16, 1971, pp. 285 – 298.

[37] On al-Maqsī see H. Suter, *op. cit.* (note 8), no. 383.

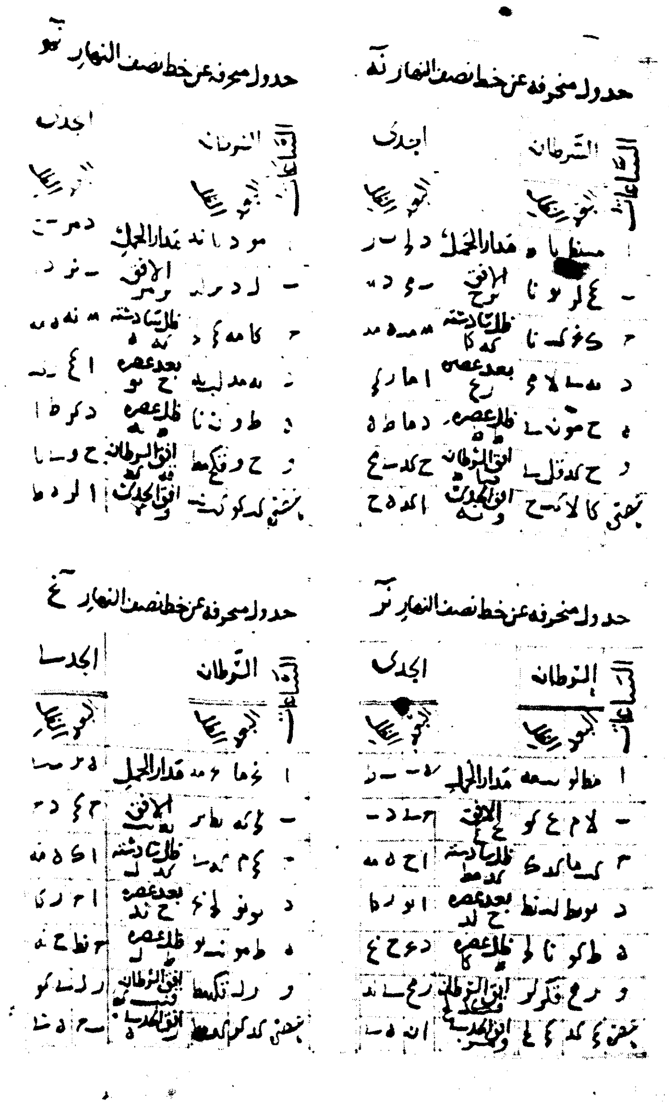

Fig. 7. Extract from the sundial tables of al-Maqsī, taken from MS Chester Beatty 4092 (fol. 148r). The four subtables mainly display the coordinates of the points on the summer and winter solsticial shadow traces corresponding to the seasonal hours of daylight and the beginning of the afternoon prayer, for use with vertical sundials erected at angles 55°, 56°, 57°, and 58° to the meridian. Reproduced with kind permission of the Librarian of the Chester Beatty Library, Dublin.

these tables. Al-Maqsi then tabulates the coordinates for sundials erected in planes which are skew to the horizontal and meridian planes. He also deals with sundials erected in the plane of the celestial equator, which were discussed by several other medieval Muslim astronomers. Later sundial tables, such as those of the 15th century astronomers Sibṭ al-Māridīnī and al-Tīzīnī, working in Cairo and Damascus respectively, use equatorial degrees as argument rather than seasonal day hours[38]. The large vertical sundials constructed on the walls of Cairo mosques facing the qibla, which were especially designed to determine the midday and afternoon prayers, were constructed using tables like those of al-Maqsī and Sibṭ al-Māridīnī[39].

Tables for marking astrolabes and quadrants

The astrolabe was used by early Muslim astronomers, and later generally replaced by numerous varieties of quadrants[40]. The transmission of the theory, construction, and use of the astrolabe to the Muslim world is very obscure[41], and we are likewise in the dark concerning the early history of the quadrant. The construction of very pretty astrolabes continued long into the period of decline of Islamic astronomy, but these were not used for any serious scientific purpose[42].

In view of our ignorance of the transmission of the astrolabe to Islam it is of some interest that the early 9th century Baghdad astronomer al-Farghānī compiled a treatise on its theory and construction as well as an extensive set of tables for drawing the circles on astrolabe plates[43]. These tables give the distance of the center of the *almucantar* circles from the center of the astrolabe, and the radius of the circle, for each degree of altitude, and for each degree of terrestrial latitude from 15° to 50°.

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[38] On these astronomers see H. Suter, *op. cit.* (note 8), nos. 445 and 450. Sibṭ al-Māridīnī's tables are discussed in C. Schoy, *Sonnenuhren* ... (see note 35).

[39] A fine example of such a sundial is preserved in the Museum of Islamic Art, Cairo (item no. 10630).

[40] On the astrolabe and quadrant in Islamic astronomy see W. Hartner, *Oriens-Occidens*, Hildesheim, 1968, pp. 287–318, and P. Schmalzl, *Zur Geschichte des Quadranten bei den Arabern*, Munich, 1929, and also L. A. Mayer, *Islamic Astrolabists and Their Works*, Geneva, 1956.

[41] Cf. O. Neugebauer, *The Early History of the Astrolabe*, "Isis", 40, 1949, pp. 240–256.

[42] For some examples dating from about 1700 onwards see O. Gingerich, D. A. King, and G. Saliba, *The 'Abd al-A'imma Astrolabe Forgeries*, "Journal for the History of Astronomy", 3, 1972, pp. 188–198.

[43] On al-Farghānī see note 4 above. His treatise on the astrolabe and associated tables exist in numerous sources and were widely known in the Islamic Middle Ages, but have never been studied in modern times.

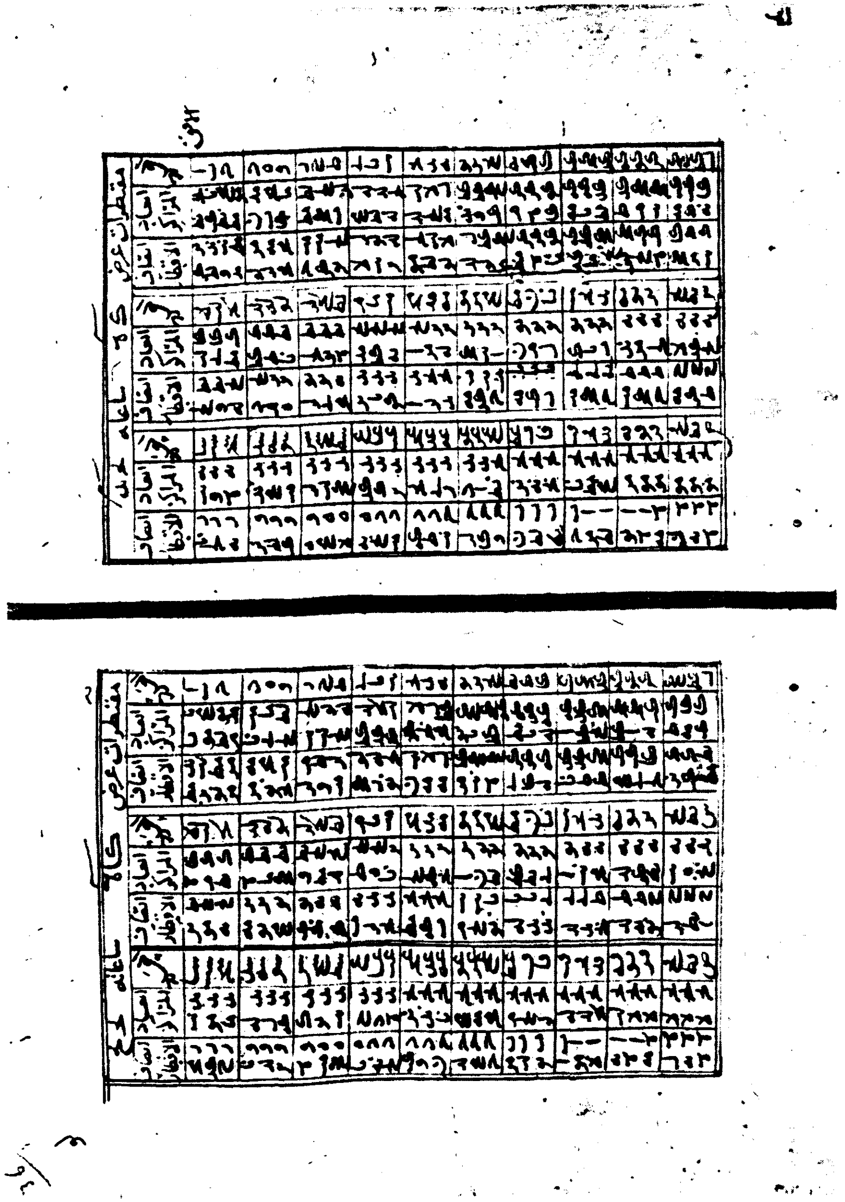

Fig. 8. Extract from al-Farghānī's astrolabe tables, taken from MS Berlin Ahlwardt 5790 (= Landberg 56, fols. 35v – 36r). These two subtables display the position and size of the almucantar circles corresponding to each degree of altitude for astrolabe plates intended to serve latitudes 20° and 21°. Reproduced with kind permission of the Director of the Orientalische Abteilung der Staatsbibliothek, Berlin.

Fig. 8 shows an extract from al-Farghānī's tables. They are easily generated from a single auxiliary function also tabulated by al-Farghānī, and in the 14th century the Egyptian al-Bakhāniqī compiled a set for latitudes 0° to 90°, for a Yemeni ruler who was probably upset that al-Farghānī's main tables could not be used for latitudes in his domain[44]. Numerous sets of tables for marking quadrants have been located in the manuscript sources, and research on these is currently in progress.

Planetary equation tables

In planetary astronomy, the Muslim scholars also compiled extensive tables separately from those in the *zījes*, particularly for the planetary equations[45]. A rather interesting set containing over 30,000 entries is attributed to Ibn Yūnus[46]: it displays the lunar equation as a double-entry function of the double elongation and the mean anomaly, both of which arguments can be taken directly from the mean motion tables. The underlying theory is entirely Ptolemaic, but the parameters are those of Ibn Yūnus. The development of tables of this kind can be traced down to those of the Damascus astronomer al-Ṣāliḥī, who lived about the year 1500: his tables display all of the planetary equations for arguments which can be taken directly from the mean motion tables. The underlying parameters are those of Ulugh Beg, and the tables contain over 170,000 entries[47].

The compilation of extensive tables of the kinds described above, and the development of sophisticated computational techniques, was perhaps the major contribution of the Muslim astronomers to the development of the exact sciences, known to the West only by modern research on manuscripts. Our picture of their achievements in this field, as in others, is still incomplete, and will remain so until more of the vast quantities of available sources are studied for the first time.

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[44] Various works of al-Bakhāniqī are analyzed in my study of medieval Islamic prayer-tables (see note 26).

[45] On Islamic planetary equation tables see, for example, E. S. Kennedy, *op. cit.* (note 10), p. 142; E. S. Kennedy and H. Salam, *op. cit.* (note 12); M. Tichenor, *Late Medieval Two-Argument Tables for Planetary Longitudes*, "Journal of Near Eastern Studies", 26, 1967, pp. 126 – 128; and C. Jensen, *The Lunar Theories of Al-Baghdādī*, "Archive for History of Exact Sciences", 8, 1972, pp. 321 – 328.

[46] See further D. A. King, *A Double-Argument Table for the Lunar Equation Attributed to Ibn Yūnus*, "Centaurus", 18, 1974, pp. 129 – 146.

[47] On al-Ṣāliḥī see H. Suter, *op. cit.* (note 8), no. 454 and D. A. King, *op. cit.* (note 46), Section 1.

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Note added in proof

Since this paper was submitted for publication I have come across four new sets of tables each of which appears to have been compiled in early ninth century Iraq. These illustrate that some of the various categories of tables discussed in this paper can be traced back further than I previously thought. Firstly, an anonymous set of prayertables for Bagdad. Secondly, al-Khwārizmī's table of an auxiliary function underlying the mathematical theory of the astrolabe. Thirdly, al-Khwārizmī's book on sundials, which consists mainly of tables of coordinates for marking horizontal sundials constructed for different latitudes. Fourthly, an anonymous qibla table based on an approximate but non-trivial formula. These tables will be discussed in detail in a series of studies currently in preparation.

The Astronomy of the Mamluks

AFTER THE SO-CALLED GOLDEN AGE of Islamic science, various regional schools of astronomy with very distinctive traditions and interests developed and flourished in each of the main areas of Islamic civilization. In Mamluk Egypt and Syria there was considerable scholarly activity in astronomy; indeed, Cairo in the late thirteenth century was one of the major centers of astronomy in the Islamic world, and Damascus in the mid-fourteenth century was the leading center of astronomy in the Islamic world, and perhaps in the world in general. Mamluk astronomers worked in each of the major branches of astronomy: theoretical and computational planetary astronomy, spherical astronomy and timekeeping, instrumentation, and folk astronomy and astrology. No overview of the activities of these regional scholars exists in the modern literature. The manuscripts and instruments that constitute the main sources for our knowledge of Mamluk astronomy have been investigated only in the past twenty-five years, and, in some cases, only in the past ten years.[1] Thus the time

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* Department of Near Eastern Languages and Literatures, Hagop Kevorkian Center for Near Eastern Studies, New York University, New York, N.Y. 10003.

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[1] For an introduction to the Mamluks see E. Atil, *Renaissance of Islam: Art of the Mamluks* (Washington, D.C.: Smithsonian Institution Press, 1981). The richest collections of Mamluk scientific manuscripts are in Istanbul (Süleymaniye and Topkapi Libraries), Cairo (Egyptian National Library), Damascus (Ẓāhirīya Library), Dublin (Chester Beatty Library), Princeton (Firestone Library), Berlin (Deutsche Staatsbibliothek), and Paris (Bibliothèque Nationale). All are catalogued except the first: see Fuat Sezgin, *Geschichte des arabischen Schrifttums*, 7 vols. (Leiden: Brill, 1967–present), Vol. VI, pp. 311 ff. Basic bibliographical sources include Heinrich Suter, "Die Mathematiker und Astronomen der Araber und ihre Werke," *Abhandlungen zur Geschichte der mathematischen Wissenschaften*, 1900, *10*, and "Nachträge und Berechtigungen," *ibid.*, 1902, *14*:157–185 (rpt. Amsterdam: Oriental Press, 1982); C. Brockelmann, *Geschichte der arabischen Litteratur*, 2nd ed., 2 vols. (Leiden: Brill, 1943–1949), and *Supplementbände*, 3 vols. (Leiden: Brill, 1937–1942); and A. El-Azzawi, *History of Astronomy in Iraq* . . . (in Arabic) (Baghdad: Iraq Academy Press, 1959) (to be used with caution). See also David A. King, A Catalogue of the Scientific Manuscripts in the Egyptian National Library, 2 vols. (Cairo: General Egyptian Book Organization, 1981–1984); King, *A Survey of the Scientific Manuscripts in the Egyptian National Library* (Publication of the American Research Center in Egypt) (Malibu, Calif.: Undena Press, in press), and for astrology, Manfred Ullmann, *Die Natur- und Geheimwissenschaften im Islam* (Leiden: Brill, 1972).

The richest collections of Mamluk instruments and Ottoman instruments based on Mamluk design are in Athens (Benaki Museum), Cairo (Egyptian National Library and Museum of Islamic Art), Dublin (Chester Beatty Library), Istanbul (Kandilli Observatory), and Oxford (Museum of the History of Science). Various Ayyubid and Mamluk astronomical instruments are listed in R. T. Gunther, *The Astrolabes of the World*, 2 vols. (Oxford: Univ. Press, 1932; London: Holland Press, 1976);

is ripe for a survey of Mamluk contributions in this field, and that is the purpose of this paper.

I. THE FATIMID AND AYYUBID BACKGROUND

When the Mamluks came to power in Egypt and Syria around 1250, some 250 years had passed since the time of Ibn Yūnus, the greatest astronomer of Egyptian history. This scholar worked for the Fatimid Caliphs al-ᶜAzīz and al-Ḥakīm, and he is the only astronomer from this period in Egypt to have much enduring influence. His major work was a *zīj*, or astronomical handbook with tables, called the *Ḥākimī Zīj*, a compilation of considerable distinction. Most of this *zīj* survives and has been studied.[2] It was used in Cairo in the late Fatimid and Ayyubid periods; for instance, the astronomers in the twelfth-century al-Afḍal–al-Baṭāᵓiḥī Observatory in Cairo used it to compile annual ephemerides, or tables displaying day-by-day positions of the sun, moon, and planets, but never replaced it with a new *zīj*, even though this had been their expressed intention.[3]

In Syria, the distinguished astronomers Ḥabash al-Ḥāsib and al-Battānī worked in ninth-century Damascus and tenth-century Raqqa, respectively. However, their works were apparently not influential in later Syria, for the astronomers at al-Afḍal–al-Baṭāᵓiḥī asserted that their colleagues in Syria used the ninth-century Iraqi *Mumtaḥan Zīj* to compile their ephemerides. A new Syrian *zīj*, compiled around 1170 by Ibn al-Dahhān, is no longer extant.[4]

In spherical astronomy no works of consequence are known of in Egypt or Syria between the time of Ibn Yūnus and the advent of the Mamluks. Ibn Yūnus prepared a substantial number of tables for timekeeping by the sun and for regulating the astronomically defined times of prayer, all computed for the latitude of Cairo. His tables were available to the early Mamluk astronomers in Cairo and formed the basis of a corpus of tables for timekeeping used there from the thirteenth to the nineteenth centuries.[5] No such tables are known for Damascus or Jerusalem from the pre-Mamluk period.

The celebrated scholar Ibn al-Haytham, a late contemporary of Ibn Yūnus in Cairo, was the first Muslim astronomer known to have written extensively on the problems of Ptolemy's planetary models.[6] Thereafter al-Juzajānī, a pupil of

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see also L. A. Mayer, *Islamic Astrolabists and Their Works* (Geneva: Ernst Kundig, 1956), and the supplement in R. Ettinghausen, ed., *Aus der Welt der islamischen Kunst* (Berlin: Mann, 1959), pp. 293–296; Alain Brieux and F. Maddison, *Répertoire des facteurs d'astrolabes et de leurs oeuvres*, Vol. I: *Islam* (forthcoming); and Emilie Savage-Smith, *Survey of Islamic Celestial Globes* (Washington, D.C.: Smithsonian Institution Press, forthcoming).

[2] See s.v. "Ibn Yūnus" in *Dictionary of Scientific Biography (DSB)*, 15 vols. (New York: Scribners, 1970–1981). The standard work on *zījes* is E. S. Kennedy, "A Survey of Islamic Astronomical Tables," *Transactions of the American Philosophical Society*, N.S., 1956, *46*(2):123–177. On tables not in *zījes* see David A. King, "On the Astronomical Tables of the Islamic Middle Ages," *Studia Copernicana*, 1975, *13*:37–56.

[3] See A. Sayili, *The Observatory in Islam* . . . (Ankara: Türk Tarih Kurumu Basimevi, 1960), pp. 167–175.

[4] See s.vv. "Ḥabash" and "al-Battānī" in *DSB* and *The Encyclopaedia of Islam*, 2nd ed., 4 vols. (Leiden: Brill, 1960–). On the *zījes* see Kennedy, *Survey*, 51 (*Mumtaḥan*) and 89 (Ibn al-Dahhān).

[5] See David A. King, "Ibn Yūnus' *Very Useful Tables* for Reckoning Time by the Sun," *Archives for History of Exact Sciences*, 1973, *10*:342–394.

[6] See s.v. "Ibn al-Haytham" in *DSB*.

Ibn Sīnā, also considered these problems, but not until the mid-thirteenth and fourteenth centuries did Muslim astronomers really come to grips with them.

Mention should also be made here of the astronomical activity, albeit on a much lower level, of the early thirteenth-century Coptic scholar al-Asᶜad Ibn al-ᶜAssāl. A manuscript of a treatise on calendar conversion and computing solar and lunar positions attributed to him exists in the Egyptian National Library. The text is written in Arabic, and the tables are written in Coptic numerals; neither has been investigated yet.[7]

In the field of instrumentation some unidentified Egyptian astronomers in the eleventh or twelfth century invented the almucantar quadrant, a simplified version of the astrolabe serving a specific latitude. This was an invention of some consequence, for the astrolabe, fitted with a series of plates for different latitudes, was neither a practical device nor an accurate observational instrument. Also, being made of brass, it was expensive. The almucantar quadrant, on the other hand, could be made of wood and was an extremely practical device with which one could solve all the problems solvable with an astrolabe, for a particular latitude. The back of such a quandrant could carry a trigonometric grid called a sine quadrant for solving all manner of computational problems. The Mamluk astronomers later developed the quadrant to all conceivable limits; it virtually replaced the astrolabe in Syria and Egypt in Mamluk and Ottoman times.[8] Another development of far-reaching consequence was the introduction to Syria in the twelfth century of the universal lamina (plate) of the eleventh-century Andalusian astronomer al-Zarqāllu. This plate constituted a universal device representing a stereographic projection for the terrestrial equator and could be used to solve all the problems of spherical astronomy for any latitude. It was derived from the universal astrolabe devised by al-Zarqāllu's contemporary, ᶜAlī ibn Khalaf al-Shakkāz, which consisted essentially of two such plates that could be made to rotate over each other to correspond to any two sets of terrestrial or celestial coordinate systems. Al-Zarqāllu's plate was known in both Damascus and Cairo by the late thirteenth century, but al-Shakkāz's universal astrolabe was not.[9]

The history of astronomy in both Egypt and Syria for the two centuries before the Mamluks came to power thus remains somewhat obscure. Damascus at least seems to have been an active center of astronomy at that time. The thirteenth-century biographical dictionaries of Ibn al-Qifṭī and Ibn Abī Uṣaybiᶜa list several recent and contemporary scholars concerned with the sciences, mainly in Damascus, although they were involved more in teaching than in writing treatises. The revival of astronomy in Cairo in the early Mamluk period around 1250–1260 occurred within a few years of the deaths of a group of prominent individuals known to have been very active in astronomy in either Syria or Egypt: Athīr al-Dīn al-Abharī, ᶜAlam al-Dīn Qayṣar, Ibn al-Lubūdī, and Baylak al-Qipjāqī. Their works either have not survived or have not yet been properly studied. At least the first three had an academic connection with Iraq. Athīr al-Dīn wrote a compendium of astronomy in Iraq that was popular in Mamluk Egypt; he also compiled the *Athīrī Zīj* in Mardin about 1250, although it appears to have had no influence in Syria thereafter. Further research is necessary to establish the influence of these individuals in later Mamluk astronomy and Islamic astronomy in general.[10] The mid-thirteenth century astronomers Muᵓayyad al-Dīn al-ᶜUrḍī (1) and Muḥiy al-Dīn al-Maghrībī (2), who were later with the observatory at Maragha in Iran, were also active at this time; their work is discussed in the appropriate sections below.

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[7] See King, *Survey*, C10.

[8] See the now outdated P. Schmalzl, *Zur Geschichte des Quadranten bei den Arabern* (Munich: Salesianische Offizin, 1929).

[9] See David A. King, "On the Early History of the Universal Astrolabe . . . ," *Journal for the History of Arabic Science*, 1979, *3*:244–257.

II. WHO WERE THE MAMLUK ASTRONOMERS?

Of the Mamluk astronomers whose names have been preserved and for whom we have biographical information, most were associated with religious institutions.[11] One office associated with the Mamluks is that of *muwaqqit* (less frequently, *miqātī*). The appellation is not attested before the time of the Mamluks, and the early development of the institution is obscure. The *muwaqqit*s were astronomers associated with the major mosques and madrasas whose particular concern was astronomical timekeeping and the regulation of the times of prayer.[12] The office demanded more scientific knowledge than that of muezzin, although there are indications that in the early centuries of Islam it was the muezzins who regulated the times of prayers even in major mosques, using the techniques of folk astronomy such as observing shadow lengths by day and the risings of the lunar mansions by night. Ibn al-Ukhuwwa, in his manual of trades written about 1300, observed that muezzins were chosen for their piety and their voices rather than their competence in astronomy, but it is curious that he does not mention *muwaqqit*s at all. References are preserved in the astronomical manuscript sources to a family of *muwaqqit*s who worked at the Mosque of ᶜAmr in Fustat in the thirteenth century, and thereafter only individual *muwaqqit*s associated with specific mosques in Cairo are known from the fifteenth century onwards. For example, al-Kawm al-Rīshī (41) and al-Wafāᵓī (55) worked as *muwaqqit*s at the Muᵓayyad Mosque in the early and mid-fifteenth century, and Sibṭ al-Māridīnī (63) at the Azhar Mosque in the latter part of the same century. We know of a series of astronomers who served as *muwaqqit*s in the Azhar Mosque until the nineteenth century, but the leading astronomers in Cairo in Ottoman times were not always associated with the Azhar Mosque. In the sixteenth century al-Minūfī, a *muwaqqit* at the Ghawrī madrasa, and in the seventeenth century ᶜAbd al-Raḥmān al-Ishkīrī, a *muwaqqit* at the Mosque of Ibn Ṭūluñ, were among the leading astronomers in Cairo.

The major school of astronomers in Damascus in the fourteenth century were *muwaqqit*s associated with the Umayyad Mosque, and various other astronomers served there in the same capacity until the nineteenth century. The most celebrated of the fourteenth-century *muwaqqit*s were Ibn al-Shāṭir (18), al-Khalīlī (19), and al-Mizzī (17). Unfortunately, we have very little information on the organization of this school. Damascus *muwaqqit*s who did serious work in the fifteenth century were al-Ḥalabī (45), al-Tīzīnī (69), and al-Ṣāliḥī (70).

Not all of the major astronomers of Mamluk Egypt and Syria were *muwaqqit*s. The Syrian astronomer al-Ḥamzāwī (67) was instead the *amīr al-ḥajj* in Aleppo, responsible for the annual pilgrimage caravans to Mecca; he wrote an astrological history of the Mamluk campaigns against the Ottomans around Adana. For many important figures, moreover, even those working in timekeeping and spherical astronomy (to some extent the domain of the *muwaqqit*s), we have no information about affiliation. The most influential of the early Mamluk astronomers, al-Marrākushī (5), an individual of Morrocan origin who worked in Cairo about 1280, is referred to in the manuscripts of his work simply as *al-shaykh al-imām*. The medieval biographical sources, as far as I am aware, do not mention him at all, nor his contemporaries al-Maqsī (6) and Najm al-Dīn al-Miṣrī (7). Of the major astronomers specializing in spherical astronomy, Ibn al-Sarrāj (15) (fourteenth-century Aleppo) and Ibn al-Majdī (44) and Ibn Abi'l-Fatḥ al-Ṣūfī (64) (fifteenth-century Cairo) are not known to have been *muwaqqit*s or to have been associated with any particular religious institution.

Finally, we do hear of astronomers sponsored or financed by the Mamluk sultans, but the only name recorded for us is that of the astrologer Ibrāhīm al-Ḥāsib (27), who worked for the Sultan al-Nāṣir Aḥmad in the mid-fourteenth century. The Egyptian historian Ibn Abi'l-Faḍāᵓil records that when the Sultan al-Nāṣir Muḥammad ibn Qalāᵓūn became ill with diarrhea, astrologers and geomancers were consulted as well as his doctors.[13] In contrast, princes at courts outside the Mamluk domains patronized and even practiced astronomy. Abu'l-Fidāᵓ (8), Ayyubid prince of Hama shortly before it fell to the Mamluks about 1300, is known to have himself compiled a *zīj*, now lost. Activity at his court, however, was without influence on Mamluk astronomy. One of his astronomers (10) compiled a commentary on the *Tadhkira* of al-Ṭūsī that Ibn al-Shāṭir does not even mention; others (9 and 11) compiled treatises on instruments of no great originality or import. Similarly, the Rasulīd Sultans of the Yemen, which had close links with the Mamluk domains of Egypt, Syria, and the Hijaz, patronized serious astronomical activity and several pursued astronomy themselves.[14] One Egyptian astronomer, al-Bakhāniqī (16), spent some time in the Yemen, working for the Sultan al-Mujāhid around 1325.

III. MAMLUK ACTIVITY IN COMPUTATIONAL ASTRONOMY

The most popular *zīj* in Mamluk Egypt until the fifteenth century was the *al-Zīj al-Muṣṭalaḥ*, compiled in the mid-thirteenth century by some Egyptian astronomers (3) as yet unidentified. The early fourteenth-century scholar Ibn al-Akfānī tells us that it was the *zīj* in use in Egypt in his day, and it was still in use there in the fifteenth century.[15] Both extant manuscripts attribute the *zīj* to Ibn

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[10] On Athīr al-Dīn al-Abharī, ᶜAlam al-Dīn Qayṣar, and Ibn al-Lubūdī see in Suter, "Mathematiker und Astronomen," nos. 364, 358, 365; on Baylak al-Qipjāqī see in *DSB*. See also Kennedy, *Survey*, 40 (*Athīrī Zīj*).

[11] The appendix lists and numbers all Mamluk astronomers of any consequence known to me, with references to the standard modern bio-bibliographical sources. The number is given periodically in boldface after each astronomer's name in the text.

[12] See David A. King, "On the Role of the *Muwaqqit* in Medieval Islamic Society," in *Proceedings of the Second International Symposium on the History of Arabic Science (Aleppo, 1979)* (forthcoming).

[13] See Manfred Ullmann, *Islamic Medicine* (Islamic Surveys, 2) (Edinburgh: Edinburgh Univ. Press, 1978), p. 112.

[14] See David A. King, *Mathematical Astronomy in Medieval Yemen* (Publications of the American Research Center in Egypt) (Malibu, Calif.: Undena Press, 1983).

[15] See Eilhard Wiedemann, *Aufsätze zur arabischen Wissenschaftsgeschichte*, 2 vols. (Hildesheim/

Yūnus. It is a highly eclectic work, based on the various *zīj*es of Ibn Yūnus, an early *zīj* by one of the thirteenth-century Maragha astronomers, and some ninth- and tenth-century Iraqi sources; as such it is, of course, of considerable historical interest.

In Syria also several *zīj*es were compiled about the mid-thirteenth century. The *Athīrī Zīj* (Mardin, ca. 1250) was mentioned above, and al-ᶜUrḍī may also have prepared a *zīj* around 1250. The *zīj* of al-Maghribī **(2)**, compiled in Damascus about this time, survives in a unique manuscript yet to be studied properly. The only Syrian *zīj* to gain popularity equivalent to that of the *Muṣṭalaḥ zīj* in Egypt, however, was Ibn al-Shāṭir's *al-Zīj al-Jadīd*, compiled in Damascus in the mid-fourteenth century. It contains solar, lunar, and planetary tables based on his new models (see Section IV) and was popular in Syria for several centuries, both in its original form and in several different recensions by later astronomers. That popularity was due not to the new models but rather to its status as the only *zīj* produced by the school of the Umayyad Mosque. It nonetheless had to compete with various Syrian recensions of other *zīj*es: one of the thirteenth-century Maragha production, the *Īlkhānī Zīj*, compiled around 1425 by Shihāb al-Dīn al-Ḥalabī **(45)**; and one of the fifteenth-century *Zīj* of Ulugh Beg of Samarqand, compiled around 1500 by al-Ṣāliḥī **(70)**.[16]

The *Zīj* of Ibn al-Shāṭir was also influential in Egypt: the *muwaqqit* al-Kawm al-Rīshī made a recension for Cairo, entitled *al-Lumᶜa*, around 1400. By this time the *Muṣṭalaḥ Zīj* was considered outdated, but the *Lumᶜa* soon had to compete with an Egyptian recension of the *Zīj* of Ulugh Beg, compiled in the late fifteenth century by the Cairo astronomer Ibn Abi'l-Fatḥ al-Ṣūfī **(64)**. The *Lumᶜa* of al-Rīshī and the *Zīj* of al-Ṣūfī were used in Cairo for several centuries. An anonymous recension of Ibn al-Shāṭir's *Zīj* was prepared for Algiers some time during the Ottoman period; otherwise, its use seems to have been restricted to Syria and Egypt.

Complementing the various *zīj*es and their recensions, certain Mamluk astronomers compiled extensive planetary equation tables to facilitate the determination of planetary positions for annual ephemerides. Solar, lunar, and planetary equations are the corrections applied to the mean positions of the planets (which are linearly related to time) to yield the actual positions. The equations for the moon and planets were generally computed from a series of auxiliary tables (adopted from Ptolemaic astronomy) using two arguments taken from the mean-motion tables. It would obviously be advantageous to have equation tables into which one could simply feed the two appropriate arguments, but such tables would inevitably be rather large. Two Mamluk manuscripts exist of the double-argument lunar equation tables attributed to Ibn Yūnus. By the late fifteenth century al-Ṣāliḥī **(70)** in Damascus had compiled extensive double-argument tables for the planetary equations in his adaptation of the *Zīj* of Ulugh Beg. Such tables, called *ḥabṭaq* and *maḥlūlāt* ("solved") in Ottoman times, were widely used in Egypt, Syria, Turkey, Iran, and India from the sixteenth century onwards.[17] (The origin of the curious term *ḥabṭaq* may lie in the terms *ᵓbqṭy* and *ᵓbṭqy* used in solar and lunar tables of the early thirteenth-century Coptic scholar al-Asᶜad Ibn al-ᶜAssāl.

Another interesting development in the computation of ephemerides took place in Mamluk Cairo. The problem of preparing annual ephemerides was facilitated by compiling a universal auxiliary table for each planet based on its periodicity, which reduced the computation essentially to determining where to plug into the auxiliary table. There is evidence that the underlying principle goes back at least to twelfth-century Iran, but Ibn al-Majdī **(44)** revived the idea in Cairo in the early fifteenth century.[18] His tables were used in Cairo for several centuries both in their original form and in various later adaptations. No Mamluk ephemerides survive, but various Ottoman examples for Cairo that do survive were clearly computed using Ibn al-Majdī's auxiliary tables.

Two complete fourteenth-century Yemeni ephemerides have recently been discovered in Cairo, and the Mamluk ephemerides were probably similar. For each day of a specific Hijra year the dates in other calendars are given, and the ecliptic longitudes of the sun, moon, and five naked-eye planets, computed by means of a *zīj* (in this case, Ibn Yūnus's *Ḥākimī Zīj*, adapted for the longitude of the Yemen), are tabulated side by side. The Yemeni ephemerides display the astrological prognostications that can be made from the relative positions of the celestial bodies on each day of the year. Some twelfth-century fragments of almanacs from the Cairo Geniza display only this astrological information.[19] I suspect that the later Mamluk ephemerides might not have included any astrological information. A treatise on ephemerides compilation by the Cairo *muwaqqit* al-Kawm al-Rīshī **(41)** deals mainly with the strictly astronomical information recorded in ephemerides.

IV. MAMLUK ACTIVITY IN THEORETICAL ASTRONOMY

When al-ᶜUrḍī **(1)** left Damascus for the Maragha observatory, he worked there on the problems of Ptolemy's planetary models with other, better-known astronomers involved in the same problems, most notably, Naṣīr al-Dīn al-Ṭūsī. Their later contemporary Quṭb al-Dīn al-Shīrāzī also wrote on planetary models and worked in Maragha. He later traveled extensively in the service of the Īlkhānids, on one occasion going to Cairo as their ambassador; if any Egyptian astronomers pursued the planetary problem, however, we do not know of them. The Maragha astronomers had not felt obliged to draw up new planetary (equation) tables based on their new models, and their models had but little influence on the Samarqand school of Ulugh Beg in the fifteenth century.[20]

New York: Olms, 1970), Vol. I, pp. 265–266. For the *Muṣṭalaḥ Zīj* see Kennedy, *Survey*, 47; King, *Survey*, C12.

[16] See Kennedy, *Survey*, 41 (al-Maghribī), 42 (al-ᶜUrḍī), 11 (Ibn al-Shāṭir), 6 (*Īlkhānī Zīj*), and 2 (Ulugh Beg).

[17] On *ḥabṭaq* see David A. King, "A Double-Argument Table for the Lunar Equation Attributed to Ibn Yūnus," *Centaurus*, 1974, *18*:129–146; and George Saliba, "Computational Techniques in a Set of Late Medieval Astronomical Tables," *J. Hist. Arabic Sci.*, 1977, *1*:24–32.

[18] See E. S. Kennedy and D. A. King, "Ibn al-Majdī's Tables for Calculating Ephemerides," *J. Hist. Arabic Sci.*, 1980, *4*:48–68.

[19] See King, *Yemen*, Pt. II, Sects. 11, 20 on the ephemerides; and Bernard R. Goldstein and David Pingree, "Astrological Almanacs from the Cairo Geniza," *Journal of Near Eastern Studies*, 1979, *38*:153–175 (Pt. I); 231–255 (Pt. II).

[20] On the Syrian work see George Saliba, "The First Non-Ptolemaic Astronomy at the Maragha School," *Isis*, 1979, *70*:571–576; Saliba, "A Damascene Astronomer Proposes a Non-Ptolemaic Astronomy" (in Arabic), *J. Hist. Arabic Sci.*, 1980, *4*:3–17; and Saliba, "Islamic Reaction to Greek

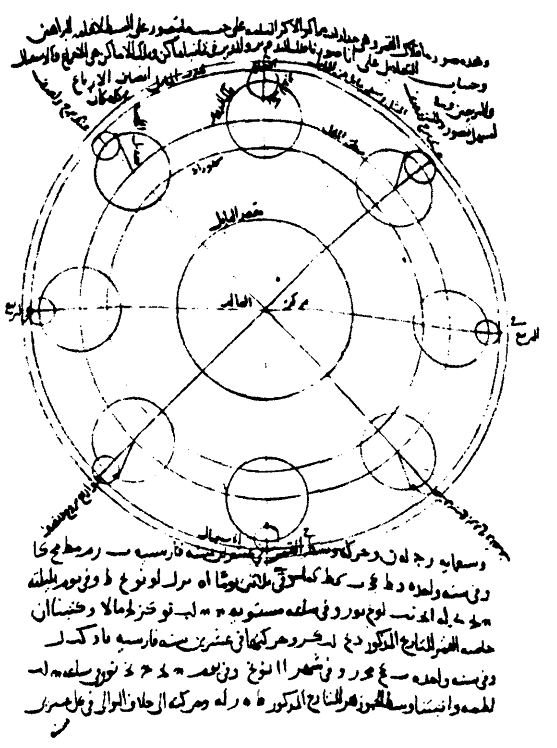

Figure 1. Ibn al-Shāṭir's new model for the moon, which represents the lunar motion in longitude as well as the varying distance of the moon from the earth. Copernicus later used the same model. Reproduced from Oxford Bodleian Marsh 139, folio 16v, with permission of the Bodleian Library.

It was a Mamluk astronomer, Ibn al-Shāṭir of Damascus (18), who played the most significant role in the development of theoretical models to account for the motions of the sun, moon, and planets and overcome the problems associated with the planetary models of Ptolemy. Ibn al-Shāṭir's first major work was a strictly Ptolemaic *zīj* for Damascus, which has unfortunately not survived. Thereafter he compiled a treatise entitled *Nihāyat al-su$^{\supset}$l fī taṣḥīḥ al-uṣūl* in which he presented the reasoning behind his new planetary models. His second *zīj*, appropriately entitled *al-Zīj al-jadīd*, the "new" *Zīj*, incorporates these models; Ibn al-Shāṭir thus has the distinction of being the first to compile planetary tables consistent with models he had devised. Building on the earlier writings of such individuals as al-^{c}Urḍī, al-Shīrāzī, and al-Ṭūsī, Ibn al-Shāṭir devised a new lunar model, which represented with a substantial measure of success both the longitude of the moon and its distance from the earth (see Fig. 1), and a series of planetary models with secondary epicycles replacing the Ptolemaic equant. This notwithstanding, his models are not known to have had any influence in Islamic astronomy after his time.[21]

Astronomy," *Proceedings of the Boston Colloquium for the Philosophy of Science, 1978* (forthcoming). On al-Ṭūsī see F. J. Ragep, "Cosmography in the 'Tadhkira' of Naṣīr al-dīn al-Ṭūsī" (Ph.D. diss., Harvard University, 1982).

[21] On Ibn al-Shāṭir's planetary models see E. S. Kennedy and I. Ghanem, eds., *The Life and Work*

When E. S. Kennedy first investigated Ibn al-Shāṭir's models some twenty-five years ago, he observed that they were mathematically equivalent to those Copernicus elaborated some 150 years after the time of Ibn al-Shāṭir. This discovery aroused considerable interest in Islamic planetary theory. A direct influence of Ibn al-Shāṭir's models on Copernicus has yet to be established, but it remains a distinct possibility. George Saliba has established that there was fairly continuous activity relating to non-Ptolemaic astronomical models in the Muslim world from the eleventh to the eighteenth century, and his current research promises to fill in many of the gaps in our present knowledge of these developments.

With the exception of the treatise of Ibn al-Shāṭir, all Mamluk treatises on theoretical astronomy or earlier treatises popular in Mamluk times were strictly Ptolemaic. A commentary was available on the treatise on astronomy of the Andalusian Jābir ibn Aflaḥ, and in Mamluk Cairo al-Juzajānī (14) compiled a commentary on the treatise by the twelfth-century Transoxanian astronomer al-Kharaqī. Tāj al-Dīn al-Tabrīzī (24), a teacher at the Turunṭāy madrasa in Cairo, wrote a short treatise dealing with arithmetic, geometry, astronomy, and astrology, extant in a unique manuscript in Cairo, but this does not appear to have been widely used. Neither does the commentary on Naṣīr al-Dīn's *Tadhkira* by ^{c}Umar al-Fārisī (10), compiled for the prince Abu'l-Fidā$^{\supset}$, although it discusses in detail certain aspects of non-Ptolemaic astronomy.

V(A). SPHERICAL ASTRONOMY

Al-Marrākushī and His Contemporaries. The major branch of astronomy practiced by the Mamluks was *^{c}ilm al-mīqāt*, the study of the apparent daily rotation of the sun and stars and its applications to timekeeping and the regulation of the times of prayer. The various tables and instruments devised by the Mamluk astronomers reflect their interest in solutions to problems of spherical astronomy for all latitudes; indeed, these universal solutions constitute their most significant contribution to the subject.[22] To some extent spherical astronomy was the domain of the *muwaqqits*, although the first Mamluk scholar of *^{c}ilm al-mīqāt*, Abū ^{c}Alī al-Marrākushī (5), was, as far as we know, not a *muwaqqit*. Since his name indicates that his family hailed from Marrakesh, al-Marrākushī is generally thought to have worked in the Maghreb; in fact he worked in Cairo, and his work appears to have been unknown in the Maghreb. His major work, *Kitāb al-Mabādi$^{\supset}$ wa-'l-ghāyāt fī ^{c}ilm al-mīqāt*, or *A Compendium ("A to Z") of Astronomical Timekeeping*, is extant in several manuscripts of Egyptian, Syrian, and Turkish provenance. The work is highly eclectic, but presents a complete survey of spherical astronomy and astronomical instruments. It had no counterpart in early or later Islamic astronomy and was very influential in later Syria as well as Egypt and Turkey. The first half, on spherical astronomy and sundial theory, was translated without commentary by J. J. Sédillot in 1834–1835, and

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of Ibn al-Shāṭir: An Arab Astronomer of the Fourteenth Century (Aleppo: Institute for the History of Arabic Science, 1976), esp. E. S. Kennedy and Victor Roberts, "The Planetary Theory of Ibn al-Shāṭir," pp. 60–68 (= *Isis*, 1959, *50*:227–235).

[22] See David A. King, "Universal Solutions to Problems of Spherical Astronomy from Mamluk Egypt and Syria," and King, "Universal Solutions in Islamic Astronomy" (both forthcoming).

the second, on instruments, was summarized in a rather haphazard fashion by his son L. A. Sédillot in 1844.[23] The treatise has attracted remarkably little attention from historians since then; it merits publication and analysis.

Al-Marrākushī does not rely at all on his illustrious Egyptian predecessor Ibn Yūnus: rather, he mentions various earlier Andalusian and Maghribi astronomers, and he seems to be using Iraqi sources, especially in his discussion of instruments. A curious omission from his discussion of several different varieties of quadrants is the almucantar quadrant, which other evidence suggests was already known in Cairo in his time. An unusual instrument made by al-Marrākushī is preserved in the Museum of the History of Science in Oxford.[24] Although shaped like an astrolabe, it is simply a plate with trigonometric grids and simple calendrical–solar longitude tables on one side and a series of circles representing altitude circles on the other. The movable parts of the instrument are missing.

Although al-Marrākushī does not mention any earlier or contemporary Egyptian astronomers, at least two other contemporary astronomers of consequence were working in Cairo: Shihāb al-Dīn al-Maqsī **(6)** and Najm al-Dīn al-Miṣrī **(7)**. Both scholars specialized in spherical astronomy, but neither mentions al-Marrākushī in their writings. Najm al-Dīn does mention al-Maqsī, but only in passing. Unfortunately, no biographical details are available for any of the three. Najm al-Dīn al-Miṣrī compiled a table for timekeeping that could be used not only for all latitudes but also for timekeeping by the sun by day and by the stars at night. Earlier astronomers in other parts of the world had compiled less extensive tables for timekeeping computed specifically for other localities such as Baghdad, Shiraz, and Maragha, and the main Mamluk tables described below were computed specifically for Cairo or Damascus or Jerusalem. Najm al-Dīn's remarkable table contains over a quarter of a million entries, and is extant in a unique manuscript (see Fig. 2).[25]

Al-Maqsī compiled two major works, a treatise on sundial theory and a set of tables for timekeeping that (unlike Najm al-Dīn's universal table) formed part of the Cairo corpus of tables.

The Main Cairo Corpus of Tables for Timekeeping. In the thirteenth and fourteenth centuries, Egyptian astronomers drew up a corpus of tables for timekeeping by the sun and for regulating the times of prayer, computed specifically for the latitude of Cairo. This corpus was widely used by Cairo *muwaqqits* until the nineteenth century, both in its original form and in various modified versions. The corpus exists in numerous manuscript copies, of which the first was rediscovered only about ten years ago. Most of the manuscripts attribute the corpus to Ibn Yūnus, and the problems of the attribution of these tables are still not completely solved.[26]

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[23] J. J. Sédillot, *Traité des instruments astronomiques des Arabes composé au treizième siècle par Aboul Hhassan* [*sic*] *de Maroc intitulé Jāmi^{c} al-mabādi$^{\supset}$ wa-l-ghāyāt*, 2 vols. (Paris: Imprimerie Royale, 1834–1835); and L. A. Sédillot, "Mémoire sur les instruments astronomiques des Arabes," *Mémoires de l'Academie Royale des Inscriptions et Belles-Lettres de l'Institut de France*, 1844, *1*:1–229.

[24] See Mayer, *Astrolabists*, p. 46, s.v. Ḥasan ibn ^{c}Alī.

[25] For more information, see King, *Tables*, pp. 44–45.

[26] See King, "Ibn Yūnus' *Very Useful Tables*" (cit. n. 4).

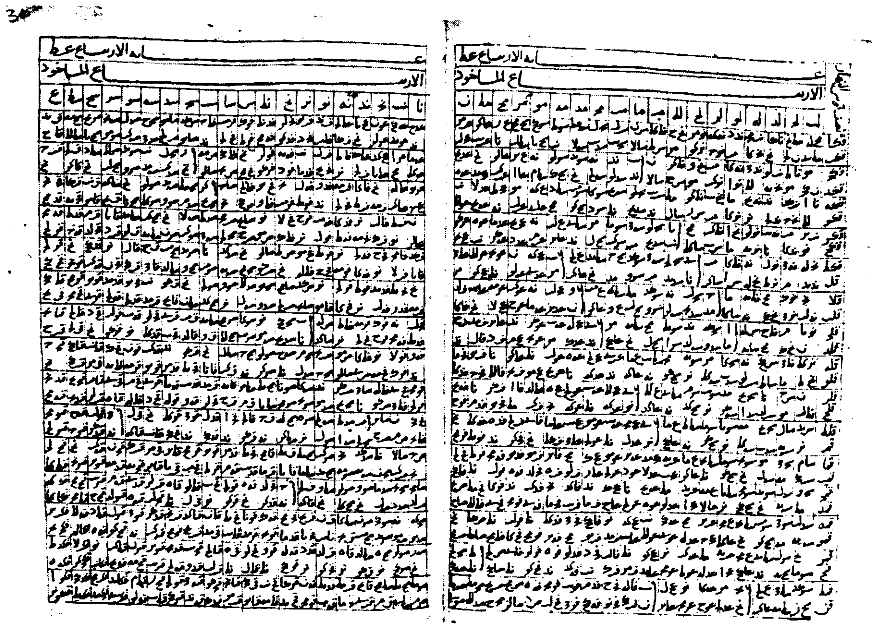

Figure 2. An extract from the universal table for timekeeping compiled by Najm al-Dīn al-Miṣrī. The table displays the time since rising of the sun or any star in terms of its observed altitude. One first feeds in the meridian altitude of the sun or star as the horizontal argument (the two pages shown here are part of the tables for argument 70°). Then one feeds in the observed altitude as the secondary horizonal argument, and the half arc of visibility of the sun or star as the vertical argument. The entry in the table is the time since rising of the sun or star, expressed in equatorial degrees and minutes (1° = 4 minutes of time). The table can be used for the sun at any time of the year or for any non-circumpolar star and for any latitude, because the first and second arguments are dependent on the latitude and declination. Reproduced from Oxford Bodleian Marsh 672, folios 33v–34r, with permission of the Bodleian Library.

The corpus contains the set of tables compiled by al-Maqsī, which was computed for the latitude of Cairo and displayed the time of day as a function of solar altitude and solar longitude (corresponding roughly to each day of the solar year). These tables, containing some 10,000 entries, were similar in conception to the set displaying the solar azimuth for the same arguments that Ibn Yūnus had prepared almost three centuries previously. A later Mamluk astronomer named Ibn al-Kattānī **(20)** used al-Maqsī's tables to derive a set displaying the hour-angle (time measured from midday) for the same arguments. Ibn al-Rashīdī **(26)** contributed other tables, and al-Bakhāniqī **(16)** put together the corpus of tables of the three functions time since sunrise, hour-angle, and azimuth (see Fig. 3), as well as tables for the prayer times, containing a grand total of over 30,000 entries.

Whatever the problems of attribution, all these tables were available in one form or another in the fourteenth century, if not before, as were others relating to the astronomically defined times of Muslim prayer. The latter displayed, for example, the time after midday at the beginning of the afternoon prayer (defined

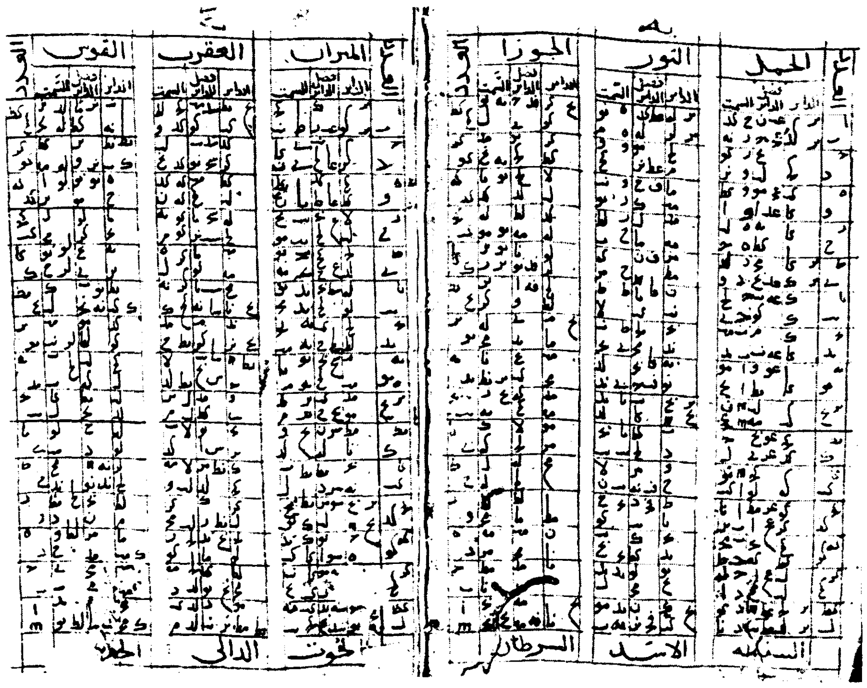

Figure 3. An extract from the tables of the three functions of time since sunrise, hour-angle, and solar azimuth for the latitude of Cairo in the edition of the main Cairo corpus of tables for timekeeping prepared by al-Bakhāniqī. These two pages serve solar altitude 15°, and entries are given for each degree of solar longitude, of which 180 suffice because the functions are symmetrical about the solstices. Reproduced from Cairo Dār al-Kutub mīqāt *690, folios 15v–16r, with permission of the Egyptian National Library.*

in terms of shadow lengths) and the duration of morning and evening twilight for determining the times of the daybreak and nightfall prayers. In late Mamluk copies of these tables other subtables appear, displaying, for example, the times when lamps on minarets should be extinguished after the nights of Ramadan, and when the muezzins should call out a benediction (*salām*) upon the Prophet, both shortly before daybreak.

The Damascus School. The Cairo tables for timekeeping served as a model for astronomers from Syria. It was to Egypt that Ibn al-Shāṭir (18) and al-Mizzī (17) came as young men to study astronomy, and other Syrian astronomers also express their indebtedness to Egyptian sources. We know that al-Mizzī studied in Cairo under Ibn al-Akfānī, although of this scholar's writings on topics other than medicine, we have only his lists of the books available in the different sciences.[27]

The major figure in the field of timekeeping at Damascus was a contemporary of Ibn al-Shāṭir named al-Khalīlī (**19**). As far as we know, no corpus of tables for timekeeping was compiled for the latitude of Damascus before the mid-fourteenth century. Al-Mizzī, no doubt inspired by the tables for Cairo that he had seen in Egypt, compiled a set of hour-angle tables and prayer-tables for Damascus, using a traditional set of parameters. Al-Khalīlī shortly thereafter recomputed the entire set for the new parameters derived from the observations of Ibn al-Shāṭir. Al-Khalīlī's tables for timekeeping were used in Damascus until the nineteenth century and exist in numerous manuscript copies.[28]

Al-Khalīlī also turned his attention to universal solutions and compiled a set of auxiliary tables for solving problems of spherical astronomy for all latitudes, based on three trigonometric functions. These auxiliary tables contain over 13,000 entries, and successive applications of the various functions tabulated lead to the solution of any problem in spherical astronomy for any latitude. Earlier Muslim astronomers, from the ninth century onwards, had compiled isolated simpler and less sophisticated sets of tables of auxiliary functions, but al-Khalīlī arrived at the final solution. His auxiliary tables were used by later Syrian, Egyptian, and Turkish astronomers.[29]

Al-Khalīlī's greatest computational achievement, however, was in the compilation of a table displaying the *qibla*, or local direction of Mecca, for each degree of latitude and longitude, based on an accurate mathematical formula.[30] Earlier astronomers from the ninth century onwards who had compiled *qibla* tables had contented themselves with much simpler approximate formulae. Again al-Khalīlī arrived at a final solution to one of the most important problems confronting the Muslim astronomers. His *qibla* table is known from only three manuscripts, each of Syrian provenance, and only one reference to it is known from the later astronomical literature.

Other Timekeeping Tables. Both the Cairo and the Damascus tables provided models for tables compiled for other localities. In the mid-fourteenth century the Egyptian astronomer Ibn al-Rashīdī (**26**) compiled two sets of timekeeping tables, one for Jerusalem and one for Mecca, after the model of the Cairo corpus. No earlier sets are known for either locality. The tables for Jerusalem comprised a set displaying the hour angle as a function of solar altitude and longitude as well as the times of prayer as functions of solar longitude. The tables for Mecca comprised only tables for regulating the times of prayer. Ibn al-Rashīdī's timekeeping tables for Jerusalem were later modified and expanded by a Jerusalem *muwaqqit* named al-Karakī (**29**), a student of al-Mizzī.[31]

Syrian astronomers prepared prayer tables for the latitudes of Tripoli and Aleppo after the model of the Damascus corpus, and the achievements of the Syrian astronomers in astronomical timekeeping inspired further activity in Cairo. In the fourteenth century a corpus of tables for timekeeping for the latitude of Tunis was prepared along the lines of the Damascus tables, and Syrian astronomers, perhaps those associated with the Umayyad Mosque in the fourteenth century, seem to have prepared the first set of prayer tables for Istanbul. All of the impressive achievements of Ottoman astronomers in timekeeping owe their inspiration to the earlier Egyptian and Syrian traditions.[32]

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[27] Wiedemann, *Aufsätze* (cit. n. 13), Vol. 1, pp. 258–266.

[28] On the Damascus corpus of tables for timekeeping see David A. King, "Astronomical Timekeeping in Fourteenth-Century Syria," in *Proceedings of the First International Symposium on the History of Arabic Science (Aleppo, 1976)*, 2 vols. (Aleppo: Institute for the History of Arabic Science, 1978), Vol. II, pp. 75–84.

[29] See David A. King, "Al-Khalīlī's Auxiliary Tables for Solving Problems of Spherical Astronomy," *Journal for the History of Astronomy*, 1973, *4*:99–110.

[30] See David A. King, "Al-Khalīlī's Qibla Table," *J. Near East. Stud.*, 1975, *35*:81–122.

[31] For the Jerusalem tables, see King, "Astronomical Timekeeping" (cit. n. 28).

V(B). INSTRUMENTATION

The Achievements of Ibn al-Sarrāj. As necessary to accurate timekeeping as tables were the many instruments used by Muslim astronomers. What al-Khalīlī achieved with tables at Damascus had been successfully attempted with instruments slightly earlier by an individual named Ibn al-Sarrāj **(15)**, who worked in Aleppo around 1325. The work of al-Marrākushī, the only earlier Mamluk astronomer to devote much time to instruments, had been highly eclectic, but that of Ibn al-Sarrāj is innovative to the extreme. Ibn al-Sarrāj, on whom we have as yet no biographical information of consequence, turned his attention to devising a series of astrolabes, quadrants, and other instruments that could be used to solve all of the problems of spherical astronomy for any latitude.[33]

Ibn al-Sarrāj actually devised two kinds of universal astrolabe. The first is simply a reinvention of the universal astrolabe usually associated with the eleventh-century Andalusian astronomer ᶜAlī ibn Khalaf al-Shakkāz. This instrument, which consisted of two *shakkāzīya* grids (stereographic projections of the altitude and azimuth circles at the terrestrial equator), was apparently not known in the Islamic east. A treatise Ibn al-Sarrāj wrote on its use has come down to us; elsewhere he relates that he invented it while in Mecca in 1325. An example of the second, more sophisticated variety, made by Ibn al-Sarrāj himself, survives in the Benaki Museum in Athens. A treatise on its use has also survived, written by the fifteenth-century Cairo *muwaqqit* al-Wafāᵓī **(55)**, who owned the very instrument now in Athens (see Fig. 4). Al-Wafāᵓī complains that Ibn al-Sarrāj had not written a treatise on the use of this astrolabe, so he takes this task upon himself. I consider Ibn al-Sarrāj's astrolabe, which is universal from five different aspects, to be the most sophisticated astrolabe from the Near East and Europe in the entire medieval and Renaissance period.

Ibn al-Sarrāj also developed several varieties of markings for the almucantar quadrant and devised various highly ingenious trigonometric grids as alternatives to the simple sine quadrant (see Fig. 5). While working on Ibn al-Sarrāj's instruments over the years, I have suspected that he wrote more than has survived in manuscript form. In the summer of 1982, I discovered in Chester Beatty Library in Dublin his autograph copy of a major work on instruments, a richly illustrated survey of all varieties of astrolabes and quadrants known to its author. Investigating it in the next few years will be a singular pleasure.

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[32] On Tunis see David A. King, "A Fourteenth-Century Tunisian Sundial . . . ," in Y. Maeyama and W. G. Saltzer, eds., *Prismata: Naturwissenschaftliche Studien—Festschrift für Willy Hartner* (Wiesbaden: Franz Steiner, 1977), pp. 187–201, esp. pp. 192–193. On the Maghreb see E. S. Kennedy and David A. King, "Indian Astronomy in Fourteenth-Century Fez . . . , Part II," *J. Hist. Arabic Sci.* (forthcoming). See also King, "Astronomical Timekeeping in Ottoman Turkey," *Proceedings of the International Symposium on the Observatories in Islam* (Istanbul: Kandilli Observatory, 1980), pp. 245–269.

[33] On Ibn al-Sarrāj (including the universal astrolabe and the ivory quadrant in the Benaki Museum) see David A. King, *The Astronomical Instruments of Ibn al-Sarrāj* (Athens: Benaki Museum, forthcoming).

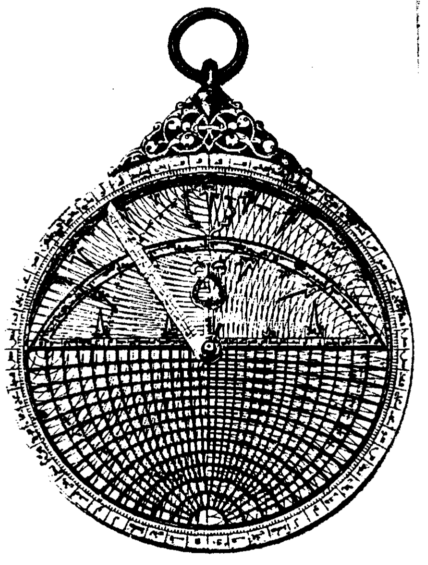

Figure 4. The universal astrolabe of Ibn al-Sarrāj. With this instrument problems of spherical astronomy can be solved for all latitudes in five different ways. First, the shakkāzīya *markings on the rete can be rotated over* shakkāzīya *markings on one of the plates. Second, the ecliptic and star pointers can be rotated over the* shakkāzīya *markings. Third, the ecliptic and star pointers can be rotated over a series of quarter plates for each 3° of latitude. Fourth, the ecliptic and star pointers can be rotated over a series of horizons for all latitudes on the mater. Fifth, numerical solutions to trigonometric problems can be obtained using a grid on the back of the instrument, together with the alidade. Courtesy of the Benaki Museum, Athens.*

The works of two other Aleppo astronomers on instruments have come down to us. Ṭībughā al-Baklamshī **(32)**, well known as the author of a treatise on archery, also wrote a treatise on the single *shakkāzīya* quadrant,[34] and his son ᶜAlī ibn Ṭībughā **(33)**, who worked as a *muwaqqit* in the Umayyad Mosque in Aleppo, wrote a substantial treatise on the sine and almucantar quadrants.

Although it was Ibn al-Sarrāj who devised the most successful varieties of instruments, the achievements in instrumentation of other Mamluk astronomers at Cairo and Damascus were also considerable. While working for the Yemeni Sultan al-Mujāhid, the early fourteenth-century Egyptian astronomer al-Bakhāniqī **(16)**, compiled an extensive set of tables of coordinates for marking curves on the plates of astrolabes for each degree of latitude from 0° to 90°. Al-Bakhāniqī thus completed the set for latitudes 15° to 50° compiled by the early ninth-century Baghdad astronomer al-Farghānī.[35] Al-Farghānī's astrolabe tables were widely known in Mamluk Egypt and Syria, and we may presume that they were used in the construction of astrolabes as well as quadrants, although not one Mamluk astrolabe other than the universal instrument of Ibn al-Sarrāj is known to survive. Ibn al-Shāṭir's **(18)** contributions to instrument design were also considerable; they included a reversed astrolabe (on which a set of horizons rotated over a fixed stereographic projection of the ecliptic and various stars), and several trigonometric grids (see Fig. 5).[36] He also made the large astrolabic

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[34] See J. Samsó Moya, "Nota acerca de cinco manuscritos sobre astrolabio," *Al-Andalus*, 1966, *31*:385–392; Samsó Moya, "Una hipótesis sobre cálculo por aproximación con el cuadrante Šakkāzī," *Al-Andalus*, 1971, *36*:117–126; and J. Samsó Moya and M. A. Catalá, "Un instrumento astronómico de raigambre Zarqālī: El cuadrante Šakkāzī de Ibn Ṭībugā," *Memorias de la Real Academia de Buenas Letras de Barcelona*, 1971, *13*(1):5–31.

[35] See King, *Tables*, pp. 53–55.

[36] See Schmalzl, *Quadrants*, pp. 100–108, reprinted in Kennedy and Ghanem, *Ibn al-Shāṭir*, pp. 27–35.

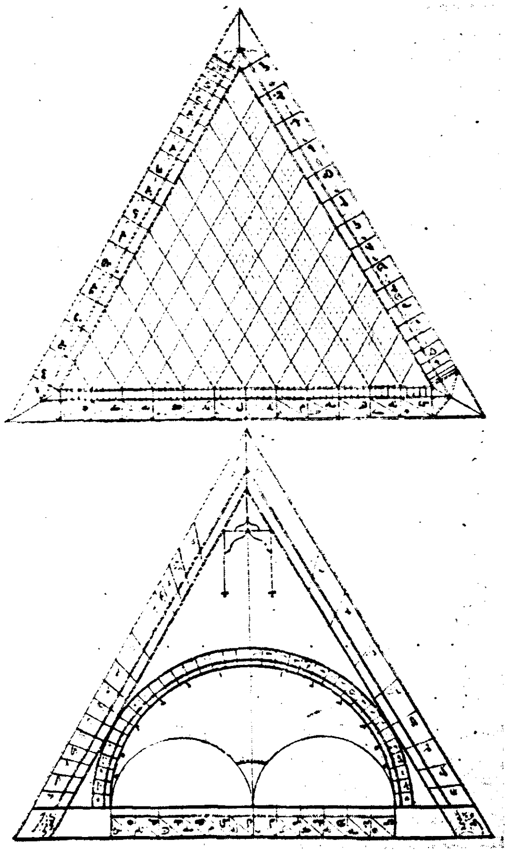

Figure 5. *Trigonometric grids devised by Ibn al-Sarrāj (below) and Ibn al-Shāṭir (above), illustrated in an eighteenth-century Egyptian manuscript. Both can be used to derive numerical solutions to the standard problems of spherical astronomy, for any latitude. Reproduced from Cairo Dār al-Kutub Muṣṭafā Fāḍil riyāḍa 40.2 with permission of the Egyptian National Library. Photograph courtesy of Owen Gingerich.*

clock described by the contemporary historian al-Ṣafadī, who saw it in the astronomer's home, and he constructed a splendid sundial for the Umayyad Mosque in Damascus.

Sundials. Sundials adorned the courts or walls of all major mosques in Egypt and Syria in the later Mamluk period. Few have survived, but three have been published in recent years.[37] One is the sundial installed in the Mosque of Ibn Ṭūlūn in the late thirteenth century. For some reason, possibly because it was defective, it was destroyed, but fortunately not beyond recognition, and Napoleon's scholars illustrated the fragments in their *Description d'Egypte*. The sundial displayed the seasonal hours and the time of the afternoon prayer. Also studied recently is the vertical sundial on one of the walls of the madrasa of

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[37] See L. Janin and D. A. King, "Le cadran solaire de la mosquée d'Ibn Ṭūlūn au Caire," *J. Hist. Arabic Sci.*, 1978, *2*:331–357; D. A. King and A. G. Walls, "The Sundial on the West Wall of the Madrasa of Sultan Qaytbay in Jerusalem," *Art and Architecture Research Papers*, July 1979, *15*:16–21; and L. Janin, "Le cadran solaire de la Mosquée Umayyade à Damas," *Centaurus*, 1972, *16*:285–298.

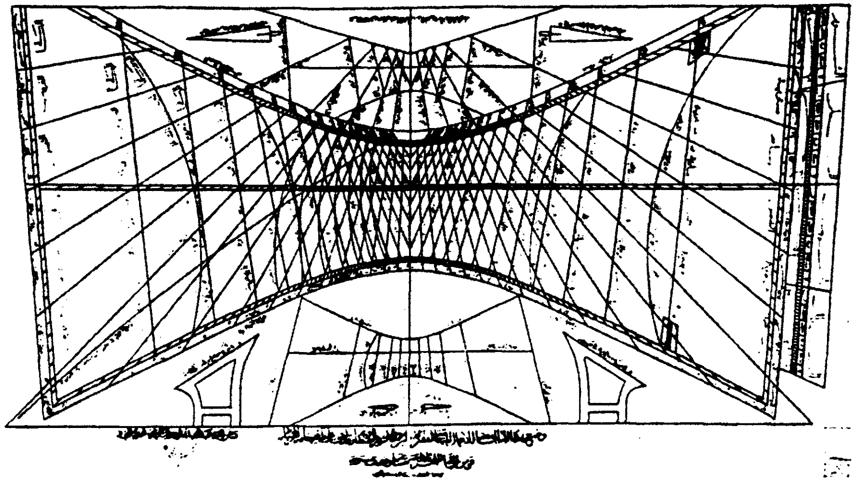

Figure 6. *The magnificent sundial that Ibn al-Shāṭir constructed in the year 1371/72 to adorn the main minaret of the Umayyad Mosque in Damascus. The sundial displays the time of day relative to sunrise, midday, and sunset and relative to the afternoon prayer. There are also special curves for times relative to daybreak and nightfall. Thus the sundial effectively measures time with respect to each of the five daily prayers. Photograph from a drawing, courtesy of Alain Brieux, Paris.*

Qāʾitbāy in Jerusalem, which displays the time remaining until the beginning of the afternoon prayer. The most impressive Mamluk sundial is, however, the one Ibn al-Shāṭir constructed for the main minaret of the Umayyad Mosque in Damascus. The surviving fragments are now on display in the National Archaeological Museum in Damascus, and an exact replica, made by the nineteenth-century *muwaqqit* al-Ṭanṭāwī, is still *in situ* on the minaret. This splendid instrument displays time in both seasonal and equinoctial hours, as well as time relative to the afternoon prayer and the prayers at nightfall and daybreak (see Fig. 6).

Mamluk treatises on sundial theory, of which several are known, give yet more information on the different varieties of sundials that were in use, including, for example, sundials skewed to both planes of the meridian and the prime vertical. Most of these treatises contain tables of coordinates for constructing sundials.[38] The earlier sundial tables of al-Maqsī (6) inspired the Damascus astronomer al-Tīzīnī (69) to compile a set of tables for marking the curves on vertical sundials for the latitudes of Cairo, Damascus, and Aleppo (see Fig. 7). Ibn Sūdūn (52) and Sibṭ al-Māridīnī (63) compiled other new sets of tables for Cairo, and Ibn al-Majdī (44) and al-Karādīsī (62) compiled new treatises on sundial theory.

Ibn al-Shāṭir also devised a universal sundial and timekeeping device called

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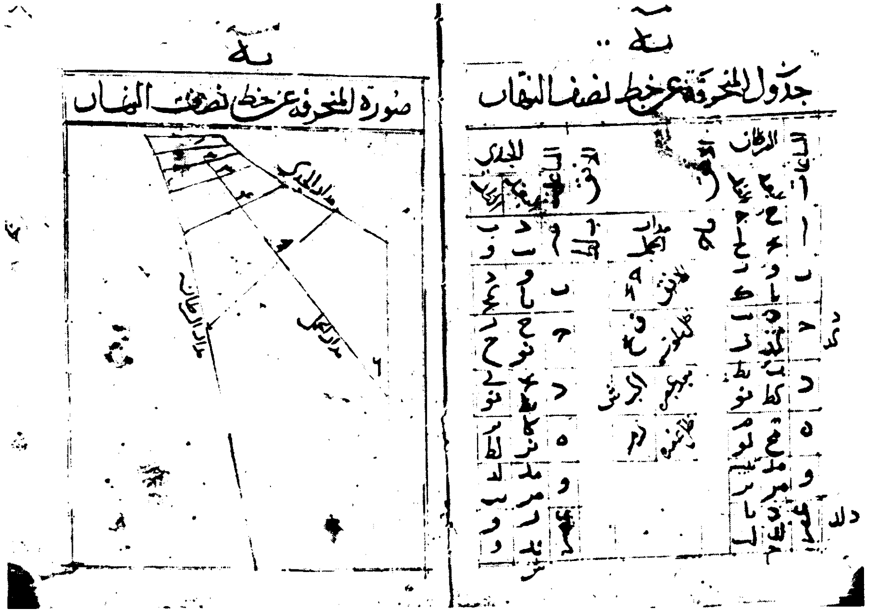
[38] On Islamic (including Mamluk) sundial theory see Carl Schoy, "Gnomonik der Araber," in E. van Bassermann-Jordan, ed., *Die Geschichte der Zeitmessung und der Uhren*, Vol. I, Part F (Berlin/Leipzig: Heinz Lafaire, 1923), and esp. Schoy, "Sonnenuhren der spätarabischen Astronomie," *Isis*, 1924, *6*:332–360 (both outdated). On Islamic sundial tables see King, *Tables*, pp. 51–53.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صورة المنحرفة عن خط نصف النهار
جدول المنحرفة عن خط نصف النهار

Figure 7. An extract from al-Maqsī's tables for constructing vertical sundials for each degree of inclination to the meridian for the latitude of Cairo. The tables display coordinates of the points of intersection of the solstitial and equinoctial shadow traces with the lines for the seasonal hours, and this particular pair of pages serves inclination 15°. Reproduced from Cairo Dār al-Kutub mīqāt *103, folios 68v–69r, with permission of the Egyptian National Library.*

ṣandūq al-yawāqīt li-ma^c rifat al-mawāqīt, "jewel box for finding the times of prayer." This instrument consisted essentially of a box containing a compass for aligning it in the meridian, fitted with a lid that could be raised to support either a polar sundial at the appropriate angle to the local horizon or a set of sights for reading the hour angle of the sun or any star. An example of this box, made by Ibn al-Shāṭir himself, survives in Aleppo. In his treatise on the use of the instrument, preserved in Berlin, Ibn al-Shāṭir failed to mention that the compass needle does not necessarily indicate true north, but then we have no idea what the magnetic declination was in Damascus in his time. We do know that the compass was in widespread use in the eastern Mediterranean in the thirteenth century, if not before.[39]

Four later contemporaries of the Damascus school, Jamāl al-Dīn al-Māridīnī (34), Abū Ṭāhir (23), al-Ghuzūlī (21), and after them al-Wafā^ī (55) continued their work in Cairo, devising new instruments and even attempting to compile new auxiliary tables. Although the Damascus astronomers had already solved most of the problems, al-Māridīnī devised a universal quadrant based on the universal astrolabe of Ibn al-Sarrāj (15) and consisting of two *shakkāzīya* quadrants; Ibn al-Ghuzūlī developed the almucantar and sinical octants; and al-Wafā^ī simplified the "jewel-box" of Ibn al-Shāṭir (18) to produce an equinoctial dial (see Fig. 8), precursor of the popular "compendium" of Renaissance Europe.[40]

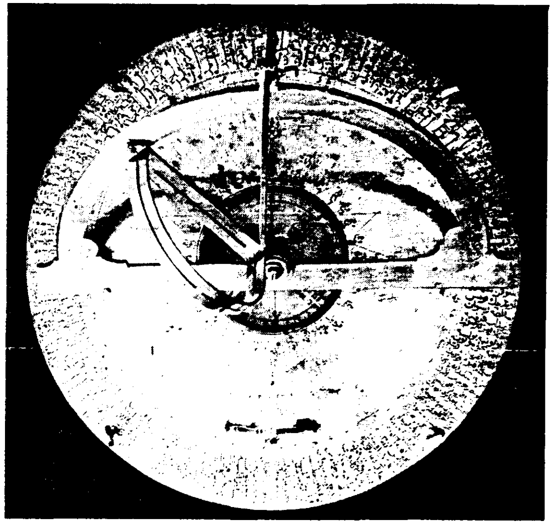

Figure 8. An example of al-Wafā^ī's equatorial semicircle. The circular base is to be set up in the cardinal directions using the compass. The semicircle is then to be elevated into the plane of the celestial equation for any latitude, and the sights aligned towards the sun: the hour-angle can then be read on the scale. Around the perimeter of the base are qiblas of different localities, and there are also markings for a horizontal sundial for the latitude of Istanbul. Courtesy of Muammer Dizer, Director, Kandilli Observatory.

By the end of the fifteenth century it was left to Ibn Abi'l-Fatḥ al-Ṣūfī (64) to write new commentaries on the use of the more sophisticated of the instruments devised by his predecessors in Mamluk Damascus and Cairo, and to Sibṭ al-Māridīnī (63) to write treatises in various recensions on the use of the simpler instruments such as the sine and almucantar quadrants. Mention should also be made of Ibn al-^cAṭṭār (42), who in the early fifteenth century compiled a treatise on the construction of all of the different kinds of quadrants known to him. This work is of considerable historical interest and has not yet been properly studied.

VI. MAMLUK ACTIVITY IN FOLK ASTRONOMY

The scholars of the Mamluk period wrote comparatively little on folk astronomy, the nonmathematical astronomical folklore of the Islamic world whose origins lie in pre-Islamic Arabia. One Sirāj al-Dunyā wa-'l-Dīn, as yet unidentified, wrote a substantial treatise not yet studied, on all aspects of the subject in Egypt around 1210. The late thirteenth-century dervish al-Dīrīnī (4) compiled a shorter treatise and yet simpler tracts were written by al-Haythamī and al-Maḥallī (50 and 51). The late fifteenth-century polymath Jalāl al-Dīn al-Suyūṭī (65) compiled a substantial treatise on the references to astronomy in the Qurᵓān and the Prophetic sayings known as the *ḥadīth*, recently studied for the first time.[41]

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[39] See L. Janin and D. A. King, "Ibn al-Shāṭir's *Ṣandūq al-Yawāqīt:* an Astronomical Compendium," *J. Hist. Arabic Sci.*, 1977, *1*:187–256; and, on the compass, Wiedemann, *Aufsätze*, Vol. I, pp. 36–37, G. R. Tibbetts, *Arab Navigation in the Indian Ocean before the Coming of the Portuguese* (London: Luzac & Co., 1971), pp. 290–294, and King, *Yemen*, Pt. II, Sect. 8.2.

[40] See D. A. King, "An Analog Computer for Solving Problems of Spherical Astronomy: The *Shakkāzīya* Quadrant of Jamāl al-Dīn al-Māridīnī," *Archives Internationales d'Histoire des Sciences*, 1974, *24*:219–242; S. Tekeli, "(The) 'Equatorial Armilla' of Iz(z) al-Din b. Muḥammad al-Wafā^ī and (the) Torquetum," *Ankara Universitesi Dil ve Tarih-Čografya Fakultesi Dergesi*, 1960, *18*:227–259; William Brice, Colin Imber, and Richard Lorch, *The Dā^ire-yi Mu^caddal of Seydī ^cAlī Re^īs* (Seminar on Early Islamic Science, monograph 1) (Manchester: Univ. Manchester, 1976) and Muammer Dizer, "The Dā^irat al-Mu^caddal in the Kandilli Observatory . . . ," *J. Hist. Arabic Sci.*, 1977, *1*(2):257–262.

An interesting development of primitive techniques of timekeeping by night using the lunar mansions is found in a thirteenth-century Egyptian muezzins' manual, extant in a unique copy in the Bodleian Library in Oxford.[42] This manual contains precisely the kind of information that Ibn al-Ukhuwwa implied all muezzins should know. The manual presents diagrams of the lunar mansions, with tables displaying the lunar mansion rising, culminating, and setting at different times of the night. Such material is not otherwise known in the Islamic sources, except in various almanacs from medieval Yemen. Later Cairo muezzins and *muwaqqits* seem to have preferred the more sophisticated tables for regulating the times of prayer described above.

VII. MAMLUK ACTIVITY IN ASTROLOGY

The Mamluk astronomers whose names we have mentioned wrote surprisingly little on astrology. All *zījes* contain some astrological material,[43] so that the major Mamluk *zījes* such as the anonymous *Muṣṭalaḥ Zīj*, the *Zīj* of Ibn al-Shāṭir, the Syrian versions of the *Īlkhānī Zīj* of al-Ṭūsī, and the Egyptian and Syrian versions of the *Zīj* of Ulugh Beg must be consulted in any investigations into this subject. The independent treatises on the theoretical aspects of astrology compiled in Mamluk Syria and Egypt are the following: a treatise by al-Marrākushī (5), known only from one surviving fragment; an eclectic work supposedly compiled in Cairo in 1358 by Ibrāhīm al-Ḥāsib al-Malikī al-Nāṣirī (27), an individual otherwise unknown to me; another such work compiled in Cairo about 1425 by Ibn al-ᶜArabānī (43); and a short treatise by Aḥmad ibn Timurbāy (72).

We may perhaps assume that the mosque associations of the *muwaqqits* prevented them from writing on astrology. Certainly there are no astrological tables in the various copies of the Cairo corpus of tables for timekeeping. Unfortunately, we have no biographical information on one Yūsuf ibn Ṭūghān (68) called *al-mīqātī*, who wrote a short astrological treatise dealing with prices. Only one individual horoscope is known to me from the entire corpus of Mamluk astronomical sources, that compiled by Ibn al-Majdī (44) for the *amīr* Naṣīr al-Dīn Abu'l-Fatḥ Muḥammad, who was born in Cairo A.H. 802 (= A.D. 1399/1400). I know of only one Mamluk astrological history, that compiled by the Aleppo astronomer al-Ḥamzāwī (67). His treatise contains a series of horoscopes for the various Mamluk campaigns against the Ottomans around Adana about 1495. The fourteenth-century Cairo scholar Ibn al-Akfānī informs us of the various astrological treatises known to him, but none was contemporary with him. The

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[41] On Islamic folk astronomy, see Ch. Pellat, article "*Anwāᵓ*," *Encyclopaedia of Islam*, 2nd. ed.; J. Ruska, article "Manāzil," in *Encyclopaedia of Islam*, 1st. ed., 4 vols. (Leiden: Brill, 1913–1934); and C. A. Nallino, *Raccolta di Scritti Editi e Inediti*, Vol. V (Rome: Istituto per l'Oriente, 1944), pp. 152–197. On al-Suyūṭī's treatise see Anton Heinen, *Islamic Cosmology: A Study of as-Suyūṭī's "al-Hayᵓa al-sanīya fi al-hayᵓa al-sunnīya"* (Beirut: in commission for Franz Steiner [Wiesbaden], 1982).

[42] See the summary in King, "Role of the *Muwaqqit*" (cit. n. 10).

[43] See Kennedy, *Survey*, pp. 144–145. The position of astrology in Islamic society is discussed in G. Saliba, "The Development of Astronomy in Medieval Islamic Society," *Arab Studies Quarterly*, 1982, *4*(3):211–225.

only Mamluk manuscript copy of such astrological treatises I know is al-Ḥamzāwī's copy of a work by the thirteenth-century scholar Muḥiy al-Dīn al-Maghribī (2) of Maragha.

It is safe to assume that the Mamluk sultans had considerable interest in astrology, even though there are so few surviving texts to prove it. No investigation of Mamluk historical documents has been conducted with this aim yet; two references to court astrologers were noted above. A *malḥama*, or book of prognostications for each month of the Syrian (solar) year, dedicated to Sultan Qāᵓitbāy, is preserved in a single manuscript in the Chester Beatty Library in Dublin. The prognostications are based on such criteria as comets, thunder, lightning, earthquakes, the physical appearance of the lunar crescents at the beginning of the Muslim months, and the like, apparently derived mainly from Hermetic material. Religious scholars, of course, frowned upon faith in such prognostications, and at least one Mamluk theologian, Ibn Qayyim al-Jawzīya of Damascus, wrote a polemic against astrology.[44]

Under the Ottomans the compilation of treatises on astrology began to flourish again, as we can judge from the number of treatises compiled by Egyptian astronomers in that period. Nevertheless, the authors of these treatises were not *muwaqqits*. The influence of astrology under the Mamluks and the Ottomans is a subject that merits investigation.

VIII. MAMLUK ACTIVITY IN MATHEMATICS

It would be inappropriate to omit from our discussion Ibn al-Hāᵓim (35), an Egyptian who worked in Jerusalem in the late fourteenth century as a teacher at the Ṣalāḥīya madrasa. Ibn al-Hāᵓim's works on arithmetic, simple algebra, and the arithmetic of inheritance had considerable influence in later Syria and Egypt. They were much commented upon in both Mamluk and Ottoman times, even by individuals, such as Sibṭ al-Māridīnī (63), whose prime interest was in astronomy. But the compilation of such commentaries, through no fault of the author of the originals, marked the end of creative mathematical activity in Syria and Egypt. It was in the Maghreb that developments of consequence were being made in mathematics, particularly with regard to algebraic notation: these Maghribi works were available to scholars in Mamluk Egypt and Syria and were also commented upon by them.[45] Even this infusion of ideas from the Maghreb does not seem to have been able to stop the steady decline of mathematics in the late Mamluk period.

IX. CONCLUDING REMARKS

Creative astronomical activity came to an end in Syria with the destruction of Damascus by the Mongols in 1402, in Egypt with the works of Sibṭ al-Māridīnī around 1500. The Ottoman astronomers found themselves heirs to the Egyptian and Syrian traditions, as well as to the works of the Samarqand astronomers, some of whom, such as Qāḍī Zāde al-Rūmī and ᶜAlī Qūshjī, had close personal ties to Ottoman Turkey. The most famous Muslim astronomer of the sixteenth century, Taqi'l-Dīn ibn Maᶜrūf, who directed the Istanbul Observatory, was a Syrian who had lived both in Egypt and in Nablus and was familiar with the rich heritage of Mamluk astronomy. In the eighteenth century the Ottomans came into contact with European astronomy, and the "*zīj*es" of the French astronomers Lalande and Cassini were translated into Turkish and their tables adapted to the longitude of Istanbul.[46] Other versions were later prepared in Arabic for Cairo and Damascus.

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[44] See J. W. Livingston, "Ibn Qayyim al-Jawziyyah: A Fourteenth Century Defense Against Astrological Divination and Alchemical Transmutation," *Journal of the American Oriental Society*, 1971, *91*:96–103.

[45] On mathematics in the Maghreb, with reference to certain Mamluk commentaries on Maghribi works, see A. Djebbar, *Enseignement et recherche mathématiques dans le Maghreb des XIII^e–XIV^e siècles* (Publications Mathématiques d'Orsay, no. 81-02)(1980).

Astronomy continued to be studied in Cairo and Damascus after Istanbul had become the center of astronomical activity in the Islamic world. Different and virtually independent schools likewise continued to flourish in the Maghreb and in Safavid Iran. The compilation of annul ephemerides and the copying of tables for timekeeping for the use of the *muwaqqit*s continued apace. Astrology, barely in evidence under the Mamluks, flourished again. But the great scientists of ninth-century Syria and tenth-century Cairo, and even the lesser, nevertheless significant astronomers of Mamluk times, had been virtually forgotten. The historian al-Jabartī tells us that the textbooks in astronomy and mathematics used in the Azhar about 1800 consisted mainly of a nontechnical digest of Ptolemaic astronomy compiled by al-Jaghmīnī in Central Asia in the early thirteenth century, various treatises on timekeeping and on instruments by Sibṭ al-Māridīnī (63), and various treatises on mathematics by Ibn al-Hāᵓim (35).[47]

The manuscript libraries of Cairo and Damascus, which contain many manuscripts copied during the Ottoman period, and even the older collections in Europe whose shelves are somewhat less cluttered with late manuscripts, bear witness to the popularity of the works of al-Jaghmīnī, Ibn al-Haᵓim and Sibṭ al-Māridīnī and the numerous commentaries on them. It was only by looking beyond these to the more exciting works of the Mamluk astronomers extant in relatively few copies that a reasonably clear picture of astronomy in Mamluk Egypt has emerged.

This present survey is of necessity preliminary. Ibn al-Shāṭir's work on planetary astronomy was investigated for the first time only twenty-five years ago and is still not published. The works of al-Maqsī, Najm al-Dīn al-Miṣrī, Ibn al-Sarrāj, al-Mizzī, al-Khalīlī, al-Māridīnī, al-Wafāᵓī, and Ibn Abi'l-Fatḥ al-Ṣūfī have been studied only in the last ten years, but none is published. Ibn al-Sarrāj's treatise on instruments was discovered after this paper was completed and remains to be properly studied. There is more work to be done, both on the technical details of various Mamluk treatises and the transmission of scientific ideas, and on the social status and institutional organization of the Mamluk astronomers. Still, we can say with conviction that the Mamluks left a heritage of astronomical treatises and instruments that was as impressive as any of their contemporaries in the Near East, where the scientific age was drawing to a close, or in Europe, where a new scientific age was beginning.

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[46] There is as yet no satisfactory overview of Ottoman astronomy. The survey in S. A. Adnan, *La science chez les Turcs Ottomans* (Paris: G. Maisonneuve, 1939) is quite inadequate.

[47] See J. Heyworth-Dunne, *An Introduction to the History of Education in Modern Egypt* (London: Luzac & Co., 1940), pp. 62–64.

APPENDIX: A LIST OF MAMLUK ASTRONOMERS

The astronomers are listed more or less chronologically; their numbers are given periodically in the text after their names. Important astronomers are marked by one asterisk, especially important ones by two.

	Astronomer, affiliation, location, and date	**References[48]**	
		Suter	King
1	**Muᵓayyad al-Dīn al-ᶜUrḍī, fl. Damascus ca. 1250; later associated with the observatory in Maragha	(pp. 147, 154)	—
2	*Muḥiy al-Dīn al-Maghribī, fl. Damascus ca. 1250; later associated with the observatory in Maragha	376	G21
3	*Anonymous compilers of the *Muṣṭalaḥ Zīj*, fl. Cairo ca. 1250	—	C12
4	Ḍiyāᵓ al-Dīn al-Dīrīnī, an itinerant dervish, fl. Egypt ca. 1275	—	C14
5	**Abū ᶜAlī al-Marrākushī, no known affiliation, fl. Cairo ca. 1280	363	C17
6	*Shihāb al-Dīn al-Maqsī, no known affiliation, fl. Cairo ca. 1280	383	C15
7	*Najm al-Dīn al-Miṣrī, no known affiliation, fl. Cairo ca. 1280	460 (confused)	C16
8	*Abū'l-Fidāᵓ, ruler and scholar, fl. Hama ca. 1300	392	—
9	*Amīn al-Dīn al-Abharī, worked for Abu'l-Fidāᵓ (8)	393	—
10	*ᶜUmar al-Fārisī, worked for Abu'l-Fidāᵓ (8)	—	C21
11	*Abū ᶜAlī al-Fārisī, no known affiliation, fl. Hama ca. 1300	—	C19
12	Badr al-Dīn ibn Jamāᶜa al-Kinānī, sometime chief *qāḍī* of Egypt and Damascus	—	C23
13	*Nāṣir al-Dīn ibn Simᶜūn, *muwaqqit* at the Mosque of ᶜAmr in Fustat ca. 1300	398	C24
14	Aḥmad al-Juzajānī, no known affiliation, fl. Cairo ca. 1325	401	—
15	**Ibn al-Sarrāj, no known affiliation, fl. Aleppo ca. 1325	508 (confused)	C26
16	*al-Bakhāniqī, no known affiliation, fl. Cairo ca. 1325; also worked in the Yemen	—	C28
17	*al-Mizzī, *muwaqqit* at the Umayyad Mosque in Damascus, died ca. 1350	406	C34
18	**Ibn al-Shāṭir, *muwaqqit* at the Umayyad Mosque in Damascus, fl. ca. 1350	416	C30
19	**al-Khalīlī, *muwaqqit* at the Umayyad Mosque in Damascus, fl. ca. 1350	418	C37
20	*Ibn al-Kattānī, no known affiliation, fl. Cairo ca. 1350	410	C32
21	*Ibn al-Ghuzūlī, no known affiliation, fl. Cairo ca. 1350	412	C33

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[48] References are by number to Suter, "Mathematiker und Astronomen der Araber" (by the number preceded by N to the "Nachträge"; see n. 1); and King, *Survey* (cit. n. 1), which has all necessary cross-references to the works of Brockelmann and Azzawi. References are also made in the following works to the following astronomers: Mayer, *Islamic Astrolabists* (cit. n. 1): **5** (p. 46), **21** (p. 53), **30** (p. 35); *Encyclopaedia of Islam*, 2nd ed. (cit. n. 4): **8**, **10** (under Ibn Djamaᶜa); *DSB* (cit. n. 2): **8**, **18**, **19** (Suppl.); Sezgin, *Geschichte des arabischen Schrifttums* (cit. n. 1): **27** (Vol. VII, p. 25); and Heinen, *Islamic Cosmology* (cit. n. 41): **65**.

22	Aḥmad ibn ᶜUmar al-Shādhilī, not identified	—	C55
23	Taqi'l-Dīn Abū Ṭāhir, not identified	—	C56
24	Tāj al-Dīn al-Tabrīzī, teacher at the Turuntāy madrasa in Cairo, fl. ca. 1350(?)	—	C22
25	al-Murshidī, no known affiliation, fl. Cairo ca. 1350(?)	500	C50
26	*Ibn al-Rashīdī, no known affiliation, fl. Cairo ca. 1360	—	C39
27	Ibrāhīm al-Nāṣirī, no known affiliation, fl. Cairo ca. 1360	—	—
28	Abu'l-Maᶜālī al-Sāᶜāti, not identified	—	C36
29	Zayn al-Dīn al-Karakī, a *muwaqqit* in Jerusalem, student of al-Mizzī (17)	—	C35
30	Aḥmad al-Ḥarīrī, no known affiliation, fl. Cairo ca. 1380	—	C45
31	Ismāᶜīl ibn Hibat Allāh al-Ḥamawī, not identified	—	C52
32	ᶜAlāᵓ al-Dīn Ṭībughā al-Baklamshī, no known affiliation, fl. Aleppo ca. 1375	—	C53
33	ᶜAlī ibn Ṭībughā al-Baklamshī, son of 32, *muwaqqit* at the Umayyad Mosque in Aleppo	—	C54
34	Jamāl al-Dīn al-Māridīnī, no known affiliation, fl. Cairo(?), perhaps Damascus, ca. 1400	421	C47
35	Ibn al-Hāᵓim, teacher at the Salāḥīya madrasa in Jerusalem, lived ca. 1355–1412	423	C58
36	ᶜAbd al-ᶜAzīz ibn Masᶜūd, no known affiliation, fl. Cairo ca. 1400	—	C46
37	Muḥammad ibn Idrīs, no known affiliation, fl. Cairo ca. 1400	—	C44
38	Zayn al-Dīn ᶜUmar al-Zuhrī, not identified	—	C79
39	Shihāb al-Dīn al-Basaṭī, not identified	—	C48
40	Sharaf al-Dīn al-Khalīlī, nephew of 19, *muwaqqit* at the Umayyad Mosque in Damascus, fl. ca. 1400	427	C38
41	Shihāb al-Dīn al-Kawm al-Rīshī, *muwaqqit* at the Mosque of al-Muᵓayyad in Cairo, fl. ca. 1410	428	C41
42	Ibn al-ᶜAṭṭār, no known affiliation, fl. Cairo ca. 1425	431	C66
43	Ibn al-ᶜArabānī, no known affiliation, fl. Cairo ca. 1425	N433	C65
44	**Ibn al-Majdī, no known affiliation, fl. Cairo ca. 1425	432	C62
45	*Shihāb al-Dīn al-Ḥalabī, *muwaqqit* at the Umayyad Mosque in Damascus, fl. ca. 1425	434	C69
46	Nūr al-Dīn ᶜAlī al-Naqqāsh, no known affiliation, fl. Cairo ca. 1425	—	C74
47	*Ibn al-Muhallabī, no known affiliation, fl. Cairo ca. 1450	—	C67
48	Sayf al-Dīn Sāṭilmish, a *muwaqqit* in Cairo, fl. ca. 1450	—	C72
49	Muḥammad al-Ghazāwī, not identified	—	C73
50	ᶜAlī ibn Muḥammad al-Haythamī, no known affiliation, fl. Cairo ca. 1450	—	C49
51	Ḥasan al-Maḥallī, no known affiliation, fl. Cairo ca. 1450	—	C85
52	Ibn Sūdūn, no known affiliation, fl. Cairo ca. 1450(?)	—	C82

53	Sūdūn al-Bashtakī, a muezzin in Cairo ca. 1450(?)	—	C81
54	*Ibn al-Mushrif, no known affiliation, fl. Cairo ca. 1450	—	C43
55	*ᶜIzz al-Dīn al-Wafāᵓī, *muwaqqit* at the Mosque of al-Muᵓayyad in Cairo, fl. ca. 1450	437	C61
56	*al-Aqfahsī, no known affiliation, fl. Cairo ca. 1450	440	C68
57	Abu'l-Baqāᵓ Yaḥyā ibn al-Jīᶜān, no known affiliation, fl. Cairo ca. 1450	—	C83
58	Zakariyāᵓ al-Bilbaysī, no known affiliation, fl. Cairo ca. 1450(?)	522	C64
59	Muḥammad ibn al-ᶜAṭṭār al-Bilbaysī, not identified	—	C80
60	al-Qaymarī, not identified	—	C42
61	Abū Bakr ibn al-Imām, not identified	—	C96
62	*al-Karādīsī, *muwaqqit* at the Ashrafīya madrasa in Cairo, fl. ca. 1460	442	C90
63	**Sibṭ al-Māridīnī, grandson of 34, *muwaqqit* at Azhar Mosque in Cairo, fl. ca. 1460	445	C97
64	**Ibn Abi'l-Fatḥ al-Ṣūfī, no known affiliation, fl. Cairo ca. 1460	447	C98
65	Jalāl al-Dīn al-Suyūṭī, celebrated author and teacher, fl. Cairo ca. 1475	449	C103
66	Yaḥyā al-Rifāᶜī, no known affiliation, fl. Cairo ca. 1475	—	C99
67	*Yūsuf ibn Qurqamās al-Ḥamzāwī, *amīr al-ḥajj* in Aleppo, fl. ca. 1475	—	C91
68	Yūsuf al-Qiṭṭajī al-Mīqātī, not identified	—	C92
69	al-Tīzīnī, *muwaqqit* at the Umayyad Mosque in Damascus, fl. ca. 1500	450	C95
70	*al-Ṣāliḥī, *muwaqqit* at the Umayyad Mosque in Damascus, fl. ca. 1500	454	C87
71	al-Qasṭalānī, no known affiliation, fl. Cairo(?) ca. 1500	458a	—
72	*al-Shihāb Aḥmad ibn Timurbāy, no known affiliation, fl. Cairo ca. 1500	—	C94
73	Muḥammad ibn Dallāl al-Wafāᵓī al-Suyūṭī, a student of al-Ṣūfī (64)	459	C101
74	ᶜAlī al-Mālaqī, a student of 73	459	C102
75	Ibn ᶜAbd al-Ghaffār, no known affiliation, fl. Mecca ca. 1515	461	C106

IV

Mathematical Astronomy in Medieval Yemen

From the ninth to the fifteenth century the astronomers of the Islamic world were pre-eminent. During these seven centuries numerous Muslim astronomers made new observations, compiled new tables, devised new instruments, and in general made progress in each of the various aspects of the science which were their concern.[1] A rather small fraction of the works that they compiled was translated into Latin and thus available to medieval Europe; the remainder was not known outside the Islamic world. These few Islamic works which were transmitted are now fairly well known to historians of science, but they hardly represent the scope of the Muslim activity in astronomy. The vast number of Islamic astronomical works that were not known in medieval Europe but which do reflect the Muslim activity in astronomy over several centuries are known mainly through modern studies of manuscripts.[2]

Medieval mathematical astronomy was concerned with such topics as the determination of the positions of the sun, moon, planets and the fixed stars; the prediction of planetary conjunctions, eclipses, and visibility of the lunar crescent; timekeeping by the sun and stars; and the computation of horoscopes and celestial configurations with astrological significance. The Islamic tradition of mathematical astronomy was based on the earlier traditions of Greek, Sasanian, and Indian astronomy, rather than the primitive folk astronomy of the Arabian peninsula. Since only a very small fraction of the works compiled by Muslim astronomers was known in medieval Europe, most of the vast corpus of Islamic astronomical literature has only become known in the West during the past 150 years, as a result of the labours of orientalists working on the manuscripts in which this heritage is in part preserved. This type of literature includes the *zīj*, i.e. astronomical handbooks containing extensive tables and instructions for solving standard problems confronting the medieval astronomer; tables for timekeeping by the sun and stars and for regulating the times of prayer, which in Islam are astronomically defined; treatises on astrology; and treatises on a wide range of other related topics. Of this literature the *zīj* and other tables for specific purposes such as timekeeping or constructing instruments are the most important category for the historian of science.[3]

There has been an active tradition in mathematical astronomy in the Yemen from the tenth century down to the present.[4] This is reflected in about one hundred medieval astronomical manuscripts of Yemeni provenance that have been located in the past few years in various libraries in Europe and the Near East, and furthermore the scope of the surviving astronomical writings suggests that other works were compiled that are now no longer extant. It must be stressed at the outset that we have very little evidence of original observations carried out in the Yemen, and that the Yemeni astronomical works are of interest to the history of science mainly because they incorporate earlier ‘Abbāsid or Fāṭimid material no longer extant in its original form. Also the Yemeni tradition of mathematical astronomy is distinct from the tradition of folk astronomy attested in the simple Yemeni almanacs on which there is already some published material.[5] This folk astronomy relates the seasons, whose passage is determined by the progress of the sun along the ecliptic, to agricultural and meteorological patterns.

The earliest Yemeni astronomer on whom we have any reliable information is the celebrated late tenth century geographer Al-Hamdānī. He compiled a *Zīj* which was used in the Yemen at least until the thirteenth century. Unfortunately this is no longer extant, but we know from a surviving fragment of al-Hamdānī's treatise on mathematical astrology that he was familiar with the earlier astronomical handbooks (*zīj*) of the astronomers of ‘Abbāsid Iraq. Most of these are likewise lost, and it is a complicated task for the historian of Islamic science to gather information on these works from such valuable sources as al-Hamdānī's chapter in his treatise on astrology dealing specifically with the different opinions of his predecessors.

Under the Rasūlid Sultans there was considerable activity in astronomy in the Yemen, some of which was conducted by the Sultans themselves. The two astronomers al-Fārisī and al-Kawāshī were apparently sponsored by the Sultan al-Muẓaffar. Al-Fārisī prepared a *Zīj* containing tables for the Yemen, based on a twelfth century Iraqi *zīj*, and al-Kawāshī prepared a *zīj* with tables specifically for Aden and Ta‘izz, recording observations which he had made himself in Egypt, and incorporating material from earlier Iraqi and Egyptian *zīj* handbooks.The Sultan al-Ashraf himself compiled an extensive treatise on the construction of astrolabes and sundials, displaying his knowledge of certain earlier Andalusian and Egyptian works on these instruments. He compiled new tables of coordinates for drawing the curves on astrolabe plates and the curves on horizontal sundials, computing these tables for the latitudes of the major cultural centres of the Yemen and the Hejaz. One of the astrolabes made by al-Ashraf with the aid of these tables is now preserved in the Metropolitan Museum of Art in New York. However, perhaps the most interesting feature of al-Ashraf's treatise is that it contains as an appendix a discussion of the magnetic compass: this is the earliest known reference to the compass in an Arabic astronomical text. Al-Ashraf also compiled an extensive treatise on astrology, in which he included astrological tables specifically computed for the latitude of Ṣan‘ā’.

The Yemeni astronomer Abu ’l-‘Uqūl worked for the Sultan al-Mu’ayyad. He compiled a *zīj* based on one of the *zīj* handbooks of the celebrated tenth century Cairo astronomer Ibn Yūnus, and included tables specifically computed for the latitudes of Aden, Ta‘izz, Zabīd, and Ṣan‘ā’. Some of the material lifted by Abu ’l-‘Uqūl from Ibn Yūnus is not extant in any of the manuscripts of Ibn Yūnus' own works.[6] Greater originality is displayed in the corpus of tables for timekeeping attributed to Abu ’l-‘Uqūl. These tables for timekeeping by the sun and stars, computed for the latitude of Ta‘izz, constitute the largest known corpus of such tables compiled for any Islamic city during the medieval period, being considerably more extensive than the corresponding tables that were prepared in the thirteenth and fourteenth centuries for such centres of astronomy as Cairo and Damascus. In fact, the Ta‘izz corpus contains over one hundred thousand entries, but it is difficult to estimate the extent to which these tables were used in later centuries since they survive in only two manuscripts whereas literally dozens of manuscripts of the Cairo and Damascus tables survive. No comparable corpus of tables for timekeeping appears to have been compiled for Ṣan‘ā’.

Each year almanacs and ephemerides were prepared for the Yemeni Sultans by their astronomers. These contained extensive calendrical and astrological information for the year in question, as well as tables of the positions of the sun, moon, and planets for each day of the year. Two of these survive in the manuscript sources: the first was prepared for Ṣan‘ā’ in 727 Hijrah (= 1326/7), and the second for Ta‘izz in 808 Hijrah (= 1405/06).

The Sultan al-Afḍal compiled an extensive compendium of astronomical treatises and tables, most of which were simpy lifted from earlier Egyptian, Syrian, and Yemeni sources. Such a compendium is of considerable interest to historians of science for the information it contains on earlier works that are no longer extant in their original form. Various other later Yemeni works of an eclectic nature survive either complete or in fragmentary form in the manuscript sources and remain to be properly studied, including *zīj* handbooks for Ṣan'ā', Zabīd, and Ta'izz.

The last Yemeni *zīj* handbooks were compiled for Ṣan'ā' in the mid-seventeenth century by the brothers al-Ḥasan and 'Abdullāh al-Sarḥī, relying entirely on earlier Yemeni *zīj* handbooks. In Ṣan'ā' today there is still a small group of elderly people who have received instruction on the *zīj* handbooks of the brothers al-Sarḥī.

Besides this sophisticated tradition of mathematical astronomy the Yemeni astronomers maintained an interest in traditional folk astronomy and simple timekeeping using shadow lengths by day and the lunar mansions by night. Thus, for example, the thirteenth century astronomer al-Fārisī wrote a treatise on these two topics as well as his *zīj*. In medieval Yemeni almanacs simple tables were sometimes given for reckoning time of night by the lunar mansions.

In the courtyard of the mosque of al-Janad north of Ta'izz there is a stone gnomon about the height of a man,* with which the time of day could be reckoned using simple rules originally adopted from Indian astronomy, and the time of the midday and afternoon prayers, both defined in terms of shadow lengths, could also be regulated. Using one eleventh-century Yemeni treatise on simple techniques of timekeeping with such a gnomon it has been possible to explain the origin of the definitions of the times of the daytime prayers in Islam.[7]

In the Yemen, as elsewhere in the Islamic world, mathematical astronomy declined from about the fifteenth century onwards. The works of the early Yemeni astronomers were thereafter forgotten and in some cases lost. Fortunately, enough Yemeni manuscripts survive in libraries in Europe and the Near East to enable us to document a substantial part of this tradition for the first time.

*This gnomon is known locally as 'Aṣā Mu'adh b. Jabal, the 'stick' of this celebrated Companion of the Prophet despatched by him to the Yemen [R.B.S.]

Notes

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1. For a brief account of Islamic astronomy see D. Pingree's article 'Ilm al-hay'ah' in the *Encyclopaedia of Islam*, 2nd edn. Additional basic information on Islamic astronomy is contained in the articles entitled Astrology, Manāzil (lunar mansions), Mīqāt (definitions of the prayer-times), Minṭaqah (zodiac), and Taysīr (aspects of mathematical astrology) in the 1st ed. of the *Encyclopaedia;* and Asṭurlāb (astrolabe) and Ḳiblah (direction of Mecca) in the 2nd. edn.

2. The basic bibliographical works dealing with the manuscript sources and the secondary literature relating to Islamic astronomy are H. Suter, 'Die Mathematiker und Astronomen der Araber und ihre Werke', *Abhandlungen zur Geschichte der mathematischen Wissenschaften,* X, 1900; C. Brockelmann, *Geschichte der arabischen Litteratur,* 2nd edn., Leiden, 1943-9, Supplementbände, 1937-42; and F. Sezgin, *Geschichte des arabischen Schrifttums,* Leiden, VI, Astronomie-Astrologie, to appear.

3. On Islamic astronomical handbooks and tables see E. S. Kennedy, 'A Survey of Islamic astronomical tables', *Transactions of the American Philosophical Society,* Philadelphia, n.s. XLVI, 1956, 123-77, and D. A. King, 'On the Astronomical Tables of the Islamic Middle Ages', *Studia Copernicana,* XII, Wroclaw, 1975, 37-56. Kennedy lists three Yemeni *zīj* handbooks namely, those of al-Hamdānī, Abu 'l-'Uqūl (described as Egyptian), and 'Abdullāh al-Sarḥī (not identified as Yemeni).The present count is sixteen.

4. Brief notices on the topic of this paper are to appear in *Bulletin de l'Institut d'Egypte*, Cairo, 1976, and R. B. Serjeant ed., *Ṣan'ā': an Arabian Islamic City*, forthcoming. A more detailed survey of all the available manuscript material entitled *Mathematical Astronomy in Medieval Yemen* is currently being submitted for publication.

5. On pre-Islamic Arabian starcults see J. Henninger, 'Über Sternkunde und Sternkult in Nord- und Zentralarabien', *Zeitschrift für Ethnologie*, LXXIX, Berlin, 1954, 82-117. On modern South Arabian astronomical folklore see R. B. Serjeant, 'Star-Calendars and an Almanac from South-West Arabia', *Anthropos,* Posieux (Fribourg), XLIX, 1954, 433-59 (with a valuable bibliography).

6. For an example of the importance of Yemeni sources for our knowledge of Ibn Yūnus' works see my article 'A Double-Argument Table for the Lunar Equation Attributed to Ibn Yūnus', *Centaurus*, XVIII, Copenhagen, 1974, 129-46.

7. A preliminary discussion is contained in my article 'A fourteenth century Tunisian sundial for regulating the times of Muslim Prayer', in Y. Maeyama and W. G. Saltzer, eds., *Prismata: Festschrift für Willi Hartner*, Wiesbaden, 1977, 187-202 and especially 193-5. A more detailed study of the origin of the prayer times is in preparation.

A Double-Argument Table for the Lunar Equation Attributed to Ibn Yūnus

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1. Introduction.
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3. On the attribution of the *ḥabṭaq* tables entitled *al-Taᶜdīl al-muḥkam* to Ibn Yūnus.
4. The lunar equation tables in the *Ḥākimī Zīj.*
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6. The *ḥabṭaq* lunar equation tables entitled *al-Taᶜdīl al-muḥkam.*
7. Ibn al-Majdī on *ḥabṭaq* tables.

1. Introduction

In this paper I describe a set of tables for finding the longitude of the moon, attributed to the tenth century Egyptian astronomer Ibn Yūnus.[1] The underlying lunar theory is that of Ptolemy, but these tables are so devised that the user is spared the calculations which are associated with Ptolemy's lunar tables. Ibn Yūnus' tables, if such they are, contain over 34,000 entries, and are preserved in a manuscript in the Egyptian National Library, Cairo. Another manuscript in the Forschungsbibliothek, Gotha, contains part of the original set. The tables are of interest as the earliest attempt by a medieval scholar to solve the computational problem of the determination of the lunar position according to the sophisticated Ptolemaic theory.[2]

Ptolemy's method for finding the lunar longitude λ involves taking the mean longitude $\bar{\lambda}$ from the mean motion tables and adjusting it by a correction δ called the lunar equation, thus:

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$$\lambda = \bar{\lambda} + \delta \tag{1}$$

The Ptolemaic lunar equation is a function of the mean lunar anomaly γ and double elongation 2η, that is, twice the elongation of the moon from the sun, both of which can easily be found from the mean motion tables for the sun and moon. In the *Almagest*, Ptolemy tabulates four auxiliary functions, here denoted by c_3, c_4, c_5, c_6 following the convention of O. Neugebauer, with which the equation can be found in two steps. The procedure is the following: first find the 'true anomaly' γ', using

$$\gamma' = \gamma + c_3(2\eta), \tag{2}$$

and then find the equation using

$$\delta(2\eta, \gamma) = c_4(\gamma') + c_5(\gamma') \cdot c_6(2\eta) \tag{3}$$

Whilst the computation of δ using these auxiliary functions is not very difficult, it can involve several interpolations as well as the additions and multiplication. For given γ, the function $\delta(2\eta, \gamma)$ varies non-linearly between $c_4(\gamma)$ at $2\eta = 0°$ and $c_4(\gamma) + c_5(\gamma)$ at $2\eta = 180°$. Note, however, that δ increases linearly between these two limits when expressed as a function of γ' and c_6. The functions c_4 and c_5 are zero for arguments 0° or 180°, since, for any value of 2η, $\delta(2\eta, \gamma)$ is zero when γ' equals either of these two values, but $\delta(2\eta, \gamma)$ is generally non-zero when γ equals 0° or 180°.

In the *Almagest*, Ptolemy tabulated the four auxiliary functions for each 6° of argument (and for each 3° near argument 90°) to two sexagesimal digits. In the *Handy Tables* the argument increment is 1° and the interpolation function c_6 is tabulated to only one digit.[3] Many medieval Muslim astronomers based their lunar equation tables on those in the *Handy Tables*, sometimes recomputing certain of them for new parameters. Thus, for example, from the early period of Islamic astronomy we have the lunar equation tables of Yaḥyā b. Abī Manṣūr (ca. 830) and al-Battānī (ca. 900), the first identical with those in the *Handy Tables* and the second slightly modified.[4] The lunar equation tables of Ḥabash (ca. 850) are adjusted so that the equation is always additive.[5]

It has already been noted that extensive double-argument tables for the lunar longitude or equation were compiled in the late Islamic period: with these tables no calculation beyond interpolation is necessary to find the true equation from the double elongation and the mean anomaly. In 1971 C. Jensen described a set of lunar tables containing 1,830 entries in the thirteenth century *Zīj* of al-Baghdādī.[6] In 1967 M. Tichenor noted the existence of a double-argument table for the lunar equation in the *Khāqānī Zīj* of the fifteenth century Samarqand astronomer al-Kāshī,[7] and more recently G. Saliba has analyzed another such table computed by the fifteenth century priest Qiryāqūs.[8]

Before discussing Ibn Yūnus' tables it is worth noting that two fifteenth century Egyptian astronomers, Ibn al-Majdī[9] and Shams al-Dīn al-Ṣūfī[10] also compiled extensive lunar equation tables. Ibn al-Majdī's lunar tables and similar tables for the sun and planets are based on the parameters of the mid-fourteenth century *Zīj* of Ibn al-Shāṭir.[11] These extensive tables are contained in a work entitled *al-Durr al-yatīm, the Unique Pearl*, which deserves study.[12] al-Ṣūfī, on the other hand, used the parameters of the fifteenth century *Zīj* of Ulugh Beg,[13] compiled in Samarqand, but his tables appear to be an extract from the more extensive lunar and planetary equation tables of the Damascus astronomer ᶜAbd al-Raḥmān al-Ṣāliḥī:[14] these are one of the largest known sets of tables from the Islamic period, containing over 170,000 entries.

2. *On the sources for Ibn Yūnus' lunar equation tables*

Ibn Yūnus' major works were the *Ḥākimī Zīj* (*Hakemite Tables*), a lengthy compendium on planetary and spherical astronomy, and at least part of the extensive tables known as *Kitāb Ghāyat al-intifāᶜ* (*Very Useful Tables*) for reckoning time by the sun and regulating the astronomically defined times of Muslim prayer.

Only three quarters of the original *Ḥākimī Zīj* is extant in the known manuscript sources. Recent research has shown, however, that considerable material due to Ibn Yūnus, including some material not contained in the extant chapters of the *Zīj*, is located in later *zījes* compiled in Egypt, Persia, and the Yemen.[15] Also, preliminary examination of these sources reveals that Ibn Yūnus compiled several *zījes* rather than just the *Ḥākimī Zīj*. For example, the Yemeni *Mukhtār Zīj*[16] is apparently based almost entirely on a *Zīj* of Ibn Yūnus other than the *Ḥākimī Zīj*.

The *Very Useful Tables*, extant in several manuscript copies, consist of about 200 pages of tables each containing 180 entries, computed with remarkable accuracy. Some of these may have been computed by the thirteenth century Egyptian astronomer al-Maqdisī.[17] The functions tabulated relate to spherical astronomy, such as the hour-angle and solar azimuth as functions of solar altitude and longitude, as well as to the times of Muslim prayer. These tables were the model for virtually all later medieval tables of this kind in Egypt and Syria.

The sources for the present study are the *Ḥākimī Zīj* itself, the Yemeni *Mukhtār Zīj*, and other manuscripts preserved in Cairo and the Yemen. I discuss three sets of tables, all of which are based on Ibn Yūnus' lunar parameters. The entries in the third set are shown in the sequel to be based on those in the second set, which in turn were derived from the entries in the first set.

(1) Regular lunar equation tables, contained in the *Ḥākimī Zīj*, MS Leiden Or. 143, pp. 175–177.
(2) Auxiliary lunar equation tables, contained in the Yemeni *Mukhtār Zīj*, MS British Museum 768 (Or. 3624), fols. 133v–151r. An abstract of these tables is contained in MS Cairo Dār al-Kutub, *miqāt* 39M, and a manuscript of an astronomical compendium by the Yemeni Sultan al-Afḍal al-ᶜAbbās ibn ᶜAlī (ca. 1375), preserved in the private library of Qadi Ismail al-Akwa, Sanaa.
(3) Tables of the true lunar equation, known as *ḥabṭaq* tables, contained in a work entitled *al-Taᶜdīl al-muḥkam*, MS Cairo Dār al-Kutub, *miqāt* 29.[18] MS Gotha Forschungsbibliothek 1410 contains a fragment of these tables.[19]

In the manuscripts all entries are written in the standard Arabic sexagesimal notation.[20]

3. *On the attribution of the ḥabṭaq tables entitled al-Taᶜdīl al-muḥkam to Ibn Yūnus*

A few words on the attribution of the third set of tables to Ibn Yūnus are necessary. In the title of the undated Cairo manuscript the Egyptian astronomer is stated as the compiler; however, this title is not in the hand of the copyist of the manuscript. It reads *Kitāb Ḥabṭaq ḥall al-shams wa-l-qamar maḥlūl l-Ibn Yūnus al-Miṣrī al-musammā bi-l-taᶜdīl al-muḥkam*, that is, *Ḥabṭaq Tables of Solar and Lunar Motion Computed by Ibn Yūnus the Egyptian and Called "The True Equation."* In the Gotha manuscript there is neither title nor ascription, but the incomplete lunar equation tables follow a set of Ibn Yūnus' solar azimuth tables.

The word *ḥabṭaq* is new to the modern literature on Islamic astronomy. However, it appears that its meaning, and the kind of tables to which it refers, were well known to medieval astronomers at least in Egypt and Syria. Ibn al-Majdī, referred to in Section 1, discusses the compilation of *ḥabṭaq* tables in some detail. Part of his discussion is presented in Section 7, and it will be clear that he had seen the tables we describe in Section 6, but that he did not know who their compiler was; that he apparently did not realize that the tables were based on Ibn Yūnus' parameters; and that he did not approve of the way in which they had been compiled.

In two other manuscripts of lunar equation tables, probably both due to the Syrian astronomer al-Sāliḥī referred to in Section 1, the tables are called *ḥabṭaq* tables.[21]

In the brief introduction on the title folio of the Cairo manuscript, reference is made to two sets of solar equation tables, one set being such that the equation is always additive. Neither of these tables is contained in the manuscript now, or in any other known Arabic sources. However, a table of an additive solar equation, based on Ibn Yūnus' value of 2;0,30° for the maximum equation, exists in three Byzantine manuscripts associated with Gregory Chioniades[22] (ca. 1300). Since this parameter was also used in the *Ilkhānī Zīj* of Naṣīr al-Dīn al-Ṭūsī[23] (ca. 1250), it may be that the Byzantine table was derived from Persia rather than from Egypt. However, the associated solar mean motion tables are based on an epoch date equivalent to 1093, and so the Byzantine solar equation tables may be those of Ibn Yūnus.

Following the introduction there is a set of solar mean motion tables attributed to Ibn Yūnus: indeed the entries are those of the *Ḥākimī Zīj* for the period from 990 to 1320 Hijra, that is from 1582 to 1902 A.D.[24] This suggests that the manuscript dates from the early part of this interval. There are no longer any lunar mean motion tables in the manuscript.

At present we have a very unclear picture of the development of astronomy in medieval Egypt, and also of the transmission of early Islamic material to Byzantine astronomers.[25] However, I see no reason yet to doubt the attribution of all of the lunar equation tables described in the sequel to Ibn Yūnus.

4. *The lunar equation tables in the Ḥākimī Zīj*

Ibn Yūnus' tables of the lunar equation in the *Ḥākimī Zīj* have the same format as those in the *Handy Tables*, and the earlier *Mumtaḥan Zīj* of

Yaḥyā b. Abī Manṣūr: the auxiliary functions are tabulated in columns according to the order c_3, c_6, c_4, and c_5 and are computed for each integral degree of argument. The functions differ from those in the *Mumtaḥan Zij* in that c_4 is computed to three sexagesimal digits rather than two, and the values of the lunar epicycle radius and deferent eccentricity, here denoted by r and s, have been changed[26] to

$$r = 5;1,14,23 \quad \text{and} \quad s = 11;7$$

as compared with the Ptolemaic values

$$r = 5;15 \quad \text{and} \quad s = 10;19.$$

A feature of the tables is that the values of the function c_3 are based on the Ptolemaic value of s: the entries are identical to those in the *Mumtaḥan Zij* and the *Handy Tables*. Likewise, Ibn Yūnus' values of the function c_6 are originally from the *Handy Tables*.

The function designated by E. S. Kennedy as c_7 is also tabulated by Ibn Yūnus: this function is a correction to be applied to the lunar orbital longitude to give the ecliptic longitude.[27] Ibn Yūnus simply copied the table of c_7 compiled by Yaḥyā, not recomputing it for his different value of the maximum lunar latitude.

Ibn Yūnus' lunar equation tables are characterized by the following maxima, for which the corresponding arguments are shown in parentheses:

c_3	13;8°	(113°–115°)
c_4	4;48,0°	(95°)
c_5	2;52°	(105°–108°)
c_7	0;6,0°	(42°–48°)

5. *The lunar equation tables in the Mukhtār and Muṣṭalaḥ Zījes*

The Yemeni *Mukhtār Zīj* contains these same lunar equation tables (MS B.M. 768, fols. 129v–132r), and also another table (fols. 133v–151r) based on them. This second table, not contained in any other source known to me, displays values of the function

$$\delta'(\gamma', c_6) = c_4(\gamma') + c_5(\gamma') \cdot c_6$$

with γ' as horizontal and c_6 as vertical arguments. Values are given for the domains

$$\gamma' = 1°, 2°, \ldots\ldots\ldots\ldots, 180°$$
$$c_6 = 0;0, 0;1, \ldots\ldots\ldots, 1;0,$$

and are computed to three sexagesimal digits. Thus the table contains nearly 11,000 entries, the computation of which is as simple as it is tedious.

The table is entitled *al-taᶜdīl al-muṣaḥḥaḥ*, "true (lunar) equation", and the vertical argument is labelled *daqā'iq al-ṣaff al-thānī*, "minutes of the second column", that is, c_6. At the head of each column are marked the arguments γ' and $(360°-\gamma')$, and the units of the entries are indicated by the standard abbreviations *j*, *q*, *y* for *daraj*, *daqā'iq* and *thawānī*, that is, degrees, minutes and seconds. The values of $c_5(\gamma')$ are written above the true anomaly arguments, and the first entry in each column, namely for $c_6 = 0$, is $c_4(\gamma')$. Clearly the entry for $c_6 = 0;n$ is simply

$$c_4 + \frac{n \cdot c_5}{60}.$$

A given pair of facing pages of the manuscript contains entries for ten consecutive values of γ', those for $c_6 = 0;1, 0;2, \ldots, 0;30$ being tabulated on the right-hand page and those for $c_6 = 0;31, 0;32, \ldots, 1;0$ on the left-hand page. The entries in this table were probably used by Ibn Yūnus to compile those in the third table described below.

MS Cairo Dār al-Kutub, *miqāt* 39M, and the manuscript of Qadi Ismail al-Akwa, Sanaa contain a table based on the one in the *Mukhtār Zīj*. The function tabulated is $\delta'(c_6, \gamma')$, with the interpolation function c_6 as horizontal argument and γ' as the vertical argument. Values are given for each 0;5 of c_6, differences for each 0;1, 0;2, 0;3, and 0;4 being given to the side of each part of the main table. In the Yemeni manuscript it is stated that this table is from the *Muṣṭalaḥ Zīj*.

This *Zīj* was a thirteenth century Egyptian compilation, based on a number of sources including Ibn Yūnus' *Ḥākimī Zīj*.[28] Neither of the extant versions of the work, MSS Paris Bibliothèque Nationale ar. 2520 and 2513, contain this table, although an additional folio in the latter (fol. 21v) does contain instructions for its use:

> To find the true equation (*al-taᶜdīl al-muḥkam*), enter the true anomaly [that is, γ'] vertically and the minutes of the interpolation factor [that is, c_6] horizontally: the entry is the true equation.

These manuscripts await detailed study to establish their precise relation to Ibn Yūnus. In both sources, however, the lunar equation tables are the same as those described in Section 4.

6. *The ḥabṭaq lunar equation tables entitled al-Taᶜdīl al-muḥkam*

The tables in the work *al-Taᶜdīl al-muḥkam* give values of the true lunar equation $\delta(2\eta, \gamma)$. With such a table there is no need to find the true anomaly γ'; indeed, the lunar equation is given immediately from the two arguments 2η and γ which can easily be found from the tables of mean lunar motion. The lunar equation is tabulated to two sexagesimal digits for the domains

$$2\eta = 1°, 2°, \ldots, 180°$$
$$\gamma = 1°, 2°, \ldots, 360°.$$

At the head of each table is marked the double elongation, 2η, referred to as *al-markaz*, literally 'center', for values between $0^s\,1°$ and $6^s\,0°$. At the foot of each table the argument $(180° - 2\eta)$ is marked. Note that the function is symmetrical about $2\eta = 6^s\,0° = 180°$. A given table may serve a single, or two, or even three consecutive values of 2η, for a reason which will become clear below. In fact, there are 95 tables rather than 180; the total number of entries is 34,200. The entries in each table are arranged in twelve columns of thirty, and the vertical argument, the mean anomaly, called *al-khāṣṣa*, is expressed in signs and degrees. The vertical argument is to be read downwards from 1° to 30° for $2\eta < 180°$ and upwards from 0° to 29° for $2\eta > 180°$. The function tabulated assumes both positive and negative values, and the sign is indicated by the letters *d* and *ṣ* for *zā'id* and *nāqiṣ*, additive and subtractive. In the instructions it is noted that one should be particularly careful in the anomaly columns for 5^s and 11^s since the equation changes direction there.

The table of $\delta(2\eta, \gamma)$ was apparently compiled from those described above, in the following way. For a given value of 2η, the regular tables of the equation in the *Ḥākimī Zīj* give the correction $c_3(2\eta)$, and also the interpolation function $c_6(2\eta)$. The value $c_3(2\eta)$ is added to each value of γ to form the true anomaly γ'. With γ' and c_6 as arguments, one finds $\delta'(\gamma', c_6)$ in the *Mukhtār* table and enters this value in the new table as a function of 2η and γ.

Notice that although γ is an integral number of degrees γ' will generally not be, and one must interpolate in the table of $\delta'(2\eta, c_6)$. By rounding the non-integral part of γ' to a convenient fraction the interpolation is much facilitated. Also, since c_6 is given to only one sexagesimal digit, several consecutive values of 2η yield the same value of c_6. In fact, those values of 2η for which

(a) $c_6(2\eta)$ is the same,

and

(b) there is a convenient approximation for the consecutive values of $c_3(2\eta)$,

have been grouped together, and a single table serves these consecutive values of 2η . This procedure reduces the total number of entries needed from 180 × 360 to 95 × 360 = 34,200, as noted above.

To illustrate this further, Column I in Table 1 shows those values of 2η for which a single table is given. Columns II and III show the corresponding values of $c_6(2\eta)$ and $c_3(2\eta)$ taken from the *Ḥākimī Zīj*. Column IV shows an approximate value of c_3 which might have been used to serve all the values of 2η. This arrangement appears at first sight rather arbitrary, but little accuracy is sacrificed, and one is spared computing another 85 × 360 entries.

The above hypothesis about the computation of the entries is borne out by the entries I have recomputed by hand. Once one has mastered the technique it is not difficult to generate the values quickly. Consider for example the entry for arguments

$$2\eta = 0^s\,18° = 18° \quad \text{and} \quad \gamma = 1^s\,13° = 43°.$$

For this value of 2η we have from the *Ḥākimī Zīj*

$$c_3 = 2;39° \quad \text{and} \quad c_6 = 0;1.$$

Using a modified value of 2;30° for c_3 (cf. Table 1), we have

$$\gamma' = 45;30°.$$

Now from the *Mukhtār* tables

$$\delta'(45;0°, 0;1) = 3;13,25°$$

and

$$\delta'(46;0°, 0;1) = 3;16,58°.$$

Table 1

Distribution of the Horizontal Argument (2η) in the Table of $\delta(2\eta, \gamma)$

Column I: Values of the argument 2η which head a given table
Column II: Corresponding value of c_6 in the *Ḥākimī Zīj*
Column III: Corresponding value of c_3 in the *Ḥākimī Zīj*
Column IV: Approximate value of c_3 to facilitate the computation of $\delta(2\eta, \gamma)$

	I	II	III	IV
1st table........................	0^s 1°	0	0; 9°	0;10°
2nd table........................	0 2	0	0;18	0;20
3rd table........................	0 3	0	0;27	0;30
	0 4	0	0;36	
4th table........................	0 5	0	0;44	0;50
	0 6	0	0;53	
	0 7	0	1; 2	
5th table........................	0 8	0	1;11	1;15
	0 9	0	1;20	
6th table........................	0 10	0	1;29	1;30
	0 11	0	1;38	
7th table........................	0 12	0	1;46	1;50
	0 13	0	1;55	
8th table........................	0 14	0; 1	2; 4	2;10
	0 15	0; 1	2;13	
9th table........................	0 16	0; 1	2;22	2;30
	0 17	0; 1	2;31	
	0 18	0; 1	2;39	
10th table........................	0 19	0; 2	2;48	2;45
11th table........................	0 20	0; 2	2;57	3;10
	0 21	0; 2	3; 5	
	0 22	0; 2	3;14	
12th table........................	0 23	0; 2	3;23	3;30
	0 24	0; 2	3;31	
13th table........................	0 25	0; 3	3;40	3;45
	0 26	0; 3	3;49	
34th table........................	2 5	0;15	9;15	9;20
	2 6	0;15	9;22	
	2 7	0;15	9;30	
35th table........................	2 8	0;16	9;37	9;40
	2 9	0;16	9;44	
92nd table........................	5 24	1; 0	2;11	2; 0
	5 25	1; 0	1;50	
93rd table........................	5 26	1; 0	1;29	1;30
94th table........................	5 27	1; 0	1; 7	1; 0
	5 28		0;45	
95th table........................	5 29	1; 0	0;23	0;10
	6 0	1; 0	0; 0	

Thus

$$\delta'(45;30°, 0;1) = 3;15°,$$

which is the entry for $\delta(18°, 43°)$.

Next consider the entry for arguments

$$2\eta = 2^s\,7° = 67° \quad \text{and} \quad \gamma = 3^s\,5° = 95°.$$

For this value of 2η the *Ḥākimī* tables have

$$c_3 = 9;15° \quad \text{and} \quad c_6 = 0;15.$$

Using a modified value of 9;20° for c_3 (cf. Table 1) we have

$$\gamma' = 104;20°.$$

From the *Mukhtār* tables

$$\delta'(104°, 0;15) = 5;27,3°$$

and

$$\delta'(105°, 0;15) = 5;26,26°,$$

so that

$$\delta'(104;20°, 0;15) = 5;27°,$$

which is the entry for $\delta(67°, 95°)$.

Again consider the entry for the arguments

$$2\eta = 5^s\,24° = 174° \quad \text{and} \quad \gamma = 10^s\,3° = 303°.$$

We have from the *Ḥākimī* tables

$$c_3 = 2;11° \quad \text{and} \quad c_6 = 1;0,$$

but we use the value 2;0° for c_3 (cf. Table 1), so that

$$\gamma' = 305°.$$

From the *Mukhtār* tables

$$\delta'(305°, 1;0) = 5;48°,$$

which is the entry for $\delta(174°, 303°)$.

Whilst the entries in the table of $\delta(2\eta, \gamma)$ can be generated with considerable facility, it does not follow that every single value was computed in this way. Clearly in such an extensive table the accuracy of a given entry is controlled by the adjacent entries. Finally, it may be that the auxiliary table of $\delta'(\gamma', c_6)$ was not used by Ibn Yūnus to compute the table of $\delta(2\eta, \gamma)$.

A program was compiled at the Yale Computer Center[29] to check the computational accuracy of Ibn Yūnus' table of $\delta(2\eta, \gamma)$, and to investigate the errors introduced by not tabulating values for each individual value of 2η. The program incorporated the instruction that the functions c_3 and c_6 be based on Ptolemaic parameters. Tables 2 and 3 show sample entries from Ibn Yūnus' tables for arguments

$$2\eta = 0^s\,16°,\ 0^s\,17° \quad \text{and} \quad 0^s\,18°$$

and

$$2\eta = 2^s\,5°,\ 2^s\,6° \quad \text{and} \quad 2^s\,7°,$$

and the corresponding recomputed values. The difference between Ibn Yūnus' entries and the recomputed values is generally zero or ±0;1°, and occasionally ±0;2° or ±0;3°. It should, however, be remembered that *al-Taᶜdīl al-muḥkam* and the recomputed values are based on a mutually inconsistent set of auxiliary functions c_3, c_4, c_5, and c_6.

Table 2

Sample Entries from *al-Taᶜdīl al-muḥkam* for $2\eta = 0^s16°, 0^s17°, 0^s18°$, Compared with Recomputed Values of $\delta(2\eta, \gamma)$ for $2\eta = 0^s16°$, $2\eta = 0^s17°$, and $2\eta = 0^s18°$

	0^s	1^s	2^s	3^s	4^s	5^s	6^s	7^s	8^s	9^s	10^s	11^s
1°.....	−0;16°	−2;29	−4;10	−4;51	−4;14	−2;20	+0;19	2;53	4;30	4;48	3;51	+2; 0
	−0;16	−2;29	−4;10	−4;51	−4;14	−2;21	+0;19	2;52	4;29	4;48	3;51	+2; 1
	−0;16	−2;30	−4;10	−4;51	−4;14	−2;20	+0;19	2;53	4;30	4;48	3;51	+2; 1
	−0;17	−2;30	−4;11	−4;51	−4;14	−2;20	+0;20	2;53	4;31	4;48	3;51	+2; 0
2°.....	−0;21	−2;34	−4;12	−4;51	−4;11	−2;15	+0;25	2;57	4;32	4;47	3;48	+1;56
	−0;20	−2;33	−4;12	−4;51	−4;11	−2;16	+0;24	2;56	4;31	4;47	3;49	+1;57
	−0;21	−2;34	−4;13	−4;51	−4;11	−2;16	+0;25	2;57	4;32	4;47	3;48	+1;56
	−0;22	−2;35	−4;13	−4;51	−4;11	−2;15	+0;26	2;58	4;32	4;48	3;48	+1;56
3°.....	−0;25	−2;38	−4;14	−4;51	−4; 8	−2;11	+0;31	3; 1	4;34	4;46	3;45	+1;52
	−0;25	−2;37	−4;14	−4;51	−4; 9	−2;11	+0;30	3; 0	4;33	4;46	3;46	+1;52
	−0;26	−2;38	−4;15	−4;51	−4; 9	−2;11	+0;30	3; 1	4;34	4;46	3;45	+1;52
	−0;26	−2;39	−4;16	−4;51	−4; 9	−2;10	+0;31	3; 2	4;34	4;47	3;45	+1;51
15°....	−1;21	−3;22	−4;38	−4;44	−3;29	−1; 9	+1;35	3;47	4;48	4;[29][a]	3; 4	+0;58
	−1;20	−3;22	−4;38	−4;44	−3;30	−1; 9	+1;34	3;46	4;48	4;30	3; 5	+0;59
	−1;21	−3;22	−4;38	−4;44	−3;29	−1; 9	+1;35	3;47	4;49	4;30	3; 5	+0;58
	−1;22	−3;23	−4;39	−4;44	−3;29	−1; 8	+1;36	3;48	4;49	4;30	3; 4	+0;58
30°....	−2;25	−4; 7	−4;50	−4;17	−2;25	+0;14	+2;48	4;28	4;49	3;54	2; 4	−0;11
	−2;25	−4; 7	−4;51	−4;17	−2;26	+0;13	+2;47	4;27	4;49	3;54	2; 5	−0;11
	−2;26	−4; 8	−4;51	−4;17	−2;25	+0;14	+2;48	4;28	4;49	3;54	2; 5	−0;12
	−2;26	−4; 8	−4;51	−4;17	−2;25	+0;15	+2;49	4;29	4;49	3;54	2; 4	−0;12

[a] MS: 39

Table 3

Sample Entries from *al-Taᶜdīl al-muḥkam* for $2\eta = 2^s5°, 2^s6°, 2^s7°$ Compared with Recomputed Values of $\delta(2\eta, \gamma)$ for $2\eta = 2^s5°$, $2\eta = 2^s6°$, and $2\eta = 2^s7°$

	0^s	1^s	2^s	3^s	4^s	5^s	6^s	7^s	8^s	9^s	10^s	11^s
1°......	−0;54°	−3;19	−4;59	−5;29	−4;28	−2; 3	+1; 6	3;51	5;21	5;17	3;56	1;41
	−0;53	−3;17	−4;59	−5;29	−4;28	−2; 3	+1; 5	3;50	5;20	5;17	3;56	1;42
	−0;54	−3;19	−5; 0	−5;30	−4;29	−2; 3	+1; 6	3;52	5;21	5;18	3;56	1;41
	−0;55	−3;20	−5; 2	−5;31	−4;30	−2; 3	+1; 7	3;53	5;23	5;19	3;56	1;41
2°......	−0;59	−3;24	−5; 1	−5;29	−4;25	−1;57	+1;12	3;56	5;22	5;15	3;53	1;36
	−0;58	−3;22	−5; 1	−5;28	−4;25	−1;57	+1;11	3;54	5;21	5;16	3;52	1;37
	−0;59	−3;23	−5; 3	−5;29	−4;25	−1;57	+1;12	3;56	5;23	5;17	3;52	1;36
	−1; 0	−3;24	−5; 4	−5;31	−4;26	−1;57	+1;13	3;57	5;24	5;18	3;53	1;36
3°......	−1; 4	−3;28	−5; 3	−5;28	−4;21	−1;51	+1;18	4; 0	5;24	5;14	3;49	1;31
	−1; 4	−3;26	−5; 3	−5;28	−4;21	−1;51	+1;17	3;59	5;22	5;14	3;48	1;32
	−1; 4	−3;27	−5; 5	−5;29	−4;22	−1;51	+1;19	4; 0	5;24	5;15	3;48	1;31
	−1; 5	−3;28	−5; 6	−5;30	−4;22	−1;51	+1;20	4; 2	5;25	5;16	3;49	1;31
15°.....	−2; 5	−4;13	−5;23	−5;13	−3;28	−0;37	+2;30	4;44	5;30	4;47	2;59	0;29
	−2; 4	−4;12	−5;23	−5;12	−3;29	−0;37	+2;29	4;43	5;30	4;48	2;58	0;30
	−2; 5	−4;13	−5;25	−5;13	−3;30	−0;36	+2;30	4;45	5;31	4;48	2;58	0;30
	−2; 6	−4;15	−5;26	−5;14	−3;30	−0;35	+2;31	4;46	5;32	4;49	2;58	0;29
30°.....	−3;15	−4;57	−5;30	−4;30	−2; 9	+1; 0	+3;47	5;19	5;18	4; 0	1;46	0;49
	−3;13	−4;56	−5;29	−4;32	−2; 9	+0;59	+3;46	5;18	5;19	3;59	1;47	0;48
	−3;14	−4;58	−5;30	−4;32	−2; 9	+1; 0	+3;47	5;20	5;20	4; 0	1;46	0;49
	−3;15	−4;59	−5;32	−4;33	−2; 8	+1; 1	+3;49	5;21	5;21	4; 0	1;46	0;50

7. *Ibn al-Majdī on ḥabṭaq tables*

In a work entitled *Kitāb al-Tashīl wa-l-taqrīb fī l-ḥall wa-l-tarkīb*, *On Simplifying the Compilation of Equation Tables and on Approximate Methods*, Ibn al-Majdī describes the compilation of *ḥabṭaq* tables. I have examined MS Paris Bibliothèque Nationale ar. 2531, fols. 80r–115r of this work.[30] Ibn al-Majdī's discussion is preliminary to a detailed description of his own lunar equation tables in the work entitled *al-Durr al-yatīm*, mentioned in Section 1.[31] The following passage is of particular relevance to the present discussion (fols. 82v–83r):

> "There is another method for finding the lunar equation called the *ḥabṭaq*. It is very well known and is easier to use than the first method: it involves preparing 180 pages followed by the half degrees of the double elongation (?) and writing at the top the arguments from 1° to $6^s0°$ and at the bottom the arguments from $11^s29°$ to $6^s0°$. The $6^s0°$ arguments are on the last page, and the sum of the arguments on the top and bottom of each page is 12^s. However, in the tables which I have come across, the compiler put on some pages one degree, on others two degrees together, and on others three degrees, completely without any proportion or order. Perhaps the person who compiled the tables prepared one page of tables instead of two or three in cases when the second equation [c_6] was the same for the different arguments and the first equation [c_3] was similar (*mutaqārib*) for these arguments.
>
> If the person who made this table had prepared it for each degree of argument as we stated before, or for each two, or three, or six degrees, in an ordered fashion, it would have been preferable and more exact to use in finding the lunar equation for a given double elongation argument. I do not know why this table was prepared in such a way, resting neither on the second equation [c_6] nor on the first equation [c_3]. It was obviously prepared for the sake of brevity and as a standby."

Thus in the early fifteenth century the leading astronomer in Egypt did not know who had compiled the *ḥabṭaq* tables, and was unable to utter any tribute to the ingenuity of his predecessor.

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NOTES

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2. Ptolemy's lunar theory is outlined in Almagest V. Cf. Manitius [1], 1, 259–288, and also Neugebauer [1], 192–198 and Kennedy [1], 141–142.
3. Cf. Halma [1], II, 78 ff and Stahlman [1], 249–254.
4. These tables are contained in MS Escorial ar. 927, fols. 20v–23r (cf. Kennedy [1], no. 51) and Nallino [1], II, 78–83 (cf. Kennedy [1], no. 55). On al-Battānī's tables see also Toomer [1], 58.
5. Cf. Kennedy-Salam [1], 496–497 for an analysis.
6. Cf. Jensen [1] and Kennedy [1], no. 3.
7. Cf. Tichenor [1] and Kennedy [1], no. 20.
8. Cf. Kennedy [1], no. 81.
9. On Ibn al-Majdī see Suter [1], no. 432; Brockelmann [1], II, 153–155 and SII, 158–159; Kennedy [1], no. 36; and Azzawi [1], 179–184.
10. On al-Ṣūfī see Suter [1], no. 447 and Kennedy [1], no. 37. I have examined the tables in MS Princeton Yahuda 3262, a work entitled *Sullam al-manāra*, in which the equation is tabulated as a function of 2η and γ for each 6° interval of 2η and each 1° of γ.
11. Cf. Kennedy [1], no. 11.
12. I have examined MS Yale 1475, an edition of the tables attributed to Sulaymān b. Ḥamza b. Bakhshīsh. MSS Cairo Dār al-Kutub, *mīqāt* 25M and 26M contain only the lunar tables. The *Zīj al-shams wa-l-qamar* attributed to Ibn al-Majdī (MS Cairo Dār al-Kutub, *mīqāt* 44M; cf. Kennedy [1], no. 36) contains only tables from the *Durr al-yatīm*. See also notes 30 and 31 below.
13. Cf. Kennedy [1], no. 12.
14. On al-Ṣāliḥī see Suter [1], no. 454. I have examined the lunar tables in MS Princeton Yahuda 4402, in which the equation is tabulated for each 2° of 2η and each 1° of γ, and MS Cairo Azhar *falak* 4386, in which the planetary equations are tabulated for each 6° of apogee distance and each 1° of mean anomaly. Both sets are part of a work entitled *al-Ṭirāz al-muᶜlam fī tashīl al-muḥkam*, which may be identical with the work *al-Durr al-naẓīm* listed by Suter. See also note 21 below.

   Two sets of tables even larger than those of al-Ṣāliḥī are the following. Firstly, the tables of Najm al-Dīn al-Miṣrī giving the time since rising for any latitude as a function of solar or stellar altitude: these are extant in MSS Cairo Dār al-Kutub *mīqāt* 132M and Oxford Bodleian Marsh 672 and contain over 250,000 entries. Secondly, the anonymous sexagesimal multiplication table extant in MS Paris Bibliothèque Nationale ar. 2552: this contains 216,000 entries.

15. On the *Ḥākimī Zīj* see Caussin [1], and King [1], currently being revised for publication. A detailed analysis of the *Very Useful Tables* is contained in King [2].
16. Cf. Kennedy [1], no. 57 (this *Zīj* is not Egyptian) and King [1], 46–47.
17. On al-Maqdisī see Suter [1], no. 383. In a paper currently in preparation I discuss the various editions of the *Very Useful Tables*, as well and describe all known medieval tables for regulating the times of Muslim prayer.
18. I have not been able to consult the actual manuscript, but have used a microfilm of it available from the Arab League, Institute of Arabic Manuscripts, Cairo. Cf. Kunitzsch [1], 21 for a brief description.
19. Only tables for arguments $1^{s}1^{\circ}$ to $2^{s}13^{\circ}$ are found in this source (fols. 37r ff.). The first part of the manuscript (fols. 1v–36v), which is in a different hand, consists of Ibn Yūnus' solar azimuth tables. Cf. King [2], Section 3.3 for an analysis of these.
20. On which see Irani [1].
21. MS Oxford Bodleian Nic. 238,6, which I have not consulted, contains lunar equation tables due to al-Ṣāliḥī of Damascus entitled *Muḥkam al-qamar* and *Ḥabṭaq al-qamar*. MS Cairo Dār al-Kutub, *mīqāt* 92 is entitled *Zīj al-ḥabṭaq fī l-ᶜamal bi-l-shams wa-l-qamar ᶜalā uṣūl al-raṣad al-jadīd* and contains an extract from a *ḥabṭaq* table, possibly al-Ṣāliḥī's, based on the parameters of Ulugh Beg.

    W. Hartner has noted the use of the term *ḥiḅtaq* in the text and tables contained in al-Dimyāṭi's commentary on al-Kawm al-Rīshi's recension of the *Zīj* of Ibn al-Shāṭir. Cf. Hartner-Ruska [1], 55–56/208–209. There are no *ḥabṭaq* tables in the manuscripts of al-Kawm al-Rīshi's work *al-Lumᶜa* which I have examined. On this author see Suter [1], no. 428.
22. Cf. Pingree [1], 143–144 and 150. I owe these references to O. Neugebauer and D. Pingree.
23. Cf. Kennedy [1], no. 6.
24. In the *Ḥākimī Zīj* there are mean motion tables for the period from the Hijra epoch (622 A.D.) to 2700 Hijra (3240 A.D.).
25. On Islamic material in Byzantine sources see, for example, Neugebauer [2], Pingree [1], and Kunitzsch [2].
26. In Chapter 56 of the *Ḥākimī Zīj* the new value of r is mentioned but Ibn Yūnus uses Ptolemy's value of s (e.g., MS Paris Bibliothèque Nationale ar. 2496, fols. 61v–62r). The new value of s is deduced from the tables in the *Ḥākimī Zīj*, and is also attributed to Ibn Yūnus in an anonymous Yemeni *Zīj* (MS Paris Bibliothèque Nationale ar. 2523, fol. 37v).
27. Cf. Kennedy - Salam [1], 495–496.
28. Cf. Kennedy [1], no. 47; King [1], 44–45; and Wiedemann [1], 189/266.
29. A grant for computer time from the Yale University Department of Near Eastern Languages and Literatures is gratefully acknowledged.
30. The work bears no title, but its contents correspond to those of a work bearing this title and extant as MS Jakarta 624, described in van Ronkel [1], 372–373. I have not been able to consult MS Cairo Dār al-Kutub, *mīqāt* 131, of Ibn al-Majdī's work.
31. In discussing another method of computing the lunar position (fol. 85r of the Paris manuscript), Ibn al-Majdī refers to some lunar tables computed by an individual named Ḥasan Shāh al-Baghdādī. He states that these had been originally computed up to 813

Hijra (= 1410 A.D.), and someone else had added extra entries to bring the tables up to date. Ibn al-Majdī's opinion of these tables was that they should be used as fly-leaves. After spending some time trying to understand how the tables had been put together he decided to compute some of the same kind using the parameters of Ibn Yūnus, to be in accordance with the *Muṣṭalaḥ Zīj*. Elsewhere in his treatise (fol. 95v) Ibn al-Majdī states that he also computed equation tables for the sun, moon, and planets using the parameters of Ibn al-Shāṭir. These equation tables of Ibn al-Majdī are of the kind contained in his work *al-Durr al-yatīm* (see note 12 above), and do not concern the present study.

Ibn al-Majdī's Tables for Calculating Ephemerides

David A. King* & E. S. Kennedy**

1. *Introduction*

This study describes a category of late medieval astronomical tables hitherto neglected by historians of science. These tables, however, were preceded by a group of related works applied to the sun and moon only, which has received attention in the literature, in item [7] in the bibliography which follows this paper. The anonymous originator of the lunar tables of [7] made use of an ancient Babylonian period relation to work out a technique for obtaining quickly a set of true longitudes of the moon. Presumably the lunar tables were known to the later Egyptian astronomer Ibn al-Majdī, who seems to have applied the same basic notion to the calculation of planetary positions also. The resulting corpus of manuscripts provides still another example of how the scientists of medieval Islam continually sought, without tampering with the underlying Ptolemaic abstract models, to ease the computations of the practising astronomer-astrologer. The general trend is amply illustrated in such papers as [10], [17], [5], [11], and [15].

Shihāb al-Dīn Abu'l-ᶜAbbās Aḥmad b. Rajab b. Ṭībughā, known as Ibn al-Majdī, was the leading astronomer of Cairo in the early fifteenth century (see [16], no. 432; [2], II, pp. 158-159 and SII, pp. 158-159; and [1], pp. 179-184). He was born in 767/1365 and died in 850/1447, and according to his biographers, excelled in Islamic law, inheritance theory, and the Arabic language, as well as in arithmetic, geometry, astronomy, and timekeeping. He belonged to the generation of astronomers following that of Ibn al-Shāṭir and al-Khalīlī of Damascus (on whom see the articles in [3]) and preceding that of Ibn Abi'l-Fatḥ al-Ṣūfī and Sibṭ al-Māridīnī of Cairo (on whom see [16], nos. 445 and 447), so that he and a few less well-known contemporaries represent the end of serious and productive activity in astronomy in medieval Egypt.

Ibn al-Majdī compiled over thirty works relating to astronomy and mathematics. Most of these are short treatises on instruments, in particular, quadrants and sundials, the best known of which was a treatise on the almucantar quadrant in ten sections. His mathematical works include a commentary on the arithmetical work called *al-Talkhiṣ* by the earlier Moroccan scholar Ibn al-Bannā' and a treatise *Kashf al-ḥaqā'iq* on sexagesimal arithmetic. More substantial works are his treatises entitled *Ghunyat al-fahīm* and *al-Jāmiᶜ al-mufīd*; these have never been studied in modern times. The most interesting of his known astronomical works is his planetary tables entitled *al-Durr al-yatīm*, "The Unique Pearl". These form the subject of the present paper.

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*New York University, Kevorkian Center for Near Eastern Studies, 50 Washington Square South, New York, N.Y 10012.

**Institute for the History of Arabic Science, University of Aleppo, Aleppo, Syria. This paper presents some of the results obtained by both authors at the American Research Center in Egypt, Cairo, The work was supported by the Smithsonian Institution, the National Seicnce Foundation (USA), the American Philosophical Society, and the Ford Foundation. Most of the sources are preserved in the Egyptian National Library, but the Beinecke Library of Yale University supplied us with a photocopy of one MS. We express gratitude to each of these institutions.

Considerable confusion obscures our understanding of Ibn al-Majdī's contribution to these tables. The manuscript sources do not explicitly state the authorship of a corpus of tables for the sun, moon, and planets, based one the method of *al-Durr al-yatīm*, but we have the distinct impression that Ibn al-Majdī himself was responsible only for the tables for the sun and moon. He laid down the numerical bases for calculating similar tables for the planets, but, as we shall show, such tables were compiled by later astronomers.

Ibn al-Majdī's solar and lunar tables and the planetary tables which were devised along the same lines are not the standard Ptolemaic variety displaying mean motions and equations (on which see [6], pp. 141-142), nor the specfically Islamic development of these in the form of *ḥabṭaq* equation tables in which one enters arguments that can be derived directly from the mean motion tables (see [11], pp. 130-131). Rather Ibn al-Majdī's tables are auxiliary tables for compiling ephemerides, that is, tables displaying solar, lunar, and planetary positions for each day of the year. As remarked above, the only known earlier example of such auxiliary tables is [7]. which probably originated in twelfth century Iran. We suspect, but cannot prove, the existence of a continuous tradition of this category of tables reaching into fifteenth century Egypt. What is more certain is that Ibn al-Majdī's tables were used extensively in Egypt until the nineteenth century.

Section 2 below describes the principle which is common to this particular technique for all the planets, and for the moon. The succeeding three sections deal in detail with the three categories of tables needed to apply the technique to the planets. A table is presented comparing spot entries from the tables with results obtained by standard Ptolemaic computations. The lunar tables having previously been described in detail (in [7]), Section 6 suffices for the moon. Here also spot checks are presented.

All of the tables contain columns for determining the day of the week (Ar. *madkhal*, Lat. *signum*) for which a longitude is being calculated. We omit all discussion of these as being irrelevant to the main topic.

The solar model is essentially simpler than those of the moon and the planets Hence the tables used by Ibn al-Majdī and his followers for calculating runs of solar longtitudes are constructed differently from those for the planets.

These are described in Sections 7 and 8, and their application in Section 9.

The general theory and structure of the tables having been dealt with, Section 10 lists the manuscript sources in which they have been found. These are numerous, and by no means have all of them been investigated by us in detail. Section 11 discusses certain of the sources and relations between them, and makes concluding remarks about the whole corpus, its duration and influence.

2. *The Basic Principle of the Planetary Tables*

Suppose that tables of the *Almagest* (or *Handy Tables*) type are available, and it is desired to calculate the true longitude of a particular planet at a given instant. The result will be

$$\lambda = \bar{\lambda} + \delta,$$

the algebraic sum of the mean longitude and the "equation", where

$$(1) \qquad \delta = -c'_3(\alpha) + c_6(\gamma') + c_8(\alpha') \cdot \begin{cases} c_5(\gamma'), & c_8 \geqslant 0, \\ c_7(\gamma'), & c_8 < 0, \end{cases}$$

$$\text{and} \qquad \gamma' = \gamma + c'_3(\alpha)\,, \qquad \alpha' = \alpha - c'_3(\alpha).$$

The functions denoted by c's are given by columns of entries in the equation tables, the subscripts indicating the order in which the columns appear. The notation used here has been slightly modified from that adopted in [13]. pp. 191-207. For a complete exposition of the underlying theory and practise, the reader is referred to [14], pp. 145-189.

The variables γ, α (the "center", the mean longitude measured from apogee), and γ (the argument of the epicyclic anomaly) are linear functions of time. For the instant in question each can be calculated by adding appropriate entries from the mean motion tables in the set at hand.

The c functions, on the other hand, are basically trigonometric, hence periodic, and their combination to form δ is tedious and involved, demanding interpolation and a proper choice of signs. An individual who wishes to compute a set of planetary positions on successive noons at, say, ten-day intervals, must repeat the entire process from scratch for each individual noon.

These difficulties are obviated for the user of the Ibn al-Majdī tables by exploiting the following facts. For each planet a large period can be found consisting of an integer number of days which measures, with considerable accuracy, an integer number of the planet's anomalistic periods. Choose a day on which the anomalistic argument, γ, is small. Then consider the set of days separated from the chosen one by integer multiples of the big period. These are indicated schematically on Figure 1 by asterisks. On each of these days also γ will be small, by virtue of the existence of the period.

Subdivide the large period into smaller intervals marked by the set of noons, in the twenty-four hours preceding each of which γ shall have passed through

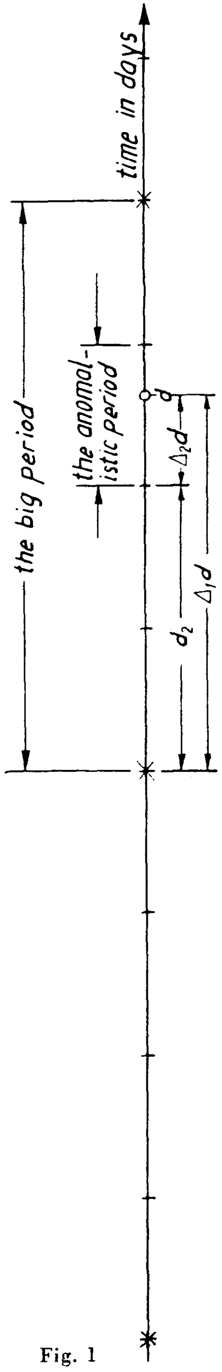

Fig. 1

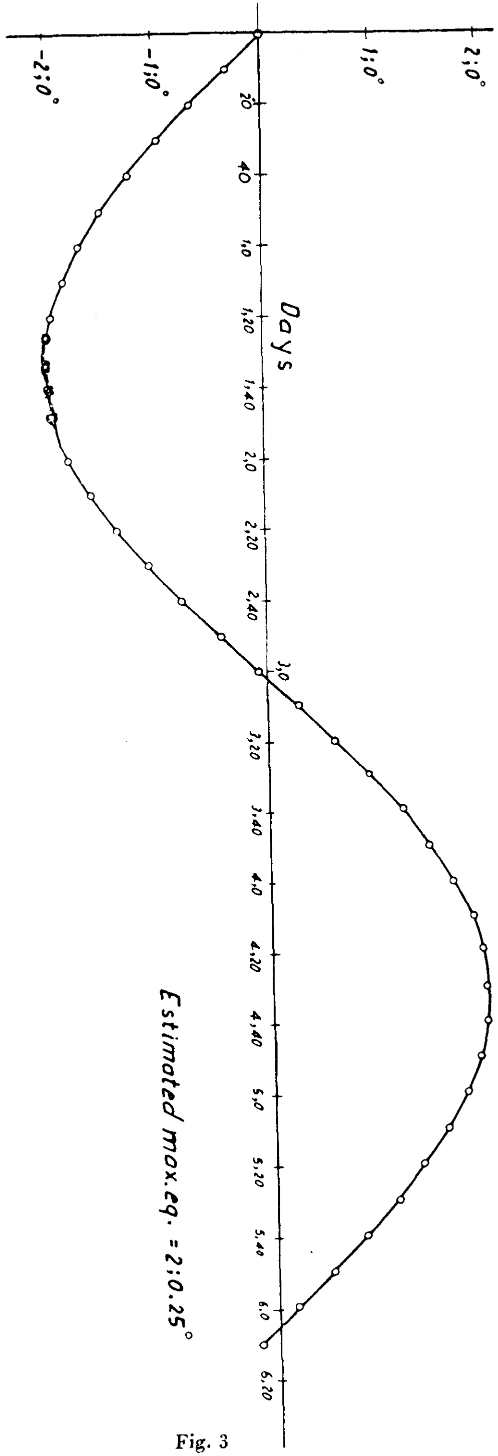

Fig. 3

360°. That is, each of the subdivisions between successive asterisks on the figure marks the first noon in a new anomalistic period. The change in γ between any such subdivision and the preceding asterisk cannot exceed the anomalistic motion in one day. Hence, because of the original positioning of the asterisks, the value of γ at all the subdivisions will be small.

Three sets of tables were calculated:

1. A *days table* to enable the user to convert a day given by a Hijra date into a number, d, the days elapsed since the Hijra epoch.

2. A *mean motions table* in two parts for each planet. The first part gives, for an appropriate span of time, the values of d at each of the noons represented by asterisks in Figure 1, together with corresponding values of the mean longitude, anomaly, and center. The second gives, for each subdivision within pairs of asterisks, the changes in d and the three variables named above.

3. A $\Delta\lambda$ table for each planet, giving the increment in longitude to be added to the mean longitude at the beginning of a particular period in order to convert it into true longitudes for a run of days thereafter. In principle there are three independent variables: (1) days elapsed within the period, (2) the values of the center, α, at the beginning of the period, and (3) the anomaly, γ, at the same time. By uirtue of the choice of asterisks, however, the possibilities for γ are very restricted, confined usually to two possibilities, 0° and 1°. For want of a better name we call this third variety of tables, "increment" tables. Note that the term "equation" would be inappropriate, since any particular $\Delta\lambda$ entry in such a table has two components, one being the planet's equation, the other the increase in mean longitude during the particular run of days.

Details are given below. What has been stated thus far makes it clear that the burden of computation has been shifted from the ephemeris maker to the calculator of the $\Delta\lambda$ tables. Once having determined a set of mean positions by a process resembling the traditional one, the user need only add a set of tabular entries to the mean longitude to produce a run of true longitudes for equally spaced intervals.

3. *The Days Table*

As stated above, one table is applied to determinations for all the planets. Its full title is *Jadwal ayyām al-masīr li'l-kawākib*, "The Days Travel Table for the Planets". Given a Hijra date, the table provides the sexagesimally-expressed number of days elapsed from the Hijra epoch to the date in question.

There are two sections, one for "summed" (*majmū^ca*) years, the second for "extended" (*mabsūṭa*) years. The argument for the summed section consists of the set

$$30(k+n), \qquad n = 0, 1, 2, \ldots, r,$$

where k and r are suitable natural numbers to cover, at intervals of the thirty-year Hijra cycle, the span of possible application of the table. The entries are the number of days elapsed from epoch until the beginning of the calendar year of the argument entry.

The extended section gives the number of days in 1,2, 3, ..., 30 Hijra years, and in the successive months of the year, ending with 5,54 (= 354) for the twelfth month of a sound year and 5,55 for a leap year.

The method of using the table is evident. For a given date, obtain three entries: (1) in the summed section, that opposite the largest argument which is less than or equal to the given year; (2) in the extended section, that opposite the excess of the given year over the entry just chosen; and (3) that opposite the month named in the given date. To the sum of these three entries add the days elapsed of the month given in the date. The resulting sum, d, is the days elapsed since the Hijra epoch.

4. *The Planetary Mean Motion Tables*

For each planet there are, as usual, two sections. The first, the "extended" one (Ar. *mabsūṭa*), gives the number of days from the Hijra epoch to the initial day of each big period tabulated. Opposite each day entry are the corresponding noon positions of mean, anomaly, and center. The second, the "summed" one (*majmū^ca*) gives the number of days elapsed from the beginning of the big period to the first day of the successive anomalistic periods. The three entries opposite each day number show the changes in the mean, anomaly, and center during the particular number of anomalistic periods. Entries are to seconds of arc, except for Saturn, Jupiter, and Mercury, for which the center has been carried to minutes only. Plate 1 displays two pages from such a table, but for the moon.

The table below shows the lengths of the two periods for each planet.

Planet	the long period		length of the anomalistic period in days
	days	number of anomalistic periods	
Saturn	6,49,36	65	6,18 or 6,19
Jupiter	2,52,51	26	6,38 or 6,39
Mars	3,27,59	16	12,59 or 13,0
Venus	2,6,31	13	9,43 or 9,44
Mercury	1,50,5	57	1,55 or 1,56

The manner of using the mean motion tables is described herewith, the various steps being illustrated schematically on Figure 1.

Having obtained d from the days table, enter the extended table for the

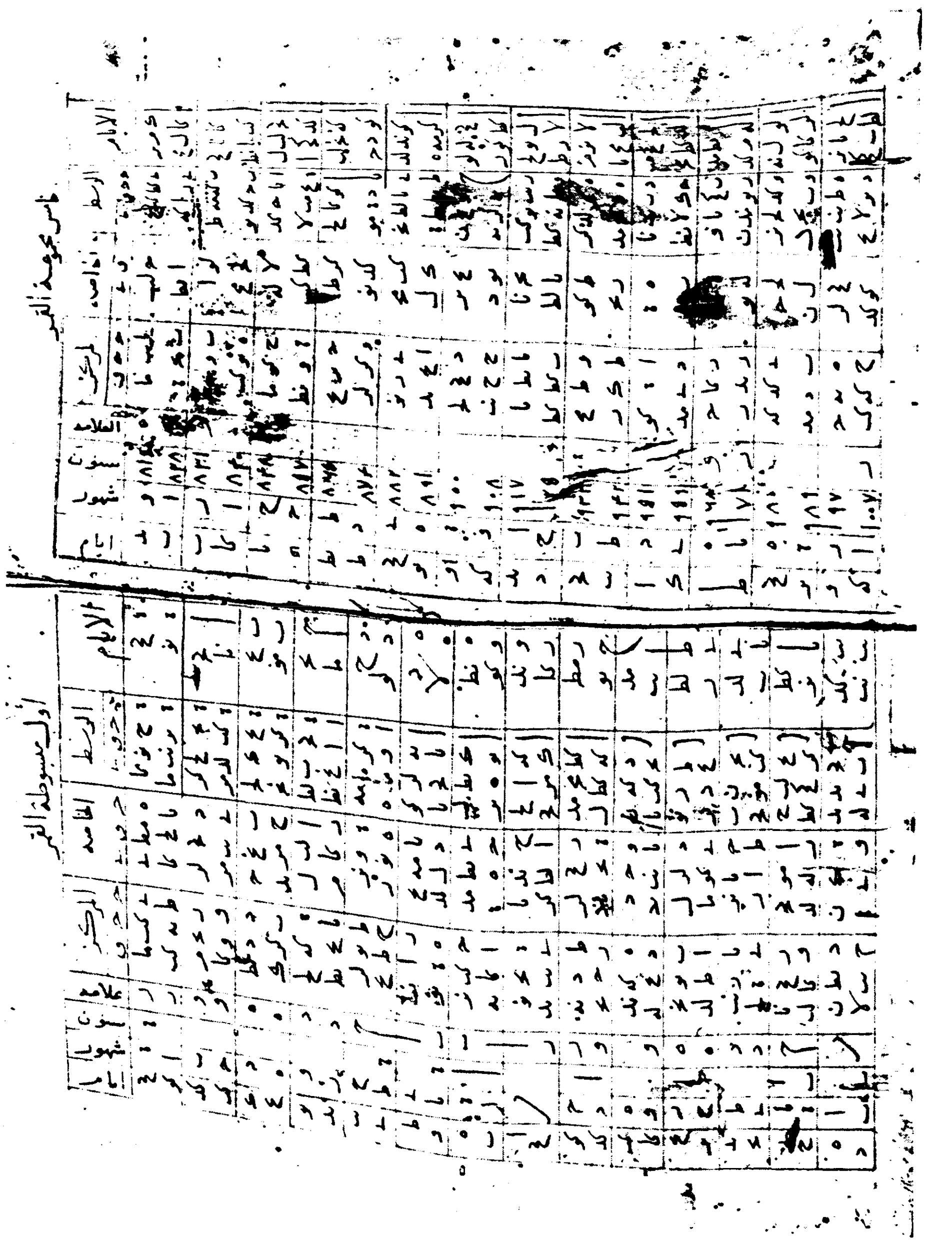

Plate 1. An extract from the *majmūʿa* and *mabsūṭa* tables for the moon in MS *A* (unfoliated).

planet in question and from the column of days obtain d_1, it being the largest entry in the column such that $d_1 \leqslant d$. Note down the triple of centries on the same line as d_1,

$$m_1, \gamma_1, \alpha_1 .$$

This process locates that one of the big periods in which d lies, and the mean longitude, anomaly, and center at that time.

Now put $\triangle_1 d = d - d_1$. This measures how far d enters into the big period.

Turn to the summed table, and find d_2, the largest argument such that $d_2 \leqslant \triangle_1 d$. This locates the initial point of the anomalistic period in which d lies. Opposite d_2 take the corresponding triple

$$\Delta m, \Delta\gamma, \Delta\alpha,$$

being the amount of change in the three variables in d_2 days.

Calculate $\Delta_2 d = \Delta_1 d - d_2$. This is the number of days by which d has entered the anomalistic period.

Then

$$m_2 = m_1 + \Delta m, \quad \gamma_2 = \gamma_1 + \Delta\gamma, \quad \alpha_2 = \alpha_1 + \Delta\alpha,$$

are the mean longitude, anomaly, and center on the initial noon of the anomalistic period in which d lies. Since each element of the triple can be thought of as a point on a circle, it should be regarded as the residue, modulo 360, of the three defining sums above.

5. *The Planetary Increment Tables*

The arguments for entering the $\Delta\gamma$ table are α_2, γ_2, and $\Delta_2 d$, calculated as described just above. In fact the user of the tables will have to content himself with approximations to these numbers, for the domain of α_2 in the tables for all the planets is $0^\circ, 6^\circ, 12, \ldots, 354^\circ$; that of γ_2 is $0^\circ, 1^\circ$ for all the planets except Mercury, which has $0^\circ, 1^\circ, 2^\circ, 3^\circ$. The domain of $\Delta_2 d$ is 10, 20, 30, ..., p, where p is the anomalistic period of the planet in days. This holds for all the planets except Mercury, where the set is 5, 10, 15, ..., p.

The entries, calculated to minutes of arc, are

$$\Delta\lambda = \Delta\bar{\lambda} + \delta = \Delta_2 d \cdot \dot{\bar{\lambda}} + \delta ,$$

the algebraic sum of the mean motion in $\Delta_2 d$ days, and δ, the equation on day d, calculated by means of the expressions marked (1) in Section 2 above. The values of the two independent variables upon which δ depends are

$$\gamma = \gamma_2 + \Delta_2 d \cdot \dot{\gamma} \quad ,$$

and

$$\alpha = \alpha_2 + \Delta_2 d \cdot \dot{\alpha} .$$

A dot over a variable denotes the variable's rate of change in degrees per day.

So the true longitude on day d (or on the day nearest d in the table) is

$$\lambda = m_2 + \Delta\lambda\,.$$

The format of the increment tables is such that on each page the column of $\Delta_2 d$'s runs down the right edge of the table. Hence, once the user has found the proper $\Delta\lambda$ for a particular day, the $\Delta\lambda$ for ten days later will be immediately below the one just found, and so on. This is particularly handy for the calculation of ephemerides. Plate 2 is an ecxerpt from an increment table, but for the moon.

An HP-67 calculator was programmed to compute the δ, thence $\Delta\lambda$, corresponding to given values of the arguments after the parameters for a particular planet had been keyed into the machine. The results for two sets of values for each planet are shown in the table below, together with the corresponding entries in the $\Delta\lambda$ table of Source H (described in Section 10 below). The agreement between text and calculation is not perfect, but it is close enough to demonstrate that our analysis is valid.

SPOT CHECK OF A PLANETARY INCREMENT TABLE
(Text values from Source *H*)

Planet	Arguments			$\Delta\lambda$		difference
	$\Delta_2 d$	α_2	γ_2	text	calc.	
Saturn	360^d	0°	0°	$9;17^\circ$	$9;19^\circ$	$0;2^\circ$
	10	120	1	356;21	$356;10^\circ$	—0;11
Jupiter	60	30	1	10;13	10;13	0;0
	360	90	0	20;25	20;25	0;0
Mars	60	12	0	37;12	37;13	0;1
	360	60	1	234;26	235;0	0;34
Venus	10	0	0	12;14	12;15	0;1
	310	192	1	275;51	273;47	—2;4
Mercury	110	0	3	101;56	101;57	0;1
	60	90	1	51;36	53;5	1;29

6. *The Lunar Tables*

The principles applied in the planetary tables are used also for the moon, the double elongation, 2η, replacing α, the center. A detailed description having already appeared in [7], a few remarks here will suffice. The long period is now 50,31 days = 1,50 anomalistic months, each of the latter running to twenty-eight and a fraction days. For the lunar increment table the domain is

$$\gamma = 0^\circ, 1^\circ, 2^\circ, \ldots, 13^\circ,$$
$$2\eta = 0^\circ, 6^\circ, 12^\circ, \ldots, 354^\circ,$$
$$\Delta_2 d = 1, 2, 3, \ldots, 28.$$

The daily motion of γ is of the order of thirteen degrees. The initial points of the long periods have been so chosen that at them γ is small. Hence at the initial points of the short intervals, noons of days in which successive anomalistic months commence, γ will never be as high as fourteen degrees (see Plate 2).

Just as in the preceding section, values of $\Delta\lambda$ were calculated for spot combinations of the arguments and compared with corresponding tabular entries found, this time in two sources. The results are displayed in the table which follows. Correspondence between text and calculation is quite good. Bodleian Arabic MS Marsh 374 is the source used for [7].

SPOT CHECK OF TWO LUNAR INCREMENT TABLES

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Arguments			$\Delta\lambda$				
γ_2	$2\eta_2$	Δ_2d	Ibn Ḥashīsh (Source *H*)	Machine Calculation	(5)–(4)	Bodleian	(5)–(7)
0°	0°	1^d	0° $11;53^\circ$	0^s $11;50^\circ$	$—0;3^\circ$	0^s $11;50^\circ$	$0;0^\circ$
0	0	10	4 5;28	4 5;24	—0;4	4 5;23	0;1
10	0	20	9 0;1	9 0;8	0;7	9 0;11	—0;3
10	186	10	4 8;46	4 8;39	—0;7	4 8;39	0;0
5	6	15	6 19;40	6 19;46	0;6	6 19;46	0;0
0	18	1	0 11;39	0 11;33	—0;6	0 11;36	—0;3
10	354	10	4 5;59	4 $5;51^\circ$	—0;8	4 5;49	0;2
10	180	20	8 28;45	8 28;59	0;14	8 28;57	0;2

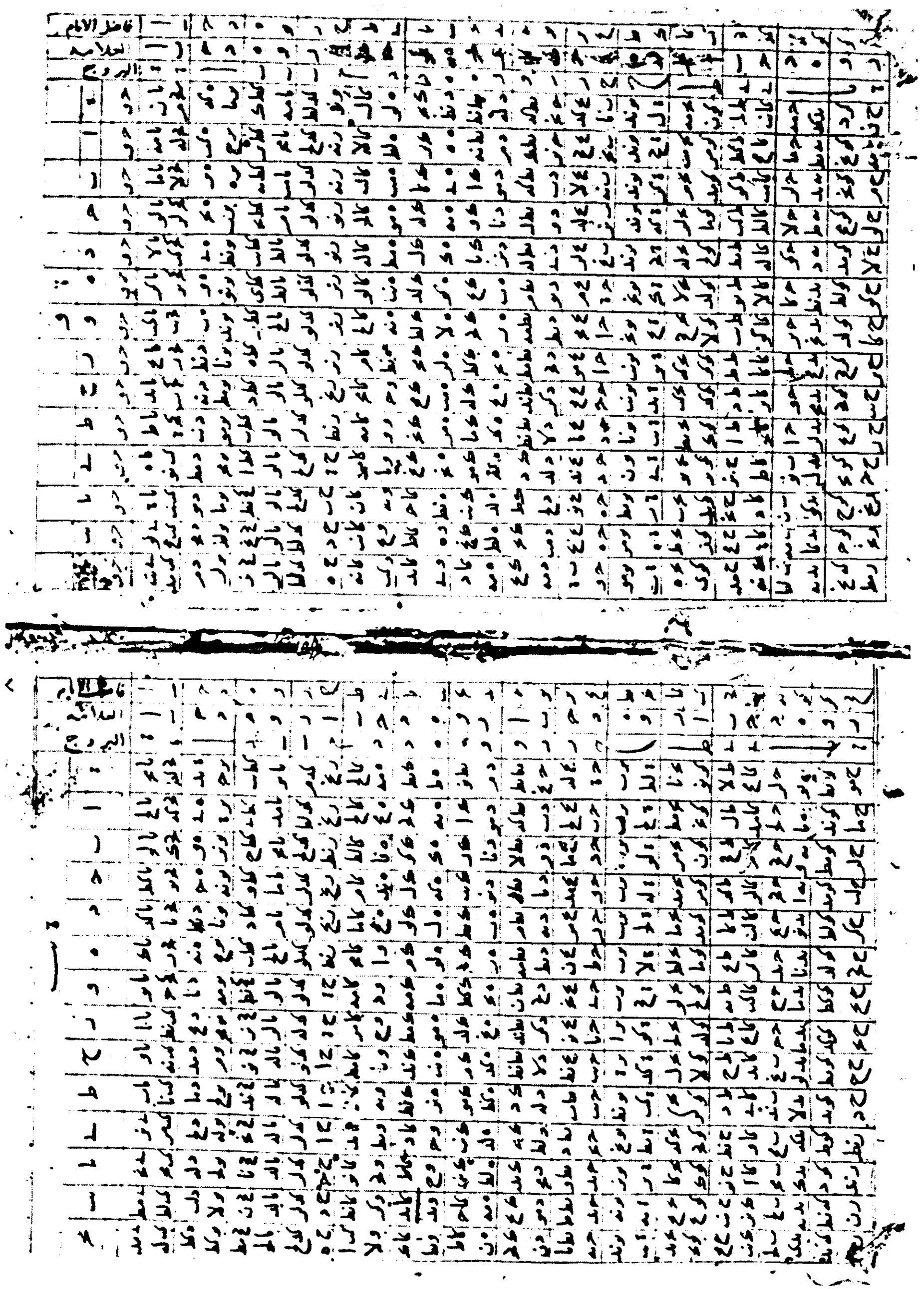

Plate 2. An extract from the increment tables for the moon in MS *A* (unfoliated).

7. *The Solar Mean Motion Table*

The sun has only one "equation" instead of the two for the planets and the moon. Hence the solar tables are set up somewhat differently from the others, and require a separate explanation. The mean motion table is in two sections of three columns each (disregarding columns for the *madkhal*), the entries being: (1) integer days, the argument of the table, (2) solar mean positions or motions, and (3) the argument of the solar anomaly. All are in sexagesimals, or zodiacal signs and degrees, the mean and anomaly being to seconds of arc. Again one section is called "summed", the other "extended". For the former, the first entry in the first column is to be regarded as the number of days passed since epoch, hence it corresponds to a specific date. Successive entries in the days column may be found by additions of 3,33,4 days = 35 "Julian" years of 365¼ days, each rounded off to the nearest integer. Corresponding entries in the mean and anomaly columns give the positions of the mean sun and its anomaly at the successive thirty-five year increments indicated. The reason for the choice of this particular interval seems to be that the anomalistic motion during this time is very nearly an integer number of revolutions, successive positions differing from each other by only 0;0,42°. Furthermore the initial date has been so chosen that the first entry for the anomalistic argument is small.

The extended section of the table has in the first column the number of days in 1,2,3,...35 Julian years, commencing with a leap year and inserting additional such every fourth thereafter. The second column gives the amount of mean solar motion in these times. The third column does the same for the anomalistic motion. The latter is very near but slightly less than a degree per day. Furthermore, the duration of the motion for all entries differs from an integer number of years by less than a day. This insures that all entries in the third column are less than unity.

8. *The Solar Increment Table*

This is called *Jadwal ta*^{c}*ādil al-shams*, "Table of the Solar Equations". It has two independent arguments. One, called the *fāḍil al-ayyām*, "excesses of days", is the set 10, 20, 30, ... , 6,10 (= 370). It takes up the first column of each page on which the table occurs. The first argument spans a year at ten-day intervals.

The second arguments is the set 0;0, 0;3, 0;6, 0;9, ..., 1;0. It is not named in the table, but, as becomes evident from texts and examples, it is a range of values of the solar anomalistic argument, γ.

As for the function tabulated, it is explained herewith by use of Figure 2. The solar true longitude is

$$\lambda = \bar{\lambda} + e ,$$

where $\bar{\lambda}$ is the mean longitude, a linear function of time, and e is the solar equation, the sinusoidal function of small amplitude hugging the horizontal axis on Figure 2. e is periodic with a period of a year. More precisely, its period is the time required for the solar anomalistic argument to run through a complete revolution. So the true longitude function consists of an ascending straight line upon which a series of identical ripples has been imposed. Since the motion takes place on a circle, perhaps it is better to think of the curve as dropping to the horizontal axis every time it reaches 360°, but this is not shown on the figure.

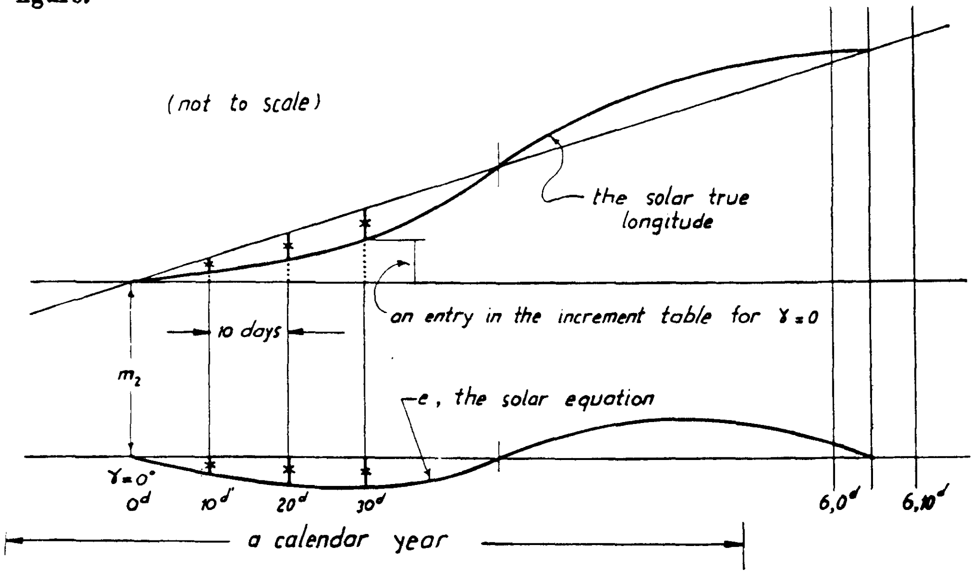

Figure 2

Consider an instant when the sun passes through its apogee. The Ptolemaic model has been so set up that at this time $\gamma = 0$ and $\lambda = \bar{\lambda} = m_2$, say. Then the entries down the first column of the equations table (for $\gamma = 0$) are represented by the set of dotted vertical segments rising from the horizontal line of height m_2 and terminated by the equation curve. That is, the entries are

$$\Delta_n\lambda = e_n + n\cdot\dot{\bar{\lambda}}\,, \qquad n = 10, 20, 30, \ldots, 6{,}10,$$

where e_n is the value of the solar equation n days after passing the apogee, and $\dot{\bar{\lambda}}$ is the rate of increase of $\bar{\lambda}$ in degrees per day.

The validity of the above assertion was demonstrated by calculating the solar mean motion in 10, 20, 30, ..., 6,10, days and subtracting the results from corresponding entries in the first column of the table. The results, when plotted in Figure 3 exhibited the characteristic form of an equation curve.

The above has to do with the entries in the first column of the equations table, when $\gamma = 0$. For the last column of the same table $\gamma = 1°$. Then the same process as described in the paragraph above was carried through for the elements of this column also. That is, the solar mean travel in intervals of ten days was subtracted from the successive entries. The set of remainders should plot as an equation curve congruent with that of Figure 2, but displaced to the left by a small amount as indicated in Figure 1. More precisely, the sun will already have passed the apogee by a degree at day zero. Since the mean and the anomaly advance at about a degree per day the horizontal displacement between the two curves should be very nearly a day.

This notion was verified by calculating the horizontal intercept of each of the two equation curves between $\Delta d = 6{,}0$ and $\Delta d = 6{,}10$. Since the curves in this neighborhood are very nearly flat, linear interpolation can be used with very little sacrifice of precision. For the $\gamma = 0$ curve it yielded an intercept at 6,5;14; for $\gamma = 1°$ at 6,4;14.

As a final test, the differences between corresponding entries in the two end columns were calculated. Since for each pair the differences in the respective γ's are precisely one degree, the results should be approximately equal to the set of first differences obtained from a solar equation table where the tabular difference is one degree. Such a table is found in the zīj of Ibn Yūnus (see the entry in [3]). First differences were calculated from it for arguments near those of the differences between column elements. In general, corresponding results were identical to seconds of arc.

The method of calculating the end columns in the equations table having been established, it remains to do the same for all the columns in between, i.e, for $\gamma = 0;3°, 0;6°, 0;9°, \ldots, 0;57°$.

For several fixed and widely separated values of Δd, plots were made of the equations table entries for the range $\gamma = 0, 0;3°, 0;6°, \ldots, 1;0°$. In all cases the resulting graphs are straight lines. That is, entries along rows were filled in by linear interpolation between the endpoints. This is reasonable, since the solar longitude function has little curvature anywhere, and the total variation along rows of the table is only a degree.

9. *Use of the Solar Tables*

Having worked out the structure of the tables, their manner of application is reasonably evident. Suppose a solar ephemeris is desired, at ten day intervals commencing from a given Hijra date. Use the days table as described in Section 3 above to obtain d, the number of days from epoch to the date in question.

Now turn to the Solar mean motion table and, as described in Section 4, determine d_1 from the extended table, and the corresponding couple

$$m_1,\ \gamma_1\ .$$

These are respectively the solar mean longitude and anomaly at the beginning of the 35-year "Julian" cycle in which the given date falls.

Put $\Delta_1 d = d - d_1$, and from the summed section of the table determine d_2, thence a second couple

$$\Delta m,\ \Delta\gamma\ ,$$

analogously to the procedure of Section 4. These two numbers are respectively the change in the mean and in the anomalistic argument from the beginning of the 35-year cycle to the first day of the "Julian" year in which the given date falls.

Calculate a third couple

$$m_2 = m_1 + \Delta m, \qquad \gamma_2 = \gamma_1 + \Delta\gamma_1\, .$$

This consists of the mean longitude and the value of the anomalistic argument on the first day of the table year. Note that by virtue of the choice of epoch for the mean motion tables and the property of the 35-year cycle, the condition $0;0^{\circ} \leqslant \gamma_2 \leqslant 1;0^{\circ}$ should always hold.

Calculate $\Delta_2 d = \Delta_1 d - d_2$, the number of days from the beginning of the table year to the given date. If $\Delta_2 d$ is divisible by ten it will appear among the "excesses of days" arguments of the equations table. Otherwise choose the day argument nearest $\Delta_2 d$. For the other argument, the anomaly, choose the table argument nearest γ_2. Find the entry in the table corresponding to these two arguments, and call it $\Delta\lambda_1$.

Then

$$\lambda_1 = m_2 + \Delta\lambda_1$$

will be the solar true longitude for the first day of the ephemeris,

$$\lambda_2 = m_2 + \Delta\lambda_2$$

for ten days later, where $\Delta\lambda_2$ is the entry under $\Delta\lambda_1$, and so on.

10. *The Sources*

Virtually all the manuscript material relating to the tradition of *al-Durr al-yatīm* is preserved in the Egyptian National Library (*Dār al-Kutub*) in Cairo, and information on all of these manuscripts is contained in the forthcoming catalog of the Cairo scientific manuscripts by the first author. Our knowledge of the tradition is impaired by the fact that most copies of the tables are anonymous or defective or both. The following manuscripts are the oldest and most reliable sources and will be hereafter referred to by the appropriate sigla.

A: MS Cairo Dār al-Kutub *miqāt* 405 (39 fols., copied *ca.* 850/1450).

This is the best available copy of the solar and lunar tables, transcribed in the elegant hand of ᶜAlī b. Ḥasan al-Baḥṭīṭī. Unfortunately the title folio, and with it the first two sets of solar tables, is missing. The latter have been added in a later inelegant hand.

B and *C*: MSS Cairo Dār al-Kutub *miqāt* M 85,1 (fols. 1r-2v, *ca.* 850/1450) and *miqāt* M 44,2 (fols. 22r-27v, *ca.* 850/1450).

These contain respectively the solar and lunar tables, copied in the distinctive hand of ᶜAlī b. Muḥammad al-Dalāmī. There is no original title on the first manuscript. The second manuscript contains a hodge-podge of fragments from later copies of the corpus.

D: MS Cairo Dār al-Kutub *miqāt* 391 (41 fols., *ca.* 1000/1600)

This is a complete copy of the solar and lunar tables in a clear hand. The title folio displays the title *Kitāb al-Durr al-yatīm fi ṣanᶜat al-taqwīm* and identifies the author as Ibn al-Majdī. He is specifically mentioned in most of the tables as the calculator. There are additional tables of the *majmūᶜa* of the moon (fols. 1v-2r), specifically attributed to Nūr al-Dīn al-Naqqāsh and ᶜIzz al-Dīn al-Wafā'ī.

E: MS Cairo Dār al-Kutub *miqāt* M 25 (30 fols., *ca.* 825/425)

This is a complete set of lunar "increment" tables in a clear and elegant hand, with a note on the first page of tables: "these are the equations of the moon based on the parameters of Ibn Yunūs computed by Ibn al-Majdī, and they are in the hand of ᶜAbd al-ᶜAziz al-Wafā'ī..."

F: MS Cairo Dār al-Kutub *miqāt* M 26 (30 fols., *ca.* 850/1446)

Another complete set of lunar increment tables, in a clear and elegant hand.

G: MS Cairo Dār al-Kutub *miqāt* 681,8 (fols. 31r-43r, *ca.* 850/1450)

A set of *majmūᶜa*, *mabsuta*, and increment tables for Venus, in a clear and elegant hand. There is no indication of any compiler.

H: MS Yale Nemoy 1453 (90 fols., 1253/1837-38)

A treatise on the use of the tables of *al-Durr al-yatīm* entitled *Ṭirāz al-ghurar fi ḥall al-durar* and compiled by an Egyptian astronomer named Sulayman b. Ḥamza b. Bakhshīsh (? or Ḥashīsh; either variant seems impprobable). This particular copy, unlike the others of the *Ṭirāz al-ghurar* that we have examined (e.g., MSS Cairo Dār al-Kutub *mīqāt* 791 (9 fols., 1072H) and *majāmīᶜ* 323, 6A (fols. 33v-34r, *ca.* 1250H), contains an extensive set of tables. See further Section 11 below.

I: MS Cairo Ṭalᶜat *miqât* 82 (37 fols., 870/1465-66)

A copy of the treatise by Ibn al-Majdi on the compilation of ephemerides called *Ghunyat al-fahīm.*

J: MS Cairo K8524 (68 fols., *ca.* 1000/1600)

A copy of the treatise by Ibn al-Majdī on the compilation of ephemerides called *Kitāb al-Tashīl wa'l-taqrīb.*

K: MS Cairo Ṭalᶜat *miqāt* 113,1 (fols. 1r-128r, 1053/1643-44)

A set of tables based on the method of *al-Durr al-yatīm* serving the sun, moon, and planets. The various tables are here attributed to their compilers; see Section 11 below.

L: MS Cairo Ḥalīm *miqāt* 16,2 (fols. 33v-37v, *ca.* 1100/1690)

A treatise on the compilation of an ephemeris for Mercury using tables in the tradition of *al-Durr al-yatīm.* The author is Shihāb al-Dīn Aḥmad al-Kutubī al-Khurfānī.

M: MS Cairo Taymūr *riyāḍa* 188 (822 pp.! *ca.* 1000/1600)

Riḍwān Efendī's own copy of his solar, lunar, and planetary tables based on the method of *al-Durr al-yatīm* and entitled *al-Durr al-naẓīm* or *al-Durr al-farīd.*

N: MS Cairo Muṣṭafā Fāḍil *miqāt* 83 (259 fols., *ca.* 1250/1835)

A later copy of Riḍwān Efendī's tables in MS M.

P: MS Cairo Dār al-Kutub *miqāt* 802 (723 fols. !, *ca.* 1000/1600)

Riḍwān Efendī's own copy of his revised version of the auxiliary tables for the planets, entitled *Kitāb Asnā al-mawāhib fī taqwīm al-kawākib.*

Q: MS Cairo Muṣṭafā Fāḍil *miqāt* 133 (90 fols., *ca.* 1150/1735)

Ramaḍan al-Khawānikī's own copy of his auxiliary tables for the moon, after the model of Riḍwān's tables in MS *P*.

R: MS Cairo Ṭalᶜat *miqāt* 113 (138 fols., 1053/1643-44)

This manuscript contains a complete set of solar, lunar and planetary tables based on the method of *al-Durr al-yatīm*. It bears the spurious title *Tashīl zīj durr al-yatīm li-Majrīṭī* [!] *bi* [*sic*]-*ṭūl Miṣr al-Muᶜizzīya*. Most of the *majmūᶜa* and *mabsūṭa* tables are attributed (see below).

S: MS Cairo Dār al-Kutub *miqāt* 504,3 (fols. 10v-14r, *ca.* 1193/1779)

An anonymous set of ephemerides for the year 1194 Hijra, apparently computed using the tables of *al-Durr al-yatīm*.

T: MS Cairo Dār al-Kutub *miqāt* 878 (25 fols. , *ca.* 1250H), plus two fragments numbered 40 and 627.

This is a set of auxiliary tables for computing solar longitudes, compiled by an Egyptian astronomer named Abū'l-Fatḥ b. ᶜAbd al-Raḥmān al-Danūshirī.

U: MS Cairo Dār al-Kutub *miqāt* 109,3 (fols 33v-39r, *ca.* 1200H).

Compiled by an anonymous Syrian astronomer, this is another set of solar tables.

11. *Discussion of the Sources*

The popularity of Ibn al-Majdī's tables in later Egypt is proven by the relatively large number of copies of these tables in various recensions, and of commentaries on the use of the tables by most of the more celebrated of later Egyptian astronomers.

Modifications were made to the solar and lunar tables already in the fifteenth century ᶜIzz al-Dīn al-Wafā'ī and Nūr al-Dīn ᶜAlī al-Naqqāsh, and the planetary tables (as in MS H) appear to date from about the year 1600 although the existence of a set of tables for Venus copied *ca.* 1450 (as in MS *G*) established that tables for the planets were also produced prior to 1600.

Commentaries on the use of the tables were written by Ibn al-Majdī himself, ᶜIzz al-Dīn al-Wafā'ī, Ibn Abi l-Fatḥ al-Ṣūfī, Ḥasan b. Khalīl al-Karādīsī, Shihāb al-Dīn Aḥmad al-Kutubī al-Khurfānī (?), Sulaymān b. Ḥamza b. Bakhshīsh, Shihāb al-Dīn Aḥmad b. Mūsā, Yaḥyā b. Muḥammad al-Khaṭṭāb, and ᶜUthmān b. Ṣāliḥ al-Wardānī as late as *ca.* 1800.

The earliest copy of planetary tables based on the method of *al-Durr al-yatīm* is MS *G*, copied *ca.* 850/1450. This contains both *majmūᶜa* and *mabsūṭa* tables and a set of increment tables for Venus. There is no indication of any compiler. The same increment tables for Venus occur in MS *R*, where a different set of *majmūᶜa*, and *mabsūṭa* tables for Venus are attributed to Shams al-Dīn Muḥammad al-Hunaydī (and yet another set is attributed to Muṣṭafā al-Fayḍī – see below).

Concerning MS *H*, several other copies of this treatise exist, but no others contain the corpus of tables. This manuscript was copied in 1253/1837-38 and the tables may not be original to the treatise of Ibn Bakhshīsh. This is confirmed by the fact that the author gives a worked example for 979 Hijra (= 1571/72), and the table of days in the following tables begin with 1110 Hijra. A complete set of *majmūᶜa* and *mabsūṭa* tables and increment tables for the sun, moon, and planets is presented in this manuscript, and the only table specifically attributed to an author is the *majmūᶜa* table for the moon which was computed by Muṣṭafā Abū'l-Itqān al-Khayyāṭ, an Egyptian astronomer who lived *ca.* 1150/1740.

We now turn to MS *R*, a corpus of tables in *ca.* 140 fols., copied in 1053/1643-44, and survey the tables it contains which relate to our subject. Firstly, there is an "extended" set of the solar increment based on Ibn al-Majdī's original tables (fols. 3r-22v), with values for each day rather than each ten days. This is followed (fol. 23r) by a table of the lunar equation when $2\eta = 6^s$ attributed to ᶜAbd al-ᶜAzīz al-Wafā'ī, with a note on how to use it. Next there is (fols. 24r-26v) a set of solar increment tables from the *Durr*, followed by (fols. 27r- 28v) the text of the treatise *al-Ṣirāṭ al-mustaqīm* by Ibn Abi'l-Fatḥ al-Ṣūfī which the copyist says is very useful although people have overlooked it. The lunar *majmūᶜa*, *mabsūṭa*, and increment tables (fols. 29r-60b) are specifically attributed to Ibn al-Majdī.

The *majmūᶜa* table for Saturn (fol. 61r) is stated to be taken from Ibn al-Majdī's work *al-Ḥall wa-l-tarkīb*; the *mabsūṭa* and increment tables (fols. 61v-68r) are unattributed.

The various *majmūᶜa* and *mabsūṭa* tables for Jupiter (fols. 69v-70r) are attributed to Ibn al-Majdī, ᶜAbd al-Raḥīm b. al-Bannā' and Muḥammad al-Hunaydī, and the increment tables (fols. 70v-76r) are attributed to Ibn al-Bannā' (see fol. 69v).

The various *majmūᶜa* and *mabsūṭa* tables for Mars (fols. 77v-78r) are attributed to Ibn al-Bannā' and Muḥammad b. al-Qalaᶜī, and the increment tables (fols. 78v-87v + 90r-101r) are attributed to Ibn al-Bannā'.

Two loose pages (fols. 88-89) contain some solar tables relating to the *Durr* in a later hand, with a note "for the latitude of Aleppo 35;50°". (The tables are in fact independent of latitude).

Only one of the *majmūᶜa* and *mabsūṭa* tables for Venus (fols. 103v-104r) is attributed, namely, to al-Hunaydī. The increment tables (fols. 104v-119r) are unattributed, but they are identical with those in the older MS *G* (see above).

The *majmū*ᶜ*a* table for Mercury (fol. 120v) is attributed to al-Hunaydī, the *mabsūṭa* table (fol. 121r) is anonymous. Likewise the increment tables (fols. 121v-128v) are unattributed.

An additional table for the *majmū*ᶜ*a* of Venus has been added in the manuscript (fol. 103r) by Muṣṭafā al-Fayḍī (*kātib muṣrif-i Sadr-i* ᶜ*Ālī*) in 1170H, and calculations in his handwriting occur elsewhere in the margins of some of the original tables. Across al-Hunaydī's *majmū*ᶜ*a* table for Venus he has written in Turkish *jumlat bu jadwal ghalaṭdir*, "all of this table is wrong". Elsewhere in the manuscript, by a table of mean positions of the lunar node (fol. 62r), the same person has written *bu jadwalak tafāwuti fāḥish olūr*, "this table has an exorbitant divergence (from the truth)". It is rare to see such critical statements in medieval Arabic manuscripts.

Each of the individuals ᶜAbd al-Raḥīm b. al-Bannā', Muḥammad al-Hunaydī, Muḥammad b. al-Qalaᶜī, and Muṣṭafā al-Fayḍī, is new to the modern literature on the history of Islamic astronomy.

Riḍwān Efendī, an astronomer who worked in Cairo *ca.* 1600, compiled a set of solar, lunar, and planetary tables based on the method of *al-Durr al-yatīm* and on the parameters of the fourteenth century *Zīj* of Ulugh Beg of Samarqand. Riḍwān's tables, extant in MS *M*, cover 822 pages of manuscript, and are copied in his own untidy hand. For the sun, moon, and five planets, there are *majmū*ᶜ*a*, *mabsūṭa*, and increment tables. The solar increment tables are as in the earlier kind with entries for each 0;3° of horizontal argument. The lunar and planetary tables give values for each 1° of horizontal argument rather than each 6°. But Riḍwān tired of computing, and, in general, values for alternate degrees are omitted; sometimes whole pages are ruled for tables but there are no entries. In all cases, values are given to three sexagesimal digits rather than two. Ridwan's tables appear in a tidier form in MS *N*, copied some 250 years after his time: here his values for the increments are rounded to two digits and the horizontal argument difference is 2°, so that the values which Riḍwān did not bother to compute have been omitted altogether.

Riḍwān seems to have had an inspiration to compile an even larger corpus of auxiliary tables, a holograph copy of which survives in MS *P*, consisting of 1446 pages of tables, although here again he succumbed to the tedium of churning out tables and left about half of the entries blank. In this version the *majmū*ᶜ*a* tables are as before, but the *mabsūṭa* tables have been incorporated into the increment tables, in a way that escapes us. MS *Q* is a unique copy of a set of lunar tables based on the same principle, compiled and copied by the Cairo astronomer Ramaḍān b. Ṣāliḥ al-Khawānikī about the year 1750. Al-Khawānikī boasts on the title folio that no one has preceded him in this, but the vast majority of entries in the ninety folios ruled for tables have been left blank.

Several dozen Egyptian ephemerides giving solar, lunar, and planetary

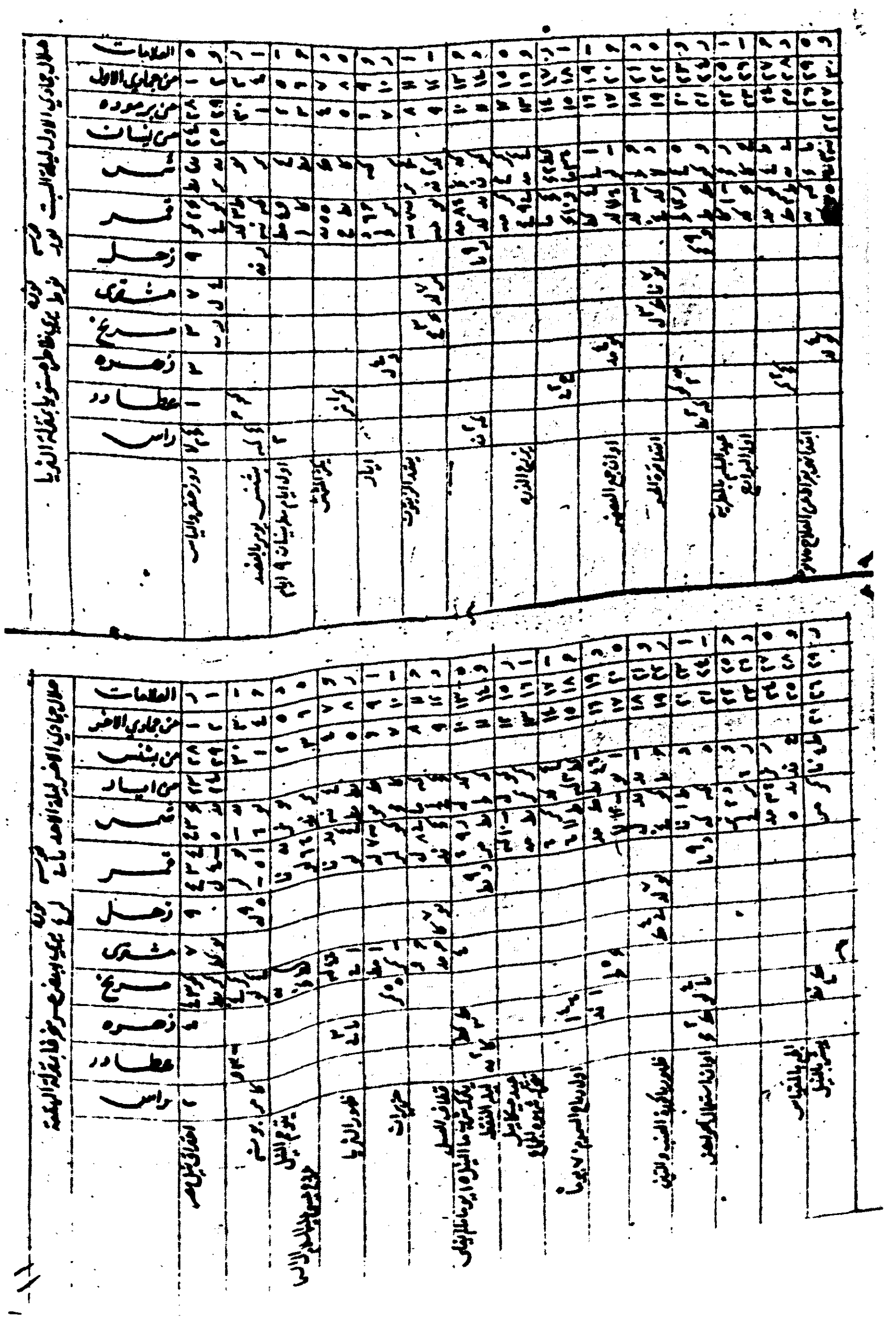

Plate 3. An extract from an anonymous set of ephemerides for the year 1194 Hijra (= 1780 A.D.), MS S, ff. 10v, 11r. These two pages serve each day of the months Jumāda I and II. Solar and lunar positions are given for each day of the year, whereas planetary positions are given for each ten days. These positions appear to have been computed using the tables described in this paper.

positions, for each day of the year, and dating from the seventeenth, eighteenth, and nineteenth centuries survive in the Egyptian National Library. An extract from one of these, contained in MS *S* and computed for the year 1194H (= 1780), is displayed in Plate 3; it is unusual in that positions are given for the sun and moon for each day, for the outer planets and Venus for each ten days, and for Mercury for each five days. It seems highly probable that this ephemeris was computed using the corpus of auxiliary tables that we have discussed in this paper. Indeed, although no surviving ephemerides contain an explicit statement that they were computed using the tables of *al-Durr al-yatīm*, we can be sure that these auxiliary tables were used extensively in Egypt from the fifteenth century onwards.

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SOME ASTRONOMICAL OBSERVATIONS FROM THIRTEENTH-CENTURY EGYPT

DAVID A. KING, New York University, and
OWEN GINGERICH, Harvard-Smithsonian Center for Astrophysics

1. *Introduction*

In this study we present a list of thirteen planetary conjunctions observed in medieval Egypt. Such lists are rare in the medieval Islamic sources, and indeed the one presented here is only the second to come to the attention of historians. The first was the list of over one hundred observations recorded by the tenth-century Egyptian astronomer Ibn Yūnus, published by the French orientalist Caussin de Perceval in 1804.[1]

The list that we present and discuss here was located by King in 1974 in a manuscript of a medieval Arabic astronomical handbook preserved in the Municipal Library in Alexandria and numbered 5577C. In 1975 an earlier copy of the same work was located in the British Library in London, numbered Or. 9116. This work belongs to the category of Islamic astronomical handbooks known in medieval Arabic as *zīj*es, of which almost 200 were compiled by Muslim astronomers during the millenium following the rise of Islamic astronomy in the eighth century.[2] The Islamic *zīj*es contain extensive tables and instructions for solving all the standard problems confronting the medieval astronomer, such as the determination of planetary and stellar positions, the computation of conjunctions and eclipses, and problems relating to time-keeping. The *zīj* preserved in Alexandria and London is entitled *Taysīr al-maṭālib fī tasyīr al-kawākib*, "The simplification of problems in finding the motions of celestial bodies", and was compiled by the thirteenth-century Yemeni astronomer Muḥammad ibn Abī Bakr al-Kawāshī.[3] The Alexandria manuscript was copied in Sanaa in the year 1730. The London manuscript, which lacks the title folio and the name of the author, can be dated to the fourteenth or fifteenth century.

Although al-Kawāshī worked in the Yemen and prepared his planetary tables for the longitude of Taiz, his *zīj* is based largely on earlier Egyptian and Iraqi material. The Yemen was an important centre of astronomy in the Middle Ages, particularly in the thirteenth century, and the numerous extant Yemeni astronomical works are of interest because some of them preserve material from earlier Iraqi, Egyptian, and North African sources that are no longer extant in their original form.[4] al-Kawāshī's *zīj*, like several other of these Yemeni sources, merits detailed study.

The fifth chapter of the introduction to al-Kawāshī's tables deals with planetary mean motions and begins with the list of the observations that the author claims to have made in order to derive the parameters on which his planetary tables are based. It seems probable that al-Kawāshī made the observations himself, although he is not mentioned in any known contemporary or later Egyptian sources.[5] The observations were made either in Qus in the Nile Valley or in Alexandria, both flourishing cities in medieval times, but no other scientific activity of consequence in either locality is known from this period. al-Kawāshī gives no indication that he actually used these observations either to check his planetary parameters or to derive new parameters. The same holds for Ibn Yūnus and his observation reports: his new parameters are introduced without explanation. The only new parameter which al-Kawāshī presents is a value of the obliquity of the ecliptic that he states to have derived himself, namely, 23;32,50°. This value is not attested in any other Islamic work that is currently known.

Our present purpose is simply to present the list of observations as it appears in the two manuscripts, giving the Arabic text (Section 2), a translation (Section 3), and a commentary (Section 4) in which we investigate each of the observation accounts using modern tables of planetary positions in medieval times. The numbering of the observations is our addition to the text.

2. *The Arabic Text*

The text of al-Kawāshī's observation list occurs on fols. 19v–20r of the London manuscript (A) and fols. 11r–11v of the Alexandria manuscript (B). It reads as follows:

قطعة من تيسير المطالب لمحمد بن ابى بكر الكواشى

أ : مخطوطة المكتبة البريطانية ٩١١٦، ق ١٩ ظ — ٢٠ و
ب : مخطوطة الاسكندرية المكتبة البلدية ٥٥٧٧ ج، ق ١١ و — ١١ ظ

الباب الخامس فى ذكر ما شاهدناه بالحس من قرانات الكواكب بعضها
لبعض (١) قارن القمر عطارد وكسفه بمدينة قوص ومكث منكسفا نحو نصف
ساعة وذلك فى الساعة الثانية من ليلة الثلثاء ثانى المحرم سنة ٦٧٢. (٢)
وقارن القمر زحل فى اول الثالثة من ليلة الاحد عاشر المحرم سنة ٦٧٦
وكان بينهما فى العرض [١] نحو شبر واحد (٣) وقارن الزهرة المشترى بكرة
نهار يوم الاثنين ثامن عشر المحرم سنة ٦٧٦ وبينهما فى العرض [١] نحو
شبر واحد تقديرا (٤) وقارن القمر المشترى وقت عشاء الآخرة ليلة السبت
خامس عشر شعبان سنة ٦٧٧ (٥) وقارن المشترى قلب الاسد بكرة نهار
يوم الخميس الثامن والعشرين من جمادى [٢] الاولى سنة ٦٧٨ وقد تقدمه
المشترى بقدر عرض اصبع واحدة تقديرا (٦) وقارن القمر المشترى فى اول
الثالثة من ليلة السبت سابع عشر رمضان سنة ٦٧٨ وهما على خط نصف

النهار وبينهما فى العرض نحو عرض اصبعين تقديرا او دون ذلك (٧) وقارن
عطارد المشترى صباح يوم الاثنين [٣] رابع عشر [٣] جمادى [٢] الاولى سنة
٦٧٩ وبينهما تقدير عرض اصبع واحدة [٤] (٨) وقارن الزهرة زحل بكرة نهار
يوم الاثنين [٥] [٦] خامس عشر [٦] شوال سنة ٦٨٠ وزحل متقدم عليها فى الطول
نحو شبر وفى الليلة الثانية تقدمت الزهرة عليه بنحو [٧] شبر (٩) وقارن المريخ
قلب الاسد بكرة نهار يوم الثلثاء تاسع ربيع الاول سنة ٦٨١ وكان شديد القرب
منه (١٠) وكسف القمر الزهرة فى ليلة السبت رابع المحرم سنة ٦٨٢ وكان
الابتداء فى اخر الساعة الثانية من تلك الليلة ومكثت مستترة الى وسط الرابعة
من الليل بالاسكندرية (١١) وقارن المريخ قلب الاسد يوم الثلثاء سابع ربيع
الاول سنة ٦٨٣ [٨] وكان تقدير القران بين الظهر والعصر (١٢) وقارن الزهرة
المريخ بكرة نهار يوم الاحد سابع جمادى [٢] الاخرة سنة ٦٨٣ (١٣) وقارن
القمر زحل بكرة يوم الجمعة خامس عشر ربيع الاخر سنة ٦٨٣ وقد تقدمه القمر
فى الطول بتقدير ثلث درجة تقديرا وبينهما فى العرض نحو درجتين او دون
ذلك

التحقيق : [١] — [١] فى الهامش فى ب [٢] فى أ : جمدى [٣] — [٣] فى
أ : ثالث عشر ، وفوق السطر بخط اخر : رابع عشر صح [٤] فى أ : واحد
[٥] فى أ : الثلاثاء ، وفوق السطر بخط اخر : الاثنين صح [٦] — [٦] فى
أ : سادس عشر ، وفوق السطر بخط اخر : خامس عشر صح [٧] فى ب : نحو
[٨] فى ب : ٦٨٢

3. *Translation*

"Chapter 5 on the planetary conjunctions which we observed.

(1) The Moon was in conjunction with Mercury and occulted it at the town of Qus. [The planet] remained occulted for about half an hour. This was during the second hour of the night of Tuesday, Muḥarram 2, in the year 672 [Hijra].

(2) The Moon was in conjunction with Saturn at the beginning of the third [hour] of the night of Sunday, Muḥarram 10, in the year 676. There was about one span latitude difference between them.
(3) Venus was in conjunction with Jupiter early in the morning of Monday, Muḥarram 18, in the year 676. There was an estimated latitude difference of about one span between them.
(4) The Moon was in conjunction with Jupiter at the time of the night [prayer] on the night of Saturday, Sha^{c}bān 15, in the year 677.
(5) Jupiter was in conjunction with Regulus early in the morning of Thursday, Jumādā I 28, in the year 678. Jupiter had an estimated latitude excess of one digit.
(6) The Moon was in conjunction with Jupiter at the beginning of the third [hour] of the night of Saturday, Ramaḍān 17, in the year 678. They were both on the meridian and their estimated latitude difference was two digits or [a little] less.
(7) Mercury was in conjunction with Jupiter on the morning of Monday, Jumādā I 14[?], in the year 679. There was an estimated latitude difference of one digit between them.
(8) Venus was in conjunction with Saturn early in the morning of Monday, Shawwāl 15, in the year 680. Saturn was ahead of [Venus] in longitude by about one span and the next night [*i.e.*, early the next morning] Venus was ahead of [Saturn] by about one span.
(9) Mars was in conjunction with Regulus early in the morning of Tuesday, Rabī^c I 9, in the year 681. It was very close to [the star].
(10) The Moon occulted Venus on the night of Saturday, Muḥarram 4, in the year 682. The beginning [of the occultation] was at the end of the second hour of that night and [Venus] remained covered until the middle of the fourth [hour] of the night at Alexandria.
(11) Mars was in conjunction with Regulus on Tuesday, Rabī^c I 7, in the year 683 [MS B has 682!]. The estimated [time of the] conjunction was between the midday and afternoon [prayers].
(12) Venus was in conjunction with Mars early in the morning of Sunday, Jumādā II 7, in the year 683.
(13) The Moon was in conjunction with Saturn early in the morning of Friday, Rabī^c II 15, in the year 683. The Moon was an estimated one-third of a degree ahead of [Saturn] in longitude, and there was about two degrees or [a little] less latitude difference between them."

4. *Commentary*

The following remarks are based on an examination of the reported observations. For each observation we have converted the dates to the Christian calendar and ascertained the details of the conjunctions or occultations.

The Hijra dates given in medieval Arabic astronomical texts may be either according to the *civil* or *astronomical* reckoning. The epoch of the Islamic Hijra calendar according to the civil reckoning is taken as Friday, 16 July 622 A.D. This was the evening of the first visibility of the lunar crescent marking the first day of the pagan Arab year in which the Prophet Muḥammad emigrated from Mecca to Medina (in Arabic, *al-hijra*, literally, "the breaking of ties"). According to the astronomical reckoning the epoch is the preceding day when the true conjunction of the Sun and Moon occurred. In medieval Arabic astronomical texts both systems are used, and we need to know the corresponding day of the week in order to convert to another calendar.[6] Modern tables display the dates in the Christian calendar corresponding to the beginning of each Hijra month from the seventh century to the present,[7] and we have relied on these. The reader should bear in mind that the Islamic day begins at sunset, so that, for example, the Arabic *laylat al-ithnayn*, literally "the night of Monday", means "the night preceding Monday morning". Also, al-Kawāshī uses seasonal hours representing twelfth divisions of the length of daytime or night-time, and hence varying throughout the year and depending on the local latitude. He expresses some of the times of day and night in terms of the times of prayer, which in Islam are astronomically defined.[8]

To check these medieval planetary observations we have used a computer program originally written by P. Huber at the Eidgenössische Technische Hochschule in Zurich (and now of Harvard) and similar to that used for the tables of B. Tuckerman.[9] We have added the coding to give topocentric positions of the Moon. For the star Regulus we have used a position computed from P. V. Neugebauer's *Sterntafeln*.[10]

The Arabic terms used for small angular distances on the celestial sphere, *aṣba^c*, "digit" (that is, finger-breadth), and *shibr*, "span", correspond to apparent elongations of about $\frac{1}{12}$° and $\frac{1}{2}$°, respectively.[11]

All of the observations are rather crudely described, and there is no need to suppose that they are anything other than naked-eye observations. Only in observations 1 and 10 are the locations Qus and Alexandria specifically mentioned. It may be that observations 1 to 9 were made in Qus, and observations 10 to 13 were made in Alexandria, *wa-llāhu a^{c}lam*, "but God knows better"; in any event, it makes scarcely any difference for the rather low precision of the given observations. Fortunately the most interesting records for a modern astronomer are observations 1 and 10. The geographical coordinates of Qus and Alexandria are as follows:

	Latitude	Longitude
Qus	25°55′	32°44′ = 2^h11^m
Alexandria	31°12′	29°54′ = 2^h00^m

The time equivalent of the longitude, together with the equation of time for the day in question, allows the apparent time of the observations to be converted into universal time (UT), which always begins at Greenwich midnight; for our purposes we are not distinguishing universal and ephemeris times. For times of sunset and sunrise we have used the tables in *The American ephemeris and nautical almanac*.

(1) Muḥarram 2, 672 Hijra *in the astronomical calendar* corresponds to Tuesday, 18 July 1273. The occultation of Mercury thus took place on the evening of Monday, 17 July 1273. Since the solar longitude was about 121°·8 on that evening, sunset occurred at about 6^h40^m after midday (= 16^h36^m UT) and one seasonal night hour equals about 55^m. Our calculations show that the occultation began almost precisely one seasonal night hour after sunset, and that the moonset half an hour later prevented observations of the conclusion of the phenomenon.

(2) Muḥarram 10, 676 Hijra *in the civil calendar* corresponds to Sunday, 13 June 1277. According to our calculations, the conjunction between Saturn and the Moon took place on the evening of Saturday, 12 June 1277. Since the solar longitude was about 88°·5 on that evening, sunset occurred at about 6^h50^m after midday (= 16^h42^m UT) and one seasonal night hour at Qus corresponded to about 52^m. The closest approach took place at 19^h 17^m UT, that is, at the end of the third seasonal night hour after sunset.

(3) Muḥarram 18, 672 Hijra in the civil calendar corresponds to Monday, 21 June 1277. The close approach of Venus and Jupiter was observed early in the morning whereas the true conjunction in longitude actually followed in the afternoon of 21 June when Venus was not visible. The latitude difference was very close to ⅔° at dawn on 21 June.

(4) Shaᶜbān 15, 677 Hijra in the astronomical calendar corresponds to Saturday, 31 December 1278. The conjunction took place on the evening of Saturday, that is, on the preceding Friday night, and was not particularly impressive because the Moon was over four degrees north of Jupiter. With such a distance the precise time of close approach would have been difficult to measure. Medieval Egyptian astronomers would have calculated nightfall (and the time of prayer) for a solar depression of 17°, which matches the conjunction with more than enough precision.

(5) Jumādā I 28, 678 Hijra in the astronomical calendar corresponds to Thursday, 5 October 1279. The positions of Jupiter and Regulus on that day were:

	Longitude	Latitude
Jupiter	139°·9	+0°·8
Regulus	139°·9	+0°·2

This latitude difference is closer to a span than the digit mentioned in the text.

(6) Ramaḍān 17, 678 Hijra in the astronomical calendar corresponds to Saturday, 20 January 1280. The conjunction is stated to have taken place with the Moon on the meridian at the beginning of the third hour of the "night of Saturday", that is, on Friday, 19 January 1280. Since the Moon was a day past full, it could not come to the meridian until after midnight, so the details of the report are faulty. The solar longitude on that date was about 307°, sunset came at 5^h35^m after midday, and the seasonal night hour was 64^m. Thus the third hour of the night began at Qus about 2^h8^m after sunset, or at 15^h37^m UT. The conjunction actually took place in the fifth seasonal hour after sunset, with the Moon 2°·5 above Jupiter (rather than the specified two digits) and our calculations show that by the time the Moon was on the meridian, it had advanced a degree in longitude beyond Jupiter.

(7) This report is slightly garbled. MS A has Jumādā I 13 with an additional note in a different hand that 14 is better. MS B has simply Jumādā I 14. The morning of Jumādā I 14, 679 Hijra, in the civil calendar corresponds to Thursday, 11 September 1280. According to our calculations the conjunction in longitude actually took place before Mercury and Jupiter were visible in the eastern dawn on the morning of 11 September 1280; the latitude difference when the planets became visible just before dawn was about ¾°, that is, more than one span rather than the one digit stated in the text.

(8) Shawwāl 15, 680 Hijra in the astronomical calendar corresponds to Monday, 26 January 1282. According to our calculations for that morning, Saturn was ahead of Venus in longitude by about ⅔° (*i.e.*, a little more than a span) and the following morning Venus was ahead of Saturn by just under ½° (*i.e.*, about a span). Their latitude difference was slightly greater than 1°. We find:

		Longitude	Latitude
1282 Jan. 26·2 (UT)	Mercury	269°·53	+2°·2
	Saturn	270°·20	0°·8
1282 Jan. 27·2	Mercury	270°·68	2°·1
	Saturn	270°·29	0°·8

The date as given in the translation thus seems secure. In MS A the day of the week is given as Tuesday, corrected to Monday, and the day of the month is given as 16, corrected to 15.

(9) Rabīᶜ I 9, 681 Hijra in the astronomical calendar corresponds to Tuesday, 16 June 1282. The text indicates only that the conjunction is close, and we find Regulus to be about a degree south of Mars:

	Longitude	Latitude
Mars	139°·8	+1°·2
Regulus	139°·9	+0°·2

(10) Muḥarram 4, 682 Hijra in the astronomical calendar corresponds to Saturday, 3 April 1283. The occultation took place on Friday, 2 April 1283. Since the solar longitude was about 20°, the second seasonal hour of the night at Alexandria ended about 1^h55^m after sunset or about $6^h20^m + 1^h50^m = 20^h15^m$ after astronomical midday, and the middle of the fourth hour of the night was about 3^h15^m after sunset or about $6^h20^m + 3^h15^m = 21^h40^m$ after midday. Although the text thus indicates an approximate duration of 1^h20^m, an occultation of Venus can last only about an hour unless the planet is moving rapidly with the Moon. Our calculations, with an accuracy of about five minutes, show that the occultation began at 19^h42^m in Alexandria and ended at 20^h42^m, with the Moon then at an altitude of 16°. It is interesting to note that the reported times agree somewhat better for Qus than for Alexandria.

(11) Rabīᶜ I 7, 683 Hijra in the astronomical calendar corresponds to Tuesday, 23 May 1284. Our calculations show that the conjunction actually took place around midday (for longitudes in Egypt) on 22 May rather than 23 May. Because the latitude difference was about 1⅓°, it would have been somewhat difficult to record by naked eye on which evening the approach was closest; since the reported event took place in the daytime, the estimated time of the conjunction was derived by interpolation or possibly by computation. Computation with the Toledan Tables, for example, would have given the same date we find provided a good position of Regulus was available. For May 22·4 (UT) we find:

	Longitude	Latitude
Mars	139°·9	+1°·37
Regulus	139°·9	+0°·2

(12) Jumādā II 7, 683 Hijra in the astronomical calendar corresponds to Sunday, 20 August 1284. According to our calculations the true conjunction in longitude took place somewhat earlier on the previous evening (19 August)

before Venus and Mars had risen, but the two planets were still very close when they became visible on the Sunday morning. Their latitude difference was about ⅓°.

(13) Rabīᶜ II 15, 683 Hijra in the astronomical calendar corresponds to Friday, 30 June 1284. According to our calculations, Saturn and the Moon came into conjunction that morning following sunrise; in other words, the phenomenon was terminated by dawn. At about dawn (June 30·11 UT) the situation was as follows:

	Longitude	Latitude
Moon	292°·12	+0°·41
Saturn	293°·85	−0°·27

In other words, the separation is about as described in the text except that the components of longitude and latitude are mixed—there is no way to get the specified 2° in latitude. The event was quite similar at Qus.

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REFERENCES

1. See D. A. King, "Ibn Yūnus", in *Dictionary of scientific biography*, xiv (New York, 1976).
2. On Islamic *zījes* see E. S. Kennedy, "A survey of Islamic astronomical tables", *Transactions of the American Philosophical Society*, n.s. xlvi (1965), 123–77.
3. More information on the author and his *zīj* is given in D. A. King, *Mathematical astronomy in medieval Yemen* (Publications of the American Research Center in Egypt; Malibu, California, 1982), Section II.7.
4. *Ibid.*, for details of this tradition.
5. For astronomy in Egypt in the thirteenth, fourteenth, and fifteenth centuries, see D. A. King, "The astronomy of the Mamluks", to appear in *Muqarnas*, ii (1983).
6. For further details see W. Hartner, "Zamān" in *The encyclopaedia of Islam* (1st ed., Leiden, 1913–34), reprinted (German version only) in *idem*, *Oriens–Occidens* (Hildesheim, 1968), 260–3.
7. *Wüstenfeld-Mahler'sche Vergleichungstabellen zur Muslimischen und Iranischen Zeitrechnung* . . . (Wiesbaden, 1961) and Robert Schramm, *Kalendariographische und Chronologische Tafeln* (Leipzig, 1908).
8. For details see D. A. King, "Ibn Yūnus' *Very useful tables* for reckoning time by the Sun", *Archive for history of exact sciences*, x (1973), 343–94, pp. 345–7. Sunset, daybreak, nightfall, and other times of religious significance can be calculated for any terrestrial latitude and solar longitude by using modern computer-generated tables similar to those used by medieval Muslim astronomers (*ibid.*, 353).
9. Bryant Tuckerman, *Planetary, lunar, and solar positions, 601 B.C. to A.D. 1*, Memoirs of the American Philosophical Society, lvi (Philadelphia, 1962); a further volume, *ibid.*, lix (Philadelphia, 1964) carries the ephemerides to 1649 A.D.
10. Paul V. Neugebauer, *Sterntafeln von 4000 vor Chr. bis zur Gegenwart nebst Hilfsmitteln zur Berechnung von Sternpositionen* (Leipzig, 1912).
11. These terms and others are used also by Ibn Yūnus but are nowhere precisely defined. See E. W. Lane, *An Arabic–English lexicon* (London, 1863; reprinted Beirut, 1968), *sub aṣbaᶜ*, *shibr*, and *rumḥ*.

VIII

Indian Astronomy in Fourteenth Century Fez: The Versified Zīj of al-Qusunṭīnī

E. S. KENNEDY* & DAVID A. KING**

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1. *Introduction*

A certain Abū'l-Ḥasan ᶜAlī b. Abī ᶜAlī al-Qusunṭīnī[1] compiled in fourteenth-century Fez a sort of miniature *zīj*, or astronomical handbook comprising tables and explanatory text,[2] which he dedicated to the Merinid Sultan Ibrāhīm al-Mustaᶜīn. This *zīj* is distinguished by the fact that the explanatory text is in verse.[3] Many mathematical and astronomical poems, some of considerable sophistication, were composed during the Islamic Middle Ages; most of these were Maghribi compilations and most are as yet unstudied in modern times.[4] The fact that al-Qusunṭīnī's *zīj* is in verse, however, is not the reason for our studying the work. Rather, it is because the zīj is the only known document

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* The American University of Beirut, Beirut, Lebanon.

** Department of Near Eastern Languages and Literatures, New York University, 50 Washington Square South, New York, New York 10003, USA.

[1] Al -Qusunṭīnī and his *zīj* are mentioned in *Suter*, no. 371; *Renaud 1*, no. 371; and *Brockelmann*, S II, pp. 364-365. (References in italics are to the bibliography at the end of the paper). The epithet al-Qusunṭīnī indicates that our author or his family was originally from Qusunṭīniya (= Constantine) on the Algerian littoral. He is not known to have compiled any other works, but we have not consulted any medieval Maghribi biographical works. He is referred to as *al-faqih*, which indicates his interest in law, and as *al-muᶜaddil*, which indicates that he was a professional time-keeper associated with a mosque and responsible for the regulation of the times of prayer.

[2] A survey of Islamic *zījes* is *Kennedy 1*.

[3] The only other *zīj* known to us which may have been written in verse is called *al-Zīj al-manẓūm*. and its arrangement in verse is implied by the title, *al-Sirr al-maktūm fi-l-ᶜamal bi'l-zīj al-manẓūm* of a work attributed to the fourteenth-century Syrian scholar Abū'l-Fidā' (*Suter 1*, no.392), and extant in a unique manuscript in Oxford. See further *Kennedy 2*, pp. 18 and 22.

[4] Some examples of the most popular scientific works in verse are the astrological poem of Ibn Abī 'l-Rijāl (on whom see *Pingree 5* and *Sezgin*, VII, pp. 186-188; the poem on algebra by Ibn al-Yasmin (*Suter 1*, no. 320); the poem on timekeeping by al-Jādarī (*Suter 1*, no. 424a); and the poem on all aspects of science by ᶜAbd al-Raḥmān al-Fāsī entitled *al-Uqnūm* (*Renaud 1*, no. 541). Each of these authors worked in the Maghrib.

extant in Arabic in which the planetary theory is essentially Indian rather than Ptolemaic. This Indian planetary theory, popular amongst certain early Muslim astronomers,[1] and not without influence in Andalusia and the Maghrib throughout the medieval period, is known to be based on pre-Ptolemaic Greek astronomical models.[2] The *zīj* of al-Khwārizmī was also based on Indian planetary theory, but it has survived only in the Latin translation of an extensive reworking of the original by al-Majrīṭī.[3]

The unique manuscript source of al-Qusunṭīnī's *zīj* is ff. 44v-63v of MS Escorial ar. 909.[4] The first part of the same manuscript contains a copy of the *zīj* of the thirteenth-century Moroccan astronomer Ibn al-Bannā', upon which, as will be seen below, al-Qusunṭīnī leans heavily. Al-Qusunṭīnī's *zīj* is reproduced in facsimile on pp. 22-41 below with kind permission of the authorities of the Biblioteca de El Escorial. The introduction is written in *rajaz* meter.

In Section 2 we attempt to put al-Qusunṭīnī in the context of astronomy in the medieval Maghrib. No clear picture of this general topic can be presented at this time. The known sources present a multiplicity of historical problems, and some of the most important sources have only recently been rediscovered and have not been studied yet.

In the next section, 3, al-Qusunṭīnī's mean motion parameters are displayed and discussed. They are seen to be from Western Arabic sources, independent of al-Khwārizmī's mean motions. The same is true of his planetary apogees, also presented.

In Section 4, however, it is shown that al-Qusunṭīnī's planetary "equation" tables are essentially the same as those of al-Khwārizmī, except that seconds of arc have been suppressed.

Section 5 is an attempt to infer from al-Qusunṭīnī's rules his method of calculating planetary true longitudes. We suggest that the solution is an algorism which, like the equation tables, is firmly in the Indian (and Sasanian Iranian, and early Islamic) tradition, but which is considerably more primitive than any related rule hitherto noted. We do not think our author originated

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[1] On the influence of Indian astronomy in early Islamic astronomy see *Pingree 1*. (Prof. Pingree informs us that there are several Sanskrit manuscripts in existence of a work entitled *Yantra Jarkali*, suggesting that al-Zarqāllu's works had some modest influence in later Indian astronomy.)

[2] See *Pingree 2* for an overview of Indian astronomy.

[3] On al-Khwārizmī see the article by G. Toomer in the *DSB*. A medieval Latin translation of al-Majrīṭī's recension of his *zīj* is published in *Suter 2*. A translation and commentary is in *Neugebauer 2*. Further insight into the original work is provided in *Goldstein 2*. On the Byzantine and medieval Latin traditions based on the *Zīj al-Sindhind*, see *Pingree 3*, pp. 151-169, and *Pingree 4*.

[4] On the manuscript see *Renaud 2*, pp. 7-10. The manuscript is of Maghribi provenance, but is not dated. It contains (1) the *zīj* of Ibn al-Bannā'; (2) the *zīj* of al-Qusunṭīnī; and (3) a commentary by Ibn al-Qunfudh (*Suter 1*, no. 422) on the astrological poem of Ibn Abī'l-Rijāl (see note 4, p. 3). Renaud gives the name as al-Qusṭanṭīnī but the text has clearly al-Qusunṭīnī.

it, but that he inherited the method from some much earlier source, probably through unknown intermediaries.

Section 6 is a detailed table of contents of the entire *zīj*. Readers who need information concerning the contents of a normal *zīj*, or conventions involving symbols, may consult *Kennedy 1*. The entries in the tables are expressed in the standard medieval Arabic alphanumerical notation.[1] Standard topics omitted by al-Qusunṭīnī are: trigonometric functions, planetary latitudes, fixed stars, geographical coordinates, and astrological functions. Of special interest is a lunar ripeness table.

2. *Brief Survey of Astronomy in the Maghrib*

The following account is the first attempt in the modern literature to outline the history of astronomy in the Maghrib.[2] The evidence indicates that such cities as Marrakesh, Tunis, Taza, and Tlemcen, were the scene of an active tradition of astronomy for several centuries. Until the available sources are investigated more thoroughly it will be difficult to establish the connections between the Andalusian and Maghribi traditions in astronomy.[3] Prof. G. Toomer, in his penetrating study of the solar theory of the eleventh-century Andalusian astronomer al-Zarqāllu, has already demonstrated the importance of Maghribi material based on earlier Andalusian sources that are no longer extant in their original form.[4]

From the first five centuries of Islam only one author is known to us from the Maghrib, namely, the astrologer Ibn Abī'l-Rijāl, who worked at the Zirid court in Tunis *ca.* 1045.[5] Thereafter we have reports of isolated measurements of the obliquity of the ecliptic conducted by an unnamed astronomer in Meknes, by Ibn Hilāl in Sebta, by al-Mirrīkh in Marrakesh, and by Ibn al-Turjumān in an unspecified location, all dating apparently from the twelfth and thirteenth centuries.[6]

The activities of al-Zarqāllu in Cordova and Toledo in the eleventh century appear to have had more influence in Europe than they had in later Islamic astronomy.[1] The Andalusian astronomer Ibn al-Kammād seems to have based his *zīj*es on the work of al-Zarqāllu, and at least one of his *zīj*es was in use in the Maghrib in the thirteenth century.[2] A Maghribi astronomer who is known to have relied on a *zīj* of Ibn al-Kammād and also on the observations of a Sicilian Jew, was Ibn Isḥāq, a Tunisian who worked in Morrocco in the early thirteenth century.[3] He compiled a *zīj* which Ibn Khaldūn tells us was widely used in the Maghrib in the fourteenth century; a copy of this work was recently discovered by the second author in Hyderabad, and awaits detailed study. Ibn Isḥāq quotes several earlier scholars whose works are no longer available in their original form: for example, in his chapter on lunar crescent visibility he cites the opinions of the earlier Andalusian astronomers Ibn Muᶜādh and Abū'l-Ḥajjāj al-Sabtī, the latter a student of Maimonides,[4] as well as others whose names are new to the modern literature (see Section 6 below).

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[1] *Cf. Irani* on this notation.

[2] The standard bio-bibliographical sources in which Maghribi astronomers and their works are listed are the following: *Suter 1*; *Renaud 1*; *Brockelmann*, II, pp. 331-332 and 615-616, and SII, pp. 364-365 and 707-709; *Azzawi*, pp. 209-221; and *Cairo Survey*, Section F. See also *Renaud 2* on astronomy in Morocco and *King 1*, pp. 192-193 on astronomy in Tunis. On Maghribi astrolabists and their works see *Gunther*, I, pp. 248-301; *Renaud 4*; *Mayer, passim*; and *Brieux & Maddison*. Maghribi contributions to mathematics are surveyed in *Djebbar*.

For catalogs of Maghribi manuscript collections see *Sezgin*, VI, pp. 329-332, 402-407, and 454-456, and on two particularly rich collections of scientific manuscripts see *Renaud 7* and *Samsó*.

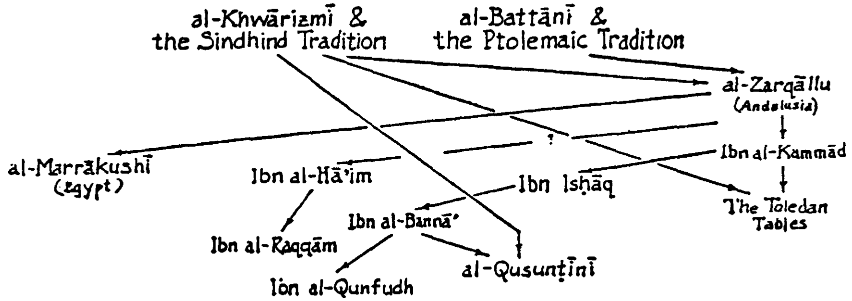
[3] On the Andalusian tradition see the numerous publications of J. Millás Vallicrosa, J. Vernet Ginés, and J. Samsó Moyá.

[4] *Cf. Toomer 2*.

[5] On Ibn Abī'l-Rijāl see note 4 to Section 1 above.

[6] These individuals are mentioned in the anonymous commentary on al-Jādarī's poem, on which see note 5 on p. 9.

In passing we should mention that the late thirteenth-century scholar Abū ᶜAlī al-Marrākushī,[5] author of an enormous compendium on spherical astronomy and instruments entitled *Jāmiᶜ al-mabādi' wa'l-ghāyāt*, hailed from the Maghrib but wrote his treatise in Cairo. Indeed, al-Marrākushī's work, which was highly influential in Egypt, Syria, and Turkey, appears to have been unknown in the Maghrib. al-Marrākushī quotes such sources as al-Zarqāllu and Ibn al-Kammād, but a thorough investigation of his sources for his writings on instruments has yet to be undertaken. Transmission and influences are indicated in the chart below.



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[1] On al-Zarqāllu see the article in the *DSB* by J. Vernet. See also *Toomer 1* on the *Toledan Tables* and *2* on al-Zarqāllu's solar theory.

[2] On Ibn al-Kammād see *Suter 1*, no. 487; *Kennedy 1*, nos. 5, 66 and 72; and *Toomer 2*, pp. 330-331.

[3] On Ibn Isḥāq see *Suter 1*, no. 356. See also *Rosenthal*, III, pp. 136-137 for the remarks of Ibn Khaldūn. The manuscript of his *zīj* is MS Hyderabad Andra Pradesh State Library no. 298 (*ca.* 200 folios, copied *ca.* 1400).

[4] On Ibn Muᶜādh see the article "al-Jayyānī" in the *DSB* by Y. Dold-Samplonius and H. Hermelink. On al-Sabtī see *Suter 1*, no. 342.

[5] On al-Marrākushī see *Suter 1*, no. 363 and *Cairo Survey*, no. C17. The first half of his treatise, which deals with spherical astronomy and sundials is translated in *Sédillot-père*. The second half, which deals with other instruments, was summarized in a rather haphazard fashion in *Sédillot-fils*.

The Moroccan scholar Ibn al-Bannā' commpiled a *zīj* in Marrakesh about the year 1300.[1] This survives in several copies but the tables have yet to be studied properly. The astronomer Ibn al-Raqqām compiled in Tunis in the early fourteenth century two *zīj*es, both of which are extant in unique manuscripts and have yet to be studied.[2] One of Ibn al-Raqqām's *zīj*es is said by the author to be based on another by Abū'l-Ḥasan ibn ᶜAbd al-Ḥaqq called Ibn al-Hā'im, a person otherwise unknown to us. The *zīj* of al-Qusunṭīnī was not the only baby *zīj* compiled in the Maghrib. Ibn al-Qunfudh in the late fourteenth century compiled a small *zīj* for Tlemcen based on the *zīj* of Ibn al-Bannā'.[3] No other *zīj*es specifically for Fez or Tlemcen are known to us.

A recension for Algiers of the *zīj* of Ibn al-Shāṭir, the celebrated astronomer of fourteenth-century Damascus, is known from a single manuscript.[4] Considerably more influential was the *zīj* of Ulugh Beg, compiled in fifteenth-century Samarqand: Tunisian recensions were prepared by Abū ᶜAbd Allāh Muḥammad al-Tūnisī known as Sanjaq Dār and by Ḥusayn Quṣᶜa, and both survive in several copies.[5] The late fifteenth century Jewish astronomer Zacuto compiled his perpetual almanac in Salamanca.[6] These tables in a modified form were apparently used in the Maghrib (as well as in Ottoman Turkey), and the introduction to them was translated into Arabic by Andalusian astronomers.

In Marrakesh in the early thirteenth century and in Taza in the early fourteenth century, astrolabes of excellent construction were being produced.[1] In the late thirteenth and early fourteenth centuries there were constructed in Fez two astronomical clocks, of a kind known otherwise only from mid-fourteenth century Damascus. The first clock was set up in the Qarawiyyīn Mosque[2] and the second in the Bu^{c}ināniyya madrasa:[3] both were water-clocks fitted with an astrolabic rete. The first clock, in its later form, is still *in situ* although the gear mechanisms have gone, and most of the second clock has disappeared: the remains of both clocks have been investigated by Prof. Derek J. de Solla Price. Several later Maghribi astrolabes and quadrants survive in museums around the world,[4] attesting to a continuing interest in instrumentation in the Maghrib until the nineteenth century.

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[1] On Ibn al-Bannā' see the article in the *DSB* by J. Vernet; *Suter 1*, no. 399; *Renaud 1*, no. 399; *Renaud 5*; and *Cairo Survey*, no. F23. The introduction to his *zīj* is translated in *Vernet 1*.

[2] On Ibn al-Raqqām see *Suter 1*, nos. 388 and 417 (?); *Brockelmann*, SII, p. 378; and *Cairo Survey*, no. F 22. His *Shāmil Zīj* is in MS Istanbul Kandilli 249 and his *Qawīm Zīj* is in the Museo Naval de Madrid – see *Vernet 2*, pp. 297-298.

Prof. F. Sezgin has recently drawn attention to a manuscript which he lists as a Maghribi recension of the *zīj* of the eighth-century astronomer al-Fazārī (on whom see the article in *DSB* by D. Pingree), this manuscript being supposedly preserved in Rabat. See further *Sezgin*, VI, p. 123, no. 1. The *zīj*, entitled *al-Zīj al-Qawīm*, has, however, nothing to do with al-Fazārī – see already the discussion in *King 4*, p. 57. Prof. George Saliba of Columbia University informs us that the manuscript is actually in Fez, not Rabat, and that it was listed in a catalog of rare manuscripts exhibited at the Qarawiyyin in Fez in 1960. In this catalog, published in Rabat, the author is identified as Muḥammad b. Ibrāhīm, that is, Ibn al-Raqqām, not al-Fazārī. Thus this manuscript is probably another copy of the *Qawīm Zīj* of Ibn al-Raqqām.

[3] On Ibn al-Qunfudh see *Suter 1*, no. 422; *Renaud 1*, no. 422; *Cairo Survey*, no. F25; and also note 4 on p. 4 above.

[4] On the *zīj* of Ibn al-Shāṭir see *Kennedy 1*, no. 11. The Algerian recension is extant in MS Cairo DM 533.

[5] On the *zīj* of Ulugh Beg see *Kennedy 1*, no. 12. On the copies of the Tunisian recensions preserved in Cairo see *Cairo Survey*, nos. F47 and F53-55.

[6] On Zacuto see the article in the *DSB* by J. Vernet. On the manuscripts of his almanac see *Goldstein 2*, especially pp. 239-248. MS Cairo DM 1081 contains a Maghribi version of the almanac and several Arabic treatises relating to it – *cf. Cairo Survey*, no. F 31.

In the fourteenth century extensive sets of tables for time-keeping by the sun and stars and for regulating the astronomically-defined times of prayer were compiled in Tunis after the model of the tables currently in use in Damascus.[5] Another smaller set of tables for regulating the times of prayer was prepared for different localities in Morocco.[6] A sundial from fourteenth century Tunis reflects the interest of the Maghribis in times of day with special religious significance that had no counterpart in contemporary practice in Mamluk Egypt and Syria.[7] The times are not displayed on a later Tunisian sundial in the Mosque of Sīdī ^{c}Uqba in Qayrawān,[8] but yet other times are tabulated in some Ottoman prayer-tables for Algiers.[9] The position of the *mu^{c}addil* appears to have been the Maghribi equivalent to of the *muwaqqit* of the Mamluk world, that is, the astronomers associated with mosques and madrasas who were responsible for regulating the astronomically-defined times of the five, or occasionally in the Maghrib six, daily prayers.[1]

As elsewhere in the medieval Islamic world there existed in the Maghrib alongside this scientific activity in astronomy a tradition of primitive folk astronomy. The pronouncements of one Abū Miqra^c, who lived in the thirteenth century, were accorded far more respect than was warranted by their scientific content.[2]

About the year 1300 the astronomer Ibn al-Bannā' compiled an almanac of the same kind as the earlier and better-known *Calendar of Cordova.*[3] At the end of the fourteenth century a certain al-Jādarī wrote a poem on timekeeping which was much commented upon in later centuries.[4] This kind of material is worth studying for its own sake but also has special rewards for the historian of science: in an anonymous commentary on al-Jādarī's poem compiled in Tlemcen in the sixteenth century there are accounts of considerable historical interest concerning earlier Maghribi activity in measurements of the obliquity of the ecliptic (see above), trepidation, and twilight determinations.[5]

Astronomical activity in the Maghrib continued until the colonial period, but by then the great *zījes* of Ibn Isḥāq and Ibn al-Bannā' and most of the underlying theory had been long forgotten. Rather, a plethora of poems on folk astronomy and on the use of the almucantar and sine quadrants for timekeeping were the favorite reading of those who passed as astronomers.[6] As we have shown, the earlier Maghribi tradition was relatively rich and is of considerable importance to the history of Islamic astronomy. Furthermore as we have noted, most of the relevant sources have yet to be studied properly. The historical and biographical sources must also be exploited before we can gain a clearer picture of astronomy in the medieval Maghrib.

3. *Mean Motion Parameters and Apsidal Positions*

From the mean motion tables the underlying base parameters were "squeezed" by a process of successive divisions of total mean travel by the respective time spans involved. The results, in degrees per day, are tabulated below, accompanied by comments where appropriate.

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[1] See, for example, *Mayer*, p. 32 on the works of Abū Bakr b. Yūsuf of Marrakesh and Supplement, p. 294 on ^{c}Alī b. Ibrāhīm of Taza. (Another incomplete astrolabe made by him is preserved in the Musée d'Histoire des Sciences in Geneva.)

[2] See Mayer, p. 67 *sub* Muḥammad al-Ḥabbāk, p. 77 *sub* Muḥammad aṣ-Ṣinhājī, p. 73 *sub* Muḥammad b. Muḥammad b. al-^{c}Arabī, and *Azzawi*, p. 216 *sub* Ibn al-Lajā'ī, for references to the historical sources on this clock, and more recently *Price* for a thorough investigation. On the clock in Damascus see the brief remarks in the article on Ibn al-Shāṭir in *DSB* by D. A. King.

[3] See *Mayer*, p. 40 *sub* ^{c}Alī b. Aḥmad, and also *Price*.

[4] See, for example. *Mayer*, pp. 60-61 *sub* Muḥammad b. Aḥmad, and also *Janin* on a Tunisian quadrant.

[5] On these Tunisian tables see the brief remarks in *King 1*, pp. 192-193 and on the Syrian tables see *King 2*. More information is contained in the forthcoming *Studies in Astronomical Timekeeping in Medieval Islam* by the second author. The various Tunisian tables are preserved in MSS Berlin Ahlwardt 5724 and Cairo DM 689.

[6] These tables are extant in MS Cairo TR 338,2 - see *Cairo Cat.* and *Survey*, no. F35 for details.

[7] See *King 1* for a detailed description of this sundial. (On p. 189 the dimensions of the sundial should be 24 cms. × 24 cms. and not 24 × 34 as stated.) See also *King 3*, pp. 367-370 on some later Maghribi and Andalusian texts on sundial theory.

[8] *Cf. Janin*, pp. 208-211 and *King 3*, pp. 369-370 on this instrument.

[9] These tables are preserved in MS Cairo ṬFT 9, 1: see *Cairo Cat.* and *Survey*, no. F68 for details.

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[1] See, for example, *Mayer*, p. 67. Nevertheless, the term seems to relate originally to an astronomer capable of reckoning the equations (*ta^{c}dīl*) of the sun, moon, and planets.

[2] On Abū Miqra^c see *Colin & Renaud*. See also *Cairo Survey*, no. F17 and F49.

[3] Translated in *Renaud 8*.

[4] On al-Jādarī see *Suter 1*, no. 424a; *Renaud 1*, no. 424a; and *Cairo Survey*, no. F26.

[5] This commentary is extant in MSS Cairo K 4311 (defective) and also London B.L. 411, 2.

[6] On some late Maghribi astronomical works see *Renaud 2* and *7*.

sun	0;59,8,11,30,5,56, close to the value in the *Toledan Tables* (see *Toomer, 1*, p. 44) which is that of Ibn al-Bannā' (see *Vernet, 1.*).
solar apsidal motion	0;0,0,2,7,11, found with Ibn al-Bannā' due to al-Zarqāllu, see *Toomer, 2*, p.316.
moon	13;10,34,52,48, the same as al-Khwārizmī, ibn al-Bannā', and the *Toledan Tables*.
moon (anomaly)	13;3,53,56,19 essentially the value of Ptolemy and many others, including the *Toledan Tables*.
lunar nodes	– 0;3,10,46,57,52, close to Ibn al-Bannā' and the *Toeldan Tables*.
Saturn	0;2,0,27,50,55, close to Ibn al-Bannā'.
Jupiter	0;4,59,7,37,54, close to Ibn al-Bannā' and the *Toledan Tables*.
Mars	0;31,26,30,0,51,
Venus (anomaly)	0;36,59,28,13,46,16, close to the value of the Ilkhānī Zīj (cf. *Kennedy*, Zīj No.6).
Mercury (anomaly)	3;6,24,7,55
trepidation	0;0,0,53,20,31

These numbers exhibit a relation to Andalusian and Maghribi astronomy, which is not surprising. It will be seen in the next section that al-Qusunṭīnī's planetary equation tables are simplified versions of those of al-Khwārizmī, the extant version of whose zīj was transmitted via Muslim Spain. Nevertheless the mean motions above are independent of al-Khwārizmī's ultimately Indian parameters (cf. *Neugebauer 2*, p.93, and *Burckhardt.*).

On f. 50r (and again on 53r) the following list of apogee longitudes is given, with no date (a superscript *s* denotes a zodiacal sign, i.e. 30°):

Saturn	7^s 29;43°
Jupiter	5 9;43
Mars	4 2;13
sun	2 17;19
Venus	2 17;19
Mercury	6 18;24

A marginal note, apparently in the same hand as the text, says that the distance from the apogee of Mercury to that of the sun is 4^s 1;8°. In fact, since 2^s 17;19° + 4^s 1;8° = 6^s 18;27°, the statement is almost correct. Since in the Arabic alphabetical numeral system the symbols for 4 (د) and 7 (ز) are easily confused, restoration of Mercury's apogee to 6^s 18;27° would make the note correct.

For the three superior planets the distances between their apogees is exactly the same as those in al-Battānī's zīj (*Nallino*, vol. 1, p.241). But al-Qusunṭīnī's whole set of apogees is a rounded off version of Ibn al-Bannā's, which is given to seconds (*Vernet 1*).

It is worth remarking that 2^s 16;44,17°, the position given for the solar apogee at the Hijra epoch (on f. 49r, cf. *Azarquiel*, p.352) is very close to the 2^s 16;45,21° used by Ibn al-Kammād, a student of al-Zarqāllu (*Toomer 2*, p. 321). It is possible that the discrepancy is due to a difference in the calculation of precession.

4. *Planetary Equation Tables*

By and large, these tables are, for al-Qusunṭīnī, the same as the analogous ones in the Khwārizmī *zīj* (*Suter 2*, pp.132-167), except that, whereas in the latter the entries for sun and moon have been carried to seconds, in the former they have been truncated (not rounded) to minutes. Thus the solar and lunar tables have been calculated by the "method of declinations":

$$e = e_{\max} \,.\, \delta(\kappa)/\varepsilon \,,$$

where e is the equation, δ the solar declination function, κ the center (see Section 5 below and the accompanying figure), and ε is the obliquity of the ecliptic. The planetary equations of the center are:

$$e = e_{\max} \sin \kappa,$$

hence were computed by the "method of sines". The epicyclic equations are based on the standard eccentric model.

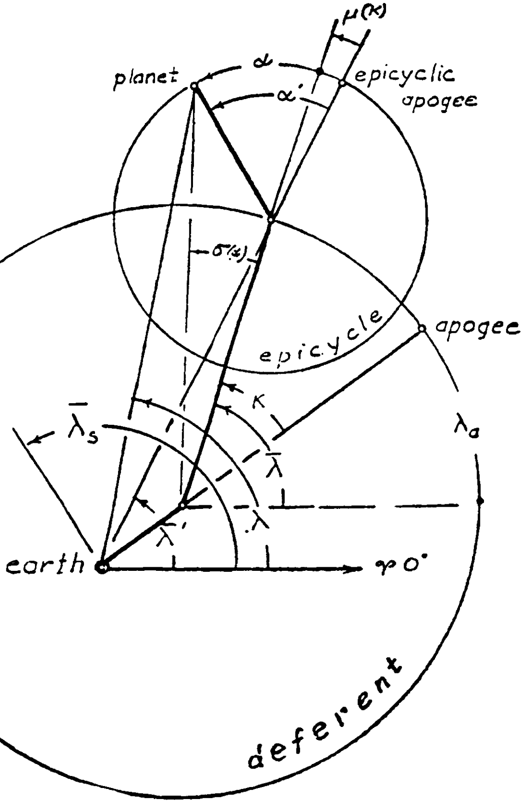
	center	epicyclic
sun	2;1[4]°	
moon	4;56	
Saturn	8;3[6]	5;44°
Jupiter	5;[6]	10;52
Mars	11;13	40;31
Venus	2;14	47;11
Mercury	4;1	21;30

where square brackets around a digit indicate restorations of scribal errors. Of these there are a good many. For instance, by plotting each of the ninety entries in the solar equation table it can be shown that about a dozen of them are erroneous.

The numbers cited above are standard parameters of Indian astronomy. The method of declinations may be from Sasanian Iran or early Islamic; it is not Ptolemaic (see *Neugebauer 2*, pp. 95-101).

5. *Calculation of True Longitudes*

Since the evidence upon which our further inferences are based is somewhat ambiguous, it will be useful to preface its presentation with a sketch of several ancient planetary models, to which al-Qusunṭīnī's is related. For all these models the orbits of the planet and the earth about the sun can be thought of as represented by two circles, the *deferent* and the *epicycle* shown in the figure below. Which circle stands for which orbit depends upon whether an inner or outer planet is being considered. Without essential loss of generality, the figure and the discussion below are taken to be for an outer planet.



An eccentric (non-equant) model
for planetary motion.

The simplest (and earliest) of the models described has the earth at the center of the deferent. The planet advances along the periphery of the epicycle at constant speed whilst the epicycle center traverses the deferent with a different constant speed. At any instant the locations of the epicycle center and the planet on the epicycle are given by two linear functions of time: $\bar{\lambda}$, the *mean longitude*, and α, the *argument* of the epicycle anomaly. Then the true longitude is

$$\lambda = \bar{\lambda} + \sigma(\alpha), \tag{1}$$

where σ is the *epicyclic equation.* Note from the picture that σ causes periodic variations in λ's rate of change; alternately λ leads $\bar{\lambda}$, then lags behind it.

At some time it was realized that (1) is too simple to yield precise predictions of position for planets. The deferent was made eccentric, its center being displaced from the earth. This caused a second periodic irregularity in the planet's motion, $\mu(\kappa)$, the *equation of the center,* where $\kappa = \bar{\lambda} - \lambda_a$ is the *center.* $\bar{\lambda}_s$ is the solar mean longitude.

The addition of the second equation greatly complicated the calculation of true longitudes. The two equations cannot simply be added algebraically to $\bar{\lambda}$ because they interact with each other in a complicated manner. For one thing, the initial point from which the argument is measured, the *epicyclic apogee,* oscillates back and forth with respect to its fixed position in the simple model. And secondly, the distance from earth to epicycle also varies. When the epicycle retires from the earth, its effect is diminished, and conversely.

Tables of the σ and μ functions were prepared, the arguments α and κ being determined from the mean motion tables. Neither equation is in principle symmetrical with respect to an α or κ of 90°. Nevertheless it was customary in Indian and Sasanian Iranian astronomy to use for μ a sine wave of amplitude $\mu_{\max}$ for each planet. In addition to the equation tables, some computational device was necessary, to give numerical effect to the interaction described above between the equations.

Indian astronomers used an ingenious if complicated technique to attain this end. Its main lines are indicated by the expression

$$\lambda = \bar{\lambda} - \mu_2 + \sigma_2, \tag{2}$$

from $\sigma_1 = \sigma(\alpha)$, $\lambda_1 = \bar{\lambda} + \frac{1}{2}\sigma_1$, $\kappa_1 = \kappa + \frac{1}{2}\sigma_1$, $\mu_1 = \mu(\kappa_1)$, $\kappa_2 = \kappa_1 - \frac{1}{2}\mu_1 + \frac{1}{2}\sigma_1$, etc. The general idea was to merge the effects of the two equations by successively introducing half of the one into the determination of the other. There were variants of the basic approach, some rules halving only one equation, and some neither. Details will be found in *Neugebauer 1,* and *2,* pp. 23-30.

A basic improvement was effected by Ptolemy's introduction (ca. 150 A.D.) of the *equant,* a device to introduce a periodic variation in the speed of the epicycle center along the deferent. After suitable modification of the μ functions, Ptolemaic longitudes are calculated by the expression

$$\text{(3)} \qquad \lambda = \bar{\lambda} - \mu(\varkappa) + \sigma(\alpha') + I(\varkappa') \,.\, \Delta\sigma(\alpha') \,,$$

where $\alpha' = \alpha + \mu(\varkappa)$, $\varkappa' = \varkappa - \mu(\varkappa)$, I is an interpolation function varying between ± 1, and $\Delta\sigma$ is the difference between σ calculated at minimum and maximum epicycle distances.

For the simple eccentric configuration illustrated in our figure a practical and accurate mode of determining true longitude (nowhere intimated in an extant text, so far as we know) would be to put

$$\text{(4)} \qquad \lambda = \bar{\lambda} - \mu(\varkappa) + \sigma(\alpha') + I(\varkappa) \,.\, \Delta\sigma(\alpha') \,,$$

where α', I, and $\Delta\sigma$ are as indicated above.

We will seek to show that the model intended by al-Qusunṭīnī's zīj is

$$\text{(5)} \qquad \lambda = \bar{\lambda} - \mu(\varkappa) + \sigma(\alpha') = \bar{\lambda}' + \sigma(\alpha') \,,$$

where we call $\bar{\lambda}'$ the *modified mean*. This is expression (4) with the last term missing. That is to say, it takes cognizance of the nodding of the epicyclic apogee about its mean position, but it ignores the effect on σ of the varying epicyclic distance from the earth.

Aside from the tables themselves, all the information upon which these conclusions are based is found in Section 7 of the verse introduction (beginning on folio 45r) which describes the calculation of λ for the superior planets. The next section does the same for Venus and Mercury, but adds nothing significant.

Section 7 of Qusunṭīnī's zīj is translated below. Parentheses are used to denote beginnings of lines in the text, and to interpolate explanatory material. The redundant verbiage in the text consists of words or phrases introduced to pad out the meter and the rhyme.

(f.46r:20) The first of those are Saturn and Jupiter, and after those two, Mars, indubitably. (21) Extract the mean ($\bar{\lambda}$) for that situation, for any one of them you choose (?), along the succession (of the zodiacal signs). (22) Then, without fail, subtract it from the solar mean properly; (23) there will remain for you the argument ($\alpha = \bar{\lambda}_s - \lambda$) in this operation. Retain it without fail. (24) Then subtract its apogee from the mean. There will remain for you the center ($\varkappa = \bar{\lambda} - \lambda_a$) in this style. (25) Then enter with it according to what you see for the center, (obtaining) its equation ($\mu(\varkappa)$) there, for distinguishing (it). (26) Add it to the center, then to the mean (i.e., form $\varkappa + \mu(\varkappa)$ and $\bar{\lambda} + \mu(\varkappa) = \bar{\lambda}'$, sic), for any planet you suppose as a condition. (27) But subtract it ($\mu(\varkappa)$) from its (the planet's) argument if its center is greater than six signs – obtain it, (f.46v:1) but if it is less than that number ($\varkappa < 6^s$), (do) the opposite with it, do not add continuously (i.e. form $\alpha' = \alpha + \mu(\varkappa)$ algebraically). (2) Enter with this modified argument (α') where you see it registered in the table, (3) and obtain the epicyclic equation ($\sigma(\alpha')$) at its place. Look at the argument a second time, (4) and if you have zodiacal signs exceeding six, subtract (the epicyclic equation from the modified mean). Then note (5) any modified planetary mean (here λ is intended) as you find it, in its resulting place. (6) But if the modified argument is less than your signs (i.e., if $\alpha' < 6^s$) (7) add it (the epicyclic equation) to the mean, and its place (i. e. true longitude) will be there, and note, it, and do not lose it.

Several conclusions are immediate and unequivocal. The equation functions are of Indian (or Iranian), provenance with no trace of Ptolemaic influence. On the other hand, the characteristic "halving of the equation" is conspicuously absent .The calculation of α' is described completely and correctly. The only objection to the adoption of expression (5) arises from the author's prescription of $\bar{\lambda}'$ as being $\bar{\lambda}' + \mu(\varkappa)$ instead of $\bar{\lambda}' - \mu(\varkappa)$ as it should be. We must bear in mind, however, that since negative numbers were generally unknown to medieval scientists, they were often constrained to split a rule into special cases if a function were sometimes positive and sometimes negative. The complete rule would then demand addition in one case and subtraction in the other, or vice versa. It is possible that a complete couplet has been dropped from al-Qusunṭīnī's poesy by a careless scribe. If the passage beginning with line 25 could be restored as

> Then enter with it according to what you see for the center, (obtaining) its equation ($\mu(\varkappa)$) there, for distinguishing it. [If the center is less than six signs, subtract the equation from the center, then from the mean. But if it is more than six signs] add it to the center, then to the mean...

the rule would be (5) without flaw. Or perhaps, in hammering out his doggerel, the poet inadvertently left out our restoration. At any rate we prefer not to accuse al-Qusunṭīnī of having been an originator. We suspect he obtained the algorism from a sequence of predecessors, including perhaps Maghribi, early Islamic, Indian, and pre-Ptolemaic Greek elements. The discovery of additional texts may settle the issue. Meanwhile we favor expression (5).

It is also possible that in its original form the procedure contained some sort of "halving the equation" routine, as in (5), which was dropped some where along the chain of transmission.

6. *Table of Contents of the Zīj*

folio

Introduction, with the customary praise of God, His Prophet, and the author's patron. 44v

Section (faṣl) 1, *On Foreign* (ʿajam) *Calendars* (a description of the use of tables to transform a date from the Hijra to a different calendar) 45r

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Section 2, On Determining the Day of the Week on Which a Given Arab (Hijra) *Year Begins*	45r
Section 3, On Determining the Initial Day of the Week of Months of Foreign (Calendars)	45v
Section 4, The Solar Equation	45v
Section 5, On (true) *Positions of the Moon and Its Equation*	46r
Section 6, On the Lunar Node	46r
Section 7, *On the* (longitudes) *of the Superior Planets*	46r
Section 8, On (the longitudes of) *Venus and Mercury*	46v
Section 9, Is the Planet Retrograde or in Forward Motion?	46v
Section 10, In Explanation of Trepidation	46v
Section 11, On Obtaining the Ascensions of the Signs	47r
Section 12, On the Degrees of Rising with the Equation	47r
Section 13, On How (to determine) *the Transfer* (ascendant)	47r
Section 14, On the Equalization of the Houses	47v
Section 15, On (first) *Visibility of the* (lunar) *Crescent*	47v
Section 16, On Determining the Lunar Latitude	48r
Section 17, On al-Faḍl al-Muqawwam	48r

(This seems to be a measure of the amount by which the planet has passed the last cardine, perhaps for finding its house.)

Section 18, On Determining (the astrological doctrine of) *the* Tasyīr.	48r
Section 19, On the Determination of Eclipses	48v
Table of the Solar Apsidal Motion	49r

All entries are to seconds of arc. Apsidal *motions* are given for:

1,2,3, ... , 30 days,
1,2,3, ... , 12 (Hijra) months,
1,2,3, ... , 30 (Hijra) years.

This table is practically identical with one in the zīj of Ibn al-Bannā' (*Vernet 1*) found on f. 15r in the same MS. It was published in *Millás*, p. 352, see also Section 3 above.

For the Hijra epoch *Millás* (*ibid.*) gives $2^{s}\,16;44,17^{\circ}$. In the Escorial manuscript of Ibn al-Bannā''s *zīj* (fol. 15r) there is a marginal note that the apogee in 990 Hijra (=1582) is $2^{s}\,20;10,51^{\circ}$, which is consistent with the Hijra epoch position and the motion of $3;26,33^{e}$ for 990 lunar years given in the table. A marginal note, in the same hand, to al-Qusunṭīnī's table (fol. 49r) gives the apogee in 990 Hijra as $2^{s}\,20;12,27^{\circ}$, for reasons best known to the writer of the note.

	folio
Table for Extracting the Rūmī Date (i. e. Seleucid epoch, Julian years) *from the Arab* (i. e. Hijra)	49v

For 480, 510, 540, ..., 900 H the equivalent Rūmī date is given in years, months, days, and minutes (i. e. sixtieths) of days.

For 1, 2, 3, ... 30 Hijra years,
1, 2, 3, ... 12 Hijra months,
1, 2, 3, ,,, 12 Latin months (beginning with October),

the elapsed time is given in Rūmī years, months, days, and minutes of days. The same table is in *Suter 2*, Table 3, *Sedillot-père*, p. 97, and Ibn al-Bannā'.

A Table of Signa (initial weekdays) *of the Arab* (i.e. Hijra) *Years and their Months, and the Apogees*	50r

The entries are changes in the signa for:

30, 60, 90, ..., 210 years,
1, 2, 3, ..., 30 years (not in order).
1, 2, 3, ..., 12 Hijra months.

There is a table of planetary apogees, to minutes of arc, transcribed and discussed in Section 2 above. The list is repeated at the top of f.53r. The same table is in *Suter 2*, Table 2, and Ibn al-Bannā'.

A Table of Signa of the Foreign (^{c}ajamiya) *Months in the Calendar of the Two-Horned* (Alexander, i.e. Rūmī)	50v

This is a rectangular, double argument table, in which the entries are signa, and the arguments are:

1, 2, 3, ..., 27 (Julian) years (since 28 = 7 days/week × 4, the leap cycle)

and Oct., Nov., Dec., ..., Sept.

The leap years are also indicated. This table is also in *Suter 2*, Table 3a, and Ibn al-Bannā'.

Table of the Solar Mean Motion in Hijra Years, for Noon at the City of Fez	51r

All entries are to seconds. Motions are given for:

folio

1, 2, 3, ..., 30 days ,
1, 2, 3, ..., 12 Hijra months,
1, 2, 3, ..., 30 Hijra years.

Positions are given for

600, 630, 660, ..., 990 H.

Tables of Lunar Mean Motion, Anomaly, and Nodes 51v–52v

Layout, arguments, and precision as for the sun.

Tables, Mean Motion of Saturn, Jupiter, and Mars 53r–54r

Layout, etc., as for the sun.

Tables, Anomalistic Argument of Venus and Mercury, as for the sun. 54v–55r

Table of Hourly Planetary Mean Motions 55v

Entries are to seconds, for 1, 2, 3, ... 24 hours, for the sun, moon, lunar anomaly, lunar nodes, Saturn, Jupiter, Mars, and the anomalistic arguments of Venus and Mercury.

Table of the Motion of Trepidation 56r

Layout, arguments, and precision as for the mean sun.

Table of the Solar Equation 56v

Entries are to minutes of arc for each degree of the argument. The function is discussed in Section 3 above.

Table of the Lunar Equation 56v

Same layout, arguments, and precision as for the solar equation. See Section 3 above.

Table, The Anomalistic Equation of Saturn 57r

Entries are to minutes of arc for each degree of the argument .The function is discussed in Section 3 above.

Table Equation of the Center, for Saturn 57r

Domain of the argument and precision of entries is as for the other equation of Saturn. See Section 3 above.

Tables, Equations of the Anomaly and Center, for Jupiter and Mars 57v-58r

Layout, arguments, and precision are as for Saturn.

Tables , Equations of the Anomaly and Center, for Venus and Mercury, as for Saturn. 58v–59r

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folio

Table of First Stations of the Five Planets (see *Kennedy 1*, p.142) 59v

Entries are to minutes of arc for argument 6°, 12°, 18°, ..., 180°.

Essentially this is the table of al-Battānī (*Nallino*, vol. 2, pp. 138-9), hence originally from Ptolemy's Handy Tables. See also *Suter 2*.

Table, Equation of the Trepidation Motion 60r

Argument range: $\Theta = 1^\circ, 2^\circ, 3^\circ, \ldots, 360^\circ$.

Entries, to minutes, are close to $10;45 \sin \Theta$, Thābit (in *Vernet 1*, p, 91, note 182 and p. 92, note 187) has a maximum of 10;45°, and al-Marrākushī (in *Sédillot-père*, p. 131) has 9;59°.

Table of Right Ascensions 60v

Entries are to degrees (sic) for each degree of the argument. The Function is in fact the normed right ascension function, $A_0(\lambda)-90^\circ$, commencing with Capricorn.

Table of Oblique Ascensions for (the latitute of) *Fez* 61r

Layout and precision as in the preceding table, except that this commences from Aries.

Table of Evening Lunar Crescent Visibility, 61v

transcribed below. The same table appears in Paris MS B.N. Or. 2513, f. 71v, of the thirteenth-century Egyptian *Muṣṭalaḥ Zīj*; f. 58v of an unnumbered Maghribi astronomical manuscript in the Museo Naval de Madrid; MS Cairo Dār al-Kutub MM 23, f. 9r, of a small *zīj* compiled in Cairo *ca.* 1700; MSS Milan Ambrosiana C82, front flyleaf, and Escorial ar. 966, f. 192v of a redaction of the astronomical tables of the late-fifteenth-century Spanish Jew Abraham Zacuto (see note 15 to Section 2) prepared in Istanbul in the early sixteenth century; and in MS Cairo ṬM 119, f. 1r on the title folio of an Egyptian copy of an early Iraqi astrological treatise.

The table from al-Majrīṭī's recension of al-Khwārizmī's *zīj* investigated by *Kennedy & Janjanian* is unrelated to al-Khwārizmī. It is also found in MS Hyderabad Andra Pradesh State Library 298 of the *zīj* of Ibn Isḥāq (see note 9 to Section 2) where it features as table no. 160. Here the table is attributed to an individual called al-Qallās, whose name is new to the literature. This table is computed for a latitude in northern Spain. Al-Khwārizmī's table for Baghdad is contained in

MS Paris B. N. ar. 6913, f. 102r of *al-Zīj al-Riqānī*, an eleventh-century compilation; MS Escorial ar. 927, f. 6r of the anonymous recension of the ninth-century *Mumtaḥan Zīj*; and MS Cairo ṬFF 11, f. 61r of the eleventh-century Persian astrological handbook entitled *Rauḍat al-munaijjimīn*.

Each of these tables is investigated in a forthcoming study by the second author on early Islamic tables for determining lunar crescent visibility.

ZODIACAL SIGNS	CLIMATES 1st	2d	3d	4th	5th	6th	7th
Aries	11;24	11;4	11;19	10;6	11;17	9;9	9;28
Taurus	11;11	11;24	10;33	10;21	10;12	9;18	9;28
Gemini	11;2	11;11	10;10	10;32	9;29	9;24	9;3
Cancer	11;10	11;15	11;38	10;32	12;25	12;46	12;9
Leo	13;14	13;18	13;4	15;0	16;7	16;17	13;15
Virgo	14;27	16;19	17;2	17;10	23;27	23;21	24;50
Libra	15;2	16;7	18;19	19;4	21;28	22;24	24;1
Scorpio	14;12	14;32	16;19	17;17	13;2	19;42	21;2
Sagittarius	12;0	18;39	13;18	14;42	14;31	14;2	14;31
Capricorn	11;10	11;45	11;21	11;26	11;0	11;9	11;45
Aquarius	11;3	11;47	11;2	11;4	9;15	9;7	9;15
Pisces	11;24	11;11	11;9	10;9	9;11	9;0	8;14

Table of Lunar Crescent Visibility, f. 61v

folio

Table of Eclipses 62r

There are in fact two tables, transcribed below, one for lunar, one for solar eclipses. For each there are two arguments:

1, 2, 3, ..., 11, distance between moon and node,

12, 13, 14, 15, deg./day lunar motion.

Entries give the eclipse magnitude in integer digits.
This is a garbled version of a table given by *Ibn al-Bannā'*.

Dist. betw. moon and node	True daily lunar motion 12	13	14	15
	Amount of the moon's body eclipsed			
	digits	digits	digits	digits
1	12	12	12	12
2	12	12	12	12
3	12	12	12	12
4	12	12	12	12
5	12	12	12	12
6	8	10	11	11
7	7	6	9	10
8	4	5	7	8
9	2	3	4	6
10	0	0	2	3
11	0	0	0	2

Dist betw. moon and node	True daily lunar motion 12	13	14	15
	Amount of the sun's body eclipsed			
	digits	digits	digits	digits
1	12	12	12	12
2	12	12	12	12
3	12	12	12	12
4	8	8	10	11
5	6	6	8	9
6	4	4	6	6
7	3	3	4	6
8	1	2	3	3
9	0	1	1	2
10	0	0	0	2
11	0	0	0	1

folio

Table of Lunar Latitude 62v

The argument is λ = 1°, 2°, 3°, ..., 360°.

Entries are 4;30° sin λ , to minutes, the standard Indian method. Cf. *Suter 2*, Tables 21-26.

Table of Eclipse Colors 63r

The table is in two parts. On the right the argument is: 10′, 20′, 30′, ..., 60′ of lunar latitude. Entries are colors of the moon.

On the left the argument is 5′, 10′, 15′, ..., 35′ of lunar latitude. Entries are colors of the sun.

The same table is in *Ibn al-Bannā'*.

Colophon (No date or name is given). 63v

NB — Plates on pp. 22 to 41 have been omitted in this reprint.

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al-Zarqāllu	See *Millás* and *Toomer 2*.

IX

Ibn Yūnus' Very Useful Tables for Reckoning Time by the Sun

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Part 1: Introduction

In this paper I describe a newly rediscovered set of mathematical tables attributed to the tenth century Egyptian astronomer *Ibn Yūnus*. These tables, which I call the *Very Useful Tables* after their Arabic title *Kitāb Ghāyat al-intifāʿ*, are for time-keeping by the sun and regulating the astronomically defined times of Muslim prayer.

1.1. On Ibn Yūnus and His Works[1]

Very little of consequence is known about the life of *ʿAlī b. ʿAbd al-Raḥmān* called *Ibn Yūnus*. His father *ʿAbd al-Raḥmān* was a famous historian of Egypt who died in the year 958, and *Ibn Yūnus*' known astronomical activity dates from about 975 to his death in 1009. Thus, it was during *Ibn Yūnus*' youth in Fustat that the Fatimids conquered Egypt and subsequently founded Cairo in 969. In the few biographical references which have survived, *Ibn Yūnus* is revealed as a highly eccentric figure who devoted most of his life to astronomy, astrology, and poetry.

Ibn Yūnus' major work was a *Zīj*, that is, an astronomical handbook with tables,[2] which was dedicated to the Fatimid Caliph *al-Ḥākim* although it was clearly begun in the reign of his father *al-ʿAzīz*. Unfortunately, not all of this extensive work is extant, but if we judge him by the parts of it which have survived in manuscripts in Leiden, Oxford, and Paris, *Ibn Yūnus* stands out as one of the greatest astronomers and mathematicians of the Middle Ages. His works were virtually unknown in Islamic Spain, however, and hence were unknown in medieval Europe; his importance was first realized in the West in the mid-seventeenth century when the Leiden fragment of his *Zīj* was investigated by the Dutch orientalist *J. Golius*. The Leiden manuscript contains accounts of about one hundred observations of eclipses and planetary conjunctions, observations made by *Ibn Yūnus* in Cairo and by certain of his predecessors in Damascus and Baghdad. His purpose in presenting these accounts was two-fold. Firstly, he wished to justify the need for recomputing planetary parameters by illustrating how computations with the *Mumtaḥan Zīj* of *Yaḥyā b. Abī Manṣūr*, prepared for the Abbasid caliph *al-Maʾmūn* about the year 810, were inconsistent with his own observations, and were even already inconsistent with the observations of certain reliable astronomers of the ninth century. Secondly, he simply wanted to list these observations for the benefit of future astronomers. In the first aim he was less successful than in the second: later astronomers in Egypt still used the *Mumtaḥan Zīj* alongside the *Ḥākimī Zīj*. On the other hand, his observational accounts, published by *Caussin* in 1804, have been used as a source of data in modern estimates of secular variations in the motion of the moon and planets, notably by *S. Newcomb* (1878), and most recently by *R. Newton* (1970). I have prepared a critical text and new translation of the observational accounts, publication of which awaits completion of the analysis.

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[1] Full references to the evidence for the assertions made in this Section are to be found in *King* [1], currently being revised for publication. On *Ibn Yūnus*' observational accounts see *Caussin* [1] and on modern computations based on the data contained in these accounts see *Newcomb* [1] and also *Newton* [1].

[2] See *Kennedy* [1] for a survey and preliminary classification of over one hundred Islamic *zījes*, and for information on all *zījes* mentioned in this paper.

These extensive observational accounts are unique in the Islamic astronomical literature. Consequently, one's expectations of *Ibn Yūnus'* other achievements are high. In 1817 the French orientalist *J.-J. Sédillot* completed a translation of the contents of both of the Leiden and Paris manuscripts. This translation was later lost, but fortunately not before *J.-B. Delambre* had made a brief, though not always accurate, analysis of it. Studies on *Ibn Yūnus* were resumed by the German scholar *C. Schoy* in a series of valuable papers published between 1915 and 1925. *Schoy* worked mainly from the Leiden and Oxford manuscripts, and his untimely death in 1925 brought studies of *Ibn Yūnus'* works to a halt for almost fifty years. My own studies have involved an analysis of all three manuscripts and have resulted in the location of other manuscripts containing certain material from the *Ḥākimī Zīj* hitherto deemed lost, and of other works by *Ibn Yūnus*, such as an extensive table giving the lunar equation as a function of the double elongation and the mean anomaly, and the *Very Useful Tables* for time-keeping.

Ibn Yūnus indeed displays far more scientific initiative than, say, his predecessor *al-Battānī*, by whose achievements Islamic astronomy has too frequently been judged as a mere intermediary between Hellenistic and medieval European astronomy. *Ibn Yūnus* was familiar with *al-Battānī*'s *Zīj*, a work closely modelled on *Ptolemy*'s *Almagest* and *Handy Tables*, but from references in his own *Ḥākimī Zīj*, which is several times larger than *al-Battānī*'s and far more detailed, it appears that he held the earlier *Zījes* of *Yaḥyā b. Abī Manṣūr*, *Ḥabash*, and *al-Nayrīzī*, in greater esteem. These three works are no longer extant in their original form and the surviving fragments await study. In contrast, the *Zīj* of *al-Battānī* is extant in its entirety and was published by *C. Nallino* over seventy years ago.

Ibn Yūnus was clearly an active observer of celestial phenomena, and it has often been stated that he worked in a well-equipped observatory. *A. Sayili* (1960) has shown, however, that there is no evidence for this assertion, and I have no information which would require that his conclusions regarding *Ibn Yūnus'* observations be modified.[3] Unfortunately, *Ibn Yūnus* does not describe any observations made to derive the planetary parameters on which his tables are based. Unfortunately, also, the material on parallax and eclipse theory in the *Ḥākimī Zīj* is not preserved in the known manuscript sources. *Ibn Yūnus'* planetary tables were held in considerable esteem by certain later Islamic astronomers, and they were used in Egypt, Persia, and the Yemen. Some of his parameters were adopted by *Naṣīr al-Dīn al-Ṭūsī* in the *Ilkhānī Zīj* (*ca.* 1260), and hence or directly, by *Chrysococces* (*ca.* 1350); his star tables were also used by Byzantine astronomers. The greatest influence of *Ibn Yūnus'* planetary tables appears to have been in the Yemen: each of several medieval Yemeni *Zījes* known to me contains material originally due to the Egyptian scholar.

Most of the extant material of the *Ḥākimī Zīj*, other than the observational accounts and planetary tables, deals with spherical astronomy. *Delambre*'s brief survey has until recently been the main study of *Ibn Yūnus'* impressive contribution in this branch of astronomy; my own detailed analysis, currently being revised for publication, is a continuation of the studies of *C. Schoy*. *Ibn Yūnus* generally outlines several alternative methods for solving each of the problems of spherical astronomy, but does not indicate how he derived them: it is my contention that these methods were derived from plane projections of the sphere rather than by spherical trigonometry.

The tables which are the subject of the present paper constitute a work separate from the *Ḥākimī Zīj*, but are in the main based on the theoretical material contained in the *Zīj*. Indeed, a few of the tables have their counterparts in the *Zīj*. The functions tabulated are spherical astronomical functions for time-keeping by the sun and others relating specifically to the prayer-times. In its entirety, the work consists of about 200 pages of tables, most of which contain 180 entries. It is entitled *Kitāb Ghāyat al-intifāʿ fī maʿrifat al-dāʾir wa-faḍlihi wa-l-samt min qibal al-irtifāʿ*, *Very Useful Tables for Finding the Time Since Sunrise, the Hour-Angle, and the Solar Azimuth, from the Solar Altitude.*[4] No two manuscripts of this work contain the same tables, and the attribution of every single table in the corpus to *Ibn Yūnus* is not beyond question.

Only one earlier table for finding time of day from solar altitude has been rediscovered in the manuscript sources, namely that due to the Baghdad astronomer *ʿAlī b. Amājūr* (fl. ca. 950), whose *Zīj* was known to *Ibn Yūnus*. However, no earlier tables for regulating the times of prayer have been located yet, and it may be that *Ibn Yūnus* was the first to compile such tables. The subsequent fate of his tables—they were much plagiarized and modified in Egypt—and the development of similar tables by other Islamic astronomers will be discussed in a series of papers currently in preparation.[5]

One of *Ibn Yūnus'* tables for time-keeping involves the effect of atmospheric refraction at the horizon, a topic not discussed in the extant chapters of the *Ḥākimī Zīj*. Indeed, it was previously thought that the quantitative effects of atmospheric refraction were not investigated at all by the Islamic astronomers. I do not fully understand this table, and suspect that it is not original and that it is based on a garbled version of a theory originally propounded by *Ibn Yūnus*. Several other medieval Islamic manuscripts deal with this topic: these are extant in Cairo and Damascus, but cast little light on *Ibn Yūnus'* theory.

1.2. On Islamic Time-Keeping

The science of time-keeping by the sun and stars, called *ʿilm al-miqāt* in Arabic, was one of the main concerns of Islamic astronomy. Its object was twofold: firstly, the reckoning of time for astronomical and civil purposes; and secondly, the regulation of the astronomically defined times of prayer. The astronomers associated with the principal mosques in the main centers of Islamic culture were known as *muwaqqits*, that is, time-keepers, and at the times calculated by them the muezzins would call the faithful to prayer.

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[3] See *Sayili* [1], 130–156.

[4] The azimuth tables (*cf.* Section 3.3) are referred to as *Kitāb al-Samt* when tabulated separately (as, *e.g.*, in MS E), and the hour-angle tables (*cf.* Section 3.2) are referred to as *Kitāb Faḍl al-dāʾir* when tabulated separately (as, *e.g.*, in MS C). *C. Schoy* was familiar with the azimuth tables: see his article *Samt* in EI_1.

[5] It has been possible to show that *Ibn Yūnus'* tables and modifications thereon were used in Egypt until the nineteenth century. Also, the prayer-tables used in Syria until the last century owe their original inspiration to those of the Egyptian scholar.

The most important *muwaqqits* and their works were listed by *H. Suter* in his survey of over five hundred Islamic astronomers and mathematicians and the works attributed to them.[6] *E. Wiedemann* & *F. Hauser* have translated two important Islamic treatises on clocks, and *Wiedemann* & *J. Frank* have presented a detailed analysis of the definitions of the prayer-times in Islam.[7] Tables for time-keeping by the sun and stars are not generally contained in Islamic *zījes*, whose abundance and diversity has been indicated by *E. S. Kennedy* is his survey of over one hundred known *zījes*.[8] In fact, only one medieval table for reckoning time of day from the solar altitude has been analyzed in the literature, namely by *B. R. Goldstein*.[9] It is now known that such tables, and tables for regulating the times of prayer, exist in profusion in the vast manuscript sources available for the study of Islamic astronomy. For the historian of science these tables are of interest because of their mathematical sophistication and the accuracy with which they were generally computed.

The definitions of the times for the five daily prayers in terms of the daily passage of the sun across the sky may be traced to the practices and injunctions of the Prophet *Muḥammad*.[10] Each of the prayers is to be performed within a certain interval of time. The Muslim day begins at sunset, and the time for the evening prayer begins then and lasts until nightfall, or end of evening twilight. The night prayer may be performed any time between nightfall and daybreak, or beginning of morning twilight. The interval for the morning prayer begins at daybreak and the prayer must be completed before sunrise. The time for the mid-day prayer begins when the sun has crossed the meridian and lasts until the beginning of the interval for the afternoon prayer, namely, when the shadow of an object equals its midday shadow increased by the length of the object. In some legal schools the interval for the afternoon prayer ends when the shadow has again increased by the length of the object; in other schools it ends at sunset.

The computation of the precise times of prayer involves spherical astronomy, and the application of complicated trigonometric formulae. The times of prayer expressed in modern civil time or its medieval equivalent change throughout the year and are functions of solar longitude. They vary also with terrestrial latitude, but when measured with respect to the local meridian they do not vary with terrestrial longitude for a given latitude. In medieval times the prayer-times were often determined using astrolabes or quadrants marked with special curves, and the day-time prayers could be regulated by means of special curves engraved on sundials.[11] However, for convenience, mathematical tables giving the lengths of the intervals for each of the prayers were also prepared. In the early tables, such as those of *Ibn Yūnus*, these intervals are expressed in equatorial degrees (360° = 24 hours, and 1° = 4 minutes), and it is possible to trace the development of such tables through to the almanacs popular in the Ottoman Empire in which the earliest permitted times for each prayer are given in equinoctial hours and minutes, local true solar time. Nowadays, muezzins generally use the prayer-times given in local civil time in almanacs, calendars, and pocket-diaries, which are based on tables approved by the local religious authorities.

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[6] See *Suter* [1]. Further information on some of these Islamic scholars is contained in *Brockelmann* [1].

[7] See *Wiedemann* & *Hauser* [1] and *Wiedemann* & *Frank* [1].

[8] See footnote 2 above.

[9] *Cf. Goldstein* [1].

[10] See, for example, the article *Mīḳāt* by *A. J. Wensinck* in EI_1 and the relevant *ḥadīth* (Prophetic statements) recorded by *al-Baghawī* (d. ca. 1125), translated in *Robson* [1], I, 118–126. See also *Wiedemann* & *Frank* [1] on the definitions of the prayer-times in astronomical terms.

[11] On the use of the astrolabe, quadrant, and sundial in Islamic astronomy see respectively *Hartner* [1], 287–311, *Schmalzl* [1], and *Schoy* [1] and [2].

A related problem of spherical trigonometry is the determination of the qibla, that is, the direction of Mecca, which defines the direction of prayer in Islam and is of importance for the orientation of mosques. Its determination was a favorite problem in medieval Islamic trigonometry. Several medieval methods of considerable sophistication have been analyzed,[12] but no survey has ever been made of the orientation of medieval mosques.

1.3. On the Manuscript Sources for Ibn Yūnus' Very Useful Tables

The tables exist in several manuscripts, some containing only a few tables from three main categories:

1. Tables of time as a function of solar altitude.
2. Tables of solar azimuth as a function of solar altitude.
3. Tables for the prayer-times and auxiliary functions for time-keeping.

The following manuscripts were used in the present study and are hereafter referred to by the sigla indicated:[13]

B: MS Berlin Ahlwardt 5753 (=Landberg 574)
(contains material from 1, 2, and 3 in considerable confusion)

C: MS Cairo Tīmūrīya, *riyāḍīyāt* 191
(contains 1 and 3)

D: MS Chester Beatty 3673
(contains 1, 2, and 3)

E: MS Escorial ar. 924,7
(contains 2)

G: MS Cairo Dār al-Kutub, *miqāt* 108
(contains 1, 2, and 3)

H: MS Cairo Azhar, *falak* 4382
(contains 1 and 2)

K: MS Cairo Dār al-Kutub, *miqāt* 137 M
(contains 2)

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[12] See the article *Ḳibla* by *A. J. Wensinck* & *C. Schoy* in EI_1 and also *King* [3], in which I describe a table by the fourteenth century Damascus astronomer *al-Khalīlī* giving the qibla for all latitudes and longitudes.

[13] The sigla are basically the same as those used in *King* [1], which contains a list of *Ibn Yūnus'* works and of all known related manuscripts.

These seven manuscripts are described in Appendix B. MS D, copied in the year 1371, was the major source for the present study. Sigla of certain other manuscripts consulted are listed after Appendix B. Of particular relevance are three manuscripts (MSS CE, HA, and PH) of a work entitled *Natīja al-afkār* by an individual named *al-Lādhiqī*.[14] Besides a short introduction and simple tables for calculating solar longitude, most of the numerous extant manuscripts of this work which I have inspected contain *Ibn Yūnus'* prayer-tables and tables of auxiliary spherical astronomical functions; also, certain tables related to those of *Ibn Yūnus* are to be found only in the three manuscripts of this work mentioned above. The location in the manuscript sources of all the tables analyzed in Part 3 is indicated in Appendix A.

Part 2: Preliminary Remarks on the Analysis

2.1. Notation Used in the Analysis

The following notation is used freely in the sequel. The Arabic expressions for the various functions tabulated by *Ibn Yūnus* are shown in parentheses and are taken from MS D unless otherwise indicated.

Latin

a	azimuth, measured from the prime vertical (*samt*)
$-_{\mathrm{a}}$	relates to the earliest time for the afternoon prayer
$-_{\mathrm{b}}$	relates to the end of the time for the afternoon prayer
D	semi diurnal arc (*niṣf al-qaws*)
f	half-excess of daylight, *i.e.*, excess of half daylight over 90° (*faḍl niṣf al-nahār* or *niṣf al-faḍla*)
g	ratio of Vers D to Sin h_{n} (*al-nisba*) (MS L)
h	altitude
h_{a}	solar altitude at the beginning of the time for the afternoon prayer (*irtifāʿ awwal waqt al-ʿaṣr*)
h_{b}	solar altitude at the end of the afternoon prayer
h_{e}	solar altitude at the equinoxes for a given azimuth
h_{n}	solar meridian altitude (*ghāyat al-irtifāʿ*)
h_{o}	solar altitude in the prime vertical (*al-irtifāʿ alladhī lā samt lahu*)
h_{q}	solar altitude in the azimuth of the qibla (*irtifāʿ al-shams idhā marrat bi-samt al-qibla* or *al-irtifāʿ al-muwāfiq li-samt al-qibla*)
$h_{\bar{\mathrm{q}}}$	solar altitude in the direction perpendicular to the azimuth of the qibla (*irtifāʿ al-shams ʿalā tarbīʿ al-qibla*) (MS G)
h_{r}	solar depression at daybreak
h_{s}	solar depression at nightfall
h_{v}	solar altitude in the azimuth of the ventilator (*irtifāʿ samt al-bādahanj*)
$\tilde{h}$	length of the seasonal day hours (*azmān al-sāʿāt*)
$-_{\mathrm{H}}$	relates to the ascendant (horoscopus)
k	horizontal variation (*ikhtilāf al-ufq* or *ḥiṣṣat al-samt*) (MS L)
L	terrestrial longitude
$-_{\mathrm{M}}$	relates to upper midheaven
n	length of darkness (*jawf al-layl*)
$2N$	nocturnal arc (*qaws al-layl*)
q	azimuth of the qibla, measured from the meridian (*samt al-qibla* or *inḥirāf al-qibla*)
$-_{\mathrm{q}}$	relates to the *qibla*
$-_{\bar{\mathrm{q}}}$	relates to the direction perpendicular to the *qibla*
r	duration of morning twilight (*ḥiṣṣat* (*ṭulūʿ*) *al-fajr*)
$-_{\mathrm{r}}$	relates to morning
R	sexagesimal base (60) (*al-jayb al-aʿẓam*) (MS L)
s	duration of evening twilight (*ḥiṣṣat* (*mughīb*) *al-shafaq*)
$-_{\mathrm{s}}$	relates to evening
t	hour-angle (*faḍl al-dāʾir*)
t_{a}	time from midday to the beginning of the time for the afternoon prayer (*al-dāʾir min al-zawāl ilā awwal waqt al-ʿaṣr*)
t_{b}	time from midday to the end of the afternoon prayer
t_{q}	hour-angle when the sun is in the azimuth of the qibla (*faḍl al-dāʾir li-niṣf al-nahār min irtifāʿ samt al-qibla*)
$t_{\bar{\mathrm{q}}}$	hour-angle when the sun is in the azimuth perpendicular to the azimuth of the qibla (*al-māḍī min al-zawāl ilā hayth takūn al-shams ʿalā tarbīʿ al-qibla*) (MS G)
T	time since sunrise or remaining until sunset (*dāʾir*)
T_{a}	time from the beginning of the afternoon prayer to sunset (*al-bāqī min awwal waqt al-ʿaṣr ilā l-ghurūb*)

---

[14] On *al-Lādhiqī* see *Suter* [1], no. 519. The title means *The Result of Reflection*, but the word *natīja* also means "calendar" or "prayer-tables".

T_b	time from the end of the afternoon prayer to sunset
T_q	time from sunrise to the time when the sun is in the azimuth of the qibla
T_σ	time from sunset to the *salām* (*al-dāʾir min al-ghurūb li-l-salām*) (MS G)
T_τ	time from sunset to the *ṭafy* in Ramaḍān (*al-dāʾir li-l-ṭafy fī Ramaḍān*) (MS G)
$-_v$	relates to ventilators
x	independent variable

Greek

α	right ascension (*al-maṭāliʿ fī l-falak al-mustaqīm*) (MS L)
α'	normed right ascension (*al-maṭāliʿ min awwal al-jady*) (MS HA)
α_r	oblique ascension of the ascendant at daybreak
α_s	oblique ascension of the ascendant at nightfall (*maṭāliʿ mughīb al-shafaq al-aḥmar*) (MS HA)
α_σ	oblique ascension of the ascendant at the *salām* (*maṭāliʿ al-salām*) (MS HA)
α_φ	oblique ascension for latitude φ (*al-maṭāliʿ al-baladīya*) (MS L)
β	ecliptic latitude (*ʿarḍ*) (MS L)
δ	solar declination (*mayl al-shams*)
Δ	stellar declination (*buʿd*)
ΔD	correction to half daylight for horizontal refraction (*daqāʾiq ikhtilāf mā bayn al-ufqayn*) (MS G)
Δh	distance between true and visible horizons
ε	obliquity of the ecliptic (*al-mayl al-aʿẓam*) (MS L)
ϑ	independent variable
λ	ecliptic longitude, solar longitude (*daraja, darajat al-shams*)
λ'	solar elongation from the nearer equinox (see Section 2.2)
$-_\sigma$	relates to the *salām* before daybreak
$-_\tau$	relates to the *ṭafy* in Ramaḍān
φ	terrestrial latitude (*ʿarḍ al-balad*)
ψ	rising amplitude, ortive amplitude (*saʿat al-mashriq*)

Miscellaneous

$\approx$	approximately equals
$\bar{\vartheta}$	complement to 90° (*tamām al-qaws*) (MS L)
; ,	sexagesimal semi-colon and commas
λ^*	opposite point on ecliptic (*al-naẓīr*)

The medieval functions:

$$\text{Sin, Cos, Tan, Cot, and Vers}$$

are to base $R=60$, unless otherwise indicated.[15] Their arguments were thought of as arcs on a circle radius R, not as angles.

2.2. On the Format of the Tables

All of the functions tabulated, except the arc Sine, have solar longitude as argument (*ʿadad*). Values are given to two sexagesimal digits for each integral degree of argument, unless otherwise indicated. All entries in the tables are written in standard Arabic *abjad* notation. The units are sometimes indicated at the head of the columns, thus: *j* for *daraj* (degrees), *q* for *daqāʾiq* (minutes). The sexagesimal semi-colon and comma used in the analysis have no counterpart in the manuscripts. The notation for zero is m (see Fig. X1), which may be added to the examples listed by *R. Irani*.[16]

Fig. X1 shows two facing pages of MS D, on which are tabulated the time since sunrise and hour-angle for solar altitude 14° for each degree of solar longitude. The format of these tables is of some interest, and I do not know whether it was originally devised by *Ibn Yūnus* or adopted by him from some earlier source.[17]

All of the tables in the *Very Useful Tables* are arranged as concisely as the symmetry of the tabulated function allows. (This is in contrast to the convention followed in the azimuth and altitude tables in the *Ḥākimī Zīj*, where the functions are tabulated for each degree of solar longitude, in twelve columns of thirty entries.) For functions which are symmetrical for solar longitudes

$$0° \leqq \lambda \leqq 90° \quad \text{and} \quad 90° \leqq \lambda \leqq 180°$$

and symmetrical for

$$180° \leqq \lambda \leqq 270° \quad \text{and} \quad 270° \leqq \lambda \leqq 360°,$$

---

[15] Thus $\text{Sin}\,\vartheta = 60\sin\vartheta$, $\text{Cos}\,\vartheta = 60\cos\vartheta$, $\text{Tan}\,\vartheta = 60\tan\vartheta$, $\text{Vers}\,\vartheta = 60 - \text{Cos}\,\vartheta = 60\,(1-\cos\vartheta)$, *etc.* On the medieval Islamic convention see *Wiedemann* [1], I, 564–577; *Kennedy* [1], 139–140; and *King* [1], 60–65.

[16] See *Irani* [1].

[17] This same format is found in the tables of the length of the seasonal day-hours and length of daylight in equinoctial hours contained in the manuscripts of the *Zījes* of *Yaḥyā b. Abī Manṣūr* and *Ḥabash*, both of which works were known to *Ibn Yūnus*. It has not yet been established, however, that these tables are original. *Cf. Kennedy* [1], 132 (no. 51) and 126–127 (nos. 15 and 16), on the manuscripts.

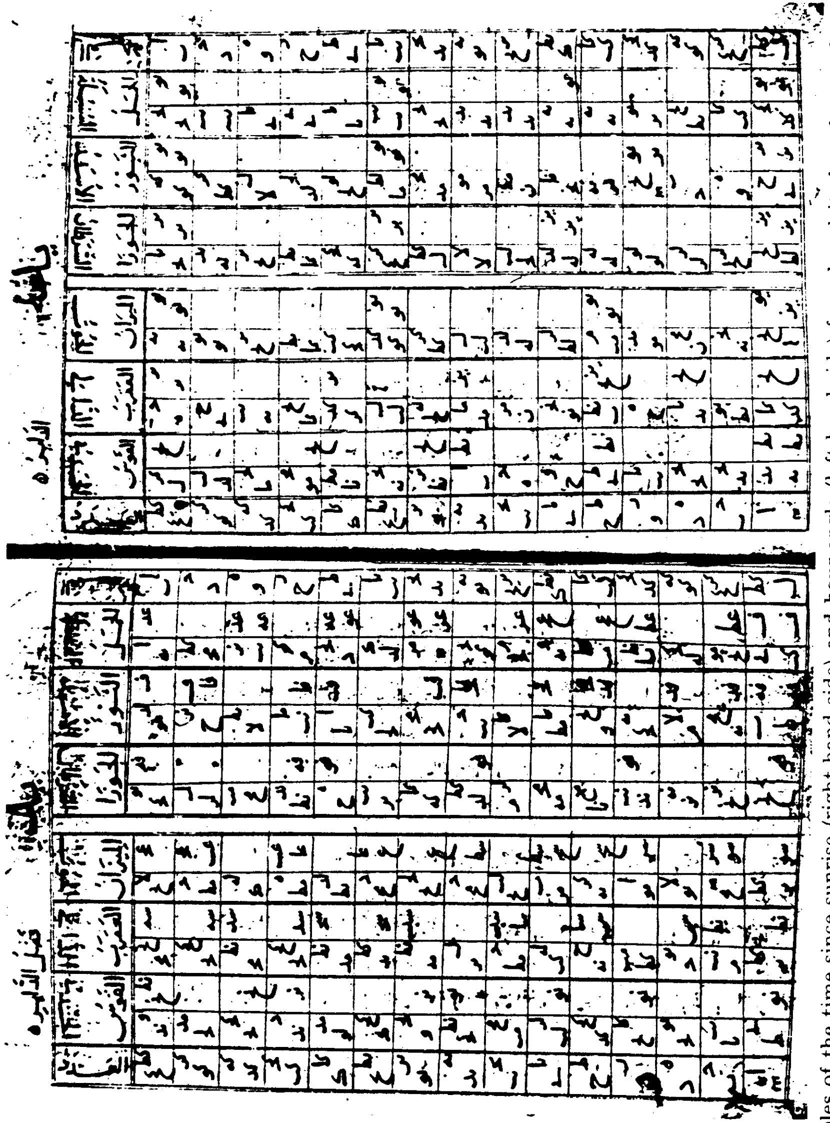

Fig. X1. Tables of the time since sunrise (right-hand side) and hour-angle (left-hand side) for solar altitude 14°, reproduced from MS D.70v–69r with kind permission of the Director of the Chester Beatty Library, Dublin.

the format of the tables is as follows, with the names of the zodiacal signs written in words. (See also Tables X1 and X2 in Section 2.3.)

Argument	♍	♌	♋	♓	♒	♑	Argument
	♈	♉	♊	♎	♏	♐	
1°							29°
2							28
3							27
⋮							⋮
28							2
29							1
30							0

The entries for the first three signs starting with Aries are to be read downwards with vertical arguments 1° to 30° in the first three columns, and those for the second three signs are to be read upwards with arguments 0° to 29° in the same columns. The entries for the solar longitudes in the southern signs are similarly arranged in the second three columns. The values of the function for the equinoxes must be found by interpolation.[18] (That this scheme of tabulation caused some confusion amongst later users is apparent, for example, in a rather corrupt version of the azimuth tables (MS B. 1v–10r) where the argument is given as 1° to 30° for all zodiacal signs.) I denote the argument in these tables by λ' ($\delta \gtrless 0$). Where the functions tabulated are symmetrical for solar longitudes in all four quadrants of the ecliptic, as is the case with the solar declination, rising amplitude, and half-excess of daylight, three columns of thirty entries each suffice to display the functions for all longitudes. I denote the argument in these tables by λ'.

2.3. On the Recomputation of the Tables

All of the mathematically significant tables in the original work have been recomputed electronically, with the same format and precision.[19] The recomputation revealed a remarkably high level of computational accuracy on the part of *Ibn Yūnus*. For each table described in Part 3 an extract is given in Part 5 in such a way as to convey some idea of the accuracy with which the tables were originally computed. *Ibn Yūnus'* values are generally accurate or in error by ± 1 or ± 2 in the second digit. In some tables larger errors occur, often in patterns. Variants between the manuscripts are, in general, attributable to copyists' errors. There are, however, exceptions to this. Tables X1 and X2 show values of a particular function tabulated both in the *Ḥākimī Zīj* and the *Very Useful Tables*: they were clearly computed separately. For each entry which is not accurately computed, I show the error in the second sexagesimal digit in square brackets after the entry. The errors are computed according to the convention

$$\text{error} = (\text{value in text}) - (\text{recomputed value}).$$

---

[18] The table of $h_n(\lambda)$ in an anonymous *Zīj* based on *Ibn Yūnus* (MS PB. 49r) has arguments 0° to 29° downwards and 1° to 30° upwards. Thus no values appear in the table for the solstices. The table of $h_a(\lambda)$ in the same *Zīj* (MS PB. 54r) has the same format as those in the *Very Useful Tables*.

[19] On the recomputation of medieval tables see, for example, *Kennedy* [2]; *Gingerich* [1]; and *King* [1], 74–75.

Table X1. Solar Azimuth for Altitude 30° as Tabulated in the *Very Useful Tables*
(MSS D. 106r and E. 45r)

	♈	♉	♊	♎	♏	♐	
1°	+18; 54°	+3; 22[d]	−7; 40 [+1]	+20; 3[g] [+1]	+37; 27	+53; 8 [+1]	29°
2	18; 21	2; 54	7; 55	20; 36	38; 2	53; 33	28
3	17; 46 [−1]	2; 27[e]	8; 9 [−1]	21; 11	38; 36	53; 58	27
4	17; 13	2; 0	8; 24	21; 45	39; 10	54; 22	26
5	16; 40	1; 35 [+1]	8; 38	22; 19	39; 44	54; 45 [−1]	25
6	16; 7	1; 8	8; 51	22; 54	40; 19 [+1]	55; 9	24
7	15; 34	0; 42	9; 5 [+1]	23; 28	40; 52	55; 32 [+1]	23
8	15; 1	+0; 17[f]	9; 17	24; 3	41; 26	55; 54 [+1]	22
9	14; 29 [+1]	−0; 9 [+1]	9; 29	24; 38	42; 0	56; 15 [+1]	21
10	13; 56[a]	0; 33	9; 40	25; 12 [−1]	42; 33	56; 35	20
11	13; 24[b] [+1]	0; 57	9; 51	25; 47 [−1]	43; 6	56; 54	19
12	12; 51	1; 22 [+1]	10; 1	26; 22	43; 40 [+1]	57; 12 [−1]	18
13	12; 20 [+1]	1; 45	10; 10 [−1]	26; 58 [+1]	44; 12	57; 30	17
14	11; 47	2; 8	10; 20	27; 33 [+1]	44; 45 [+1]	57; 47	16
15	11; 16	2; 31	10; 29	28; 7	45; 17	58; 3[n] [−1]	15
16	10; 45 [+1]	2; 53	10; 37	28; 42 [−1]	45; 48 [−1]	58; 18 [−1]	14
17	10; 13	3; 15	10; 45	29; 17 [−1]	46; 19 [−1]	58; 33[p]	13
18	9; 42	3; 37	10; 52	29; 52 [−1]	46; 51[l] [−1]	58; 45 [−1]	12
19	9; 11	3; 58	10; 58	30; 28	47; 22 [−1]	58; 59	11
20	8; 40 [−1]	4; 19	11; 4	31; 3	47; 54	59; 10	10
21	8; 10	4; 40	11; 10	31; 38	48; 24	59; 21 [+1]	9
22	7; 41 [+1]	5; 0	11; 15 [+1]	32; 13[h]	48; 54 [−1]	59; 30	8
23	7; 11	5; 20 [+1]	11; 19	32; 48	49; 26 [+2]	59; 38	7
24	6; 41	5; 39 [+1]	11; 23 [+1]	33; 23[i]	49; 55 [+1]	59; 45	6
25	6; 12[c]	5; 57	11; 25	33; 59 [+1]	50; 24 [+1]	59; 51	5
26	5; 42	6; 15	11; 28	34; 33	50; 53[m] [+2]	59; 57 [+1]	4
27	5; 14	6; 33	11; 30	35; 7 [−1]	51; 20	60; 0	3
28	4; 45	6; 51 [+1]	11; 31	35; 44 [+1]	51; 48 [+1]	60; 2 [−1]	2
29	4; 17	7; 7	11; 32	36; 17[j] [−1]	52; 14	60; 4 [−1]	1
30	+3; 50 [+1]	−7; 24 [+1]	−11; 32 [−1]	+36; 53[k] [+1]	+52; 41	+60; 5	0
	♍	♌	♋	♓	♒	♑	

[a] MS E: 56; MS D: 54
[b] MS E: 24; MS D: 26
[c] MS D: 12; MS E: 17 [*sic*]
[d] MS D: 22; MS E: 40 [*sic*]
[e] MS D: 27; MS E: 9 [*sic*]
[f] MS E: 17; MS D: 16
[g] MS D: 3; MS E: 1
[h] MS D: 13; MS E: 53 [*sic*]
[i] MS E: 23; MS D: 28 [*sic*]
[j] MS E: 17; MS D: 13 [*sic*]
[k] MS E: 53; MS D: 56 [*sic*]
[l] MS D: 51; MS E: 11 [*sic*]
[m] MS D: 53; MS E: 13 [*sic*]
[n] MS E: 3; MS D: 8 [*sic*]
[p] MS D: 33; MS E: 38 [*sic*]

Table X2. Solar Azimuth for Altitude 30° as Tabulated in the *Ḥākimī Zīj*
(MSS L. 389 and O. 124v–125v)
[N.B. Entries are given for each degree of solar longitude: the entry for the equinoxes is 19; 28°]

	♈	♉	♊	♎	♏	♐	
1°	+18; 54°	+3; 22[b]	−7; 39	+20; 1 [−1]	+37; 27	+53; 7	29°
2	18; 21	2; 54	7; 55	20; 36	38; 2	53; 33	28
3	17; 47	2; 27	8; 10	21; 11	38; 36	53; 58	27
4	17; 14 [+1]	2; 1 [+1]	8; 24	21; 45	39; 10	54; 22	26
5	16; 40	1; 34	8; 38	22; 19	39; 44	54; 46	25
6	16; 7	1; 8	8; 51	22; 54	40; 18	55; 9	24
7	15; 34	0; 42	9; 4	23; 29 [+1]	40; 52	55; 32 [+1]	23
8	15; 0 [−1]	+0; 17	9; 17	24; 3	41; 26	55; 53	22
9	14; 28	−0; 8	9; 29	24; 38[d]	42; 0	56; 14	21
10	13; 56	0; 33	9; 40	25; 13	42; 33	56; 35	20
11	13; 23	0; 57	9; 51	25; 48	43; 6	56; 54	19
12	12; 51	1; 21	10; 1	26; 23 [+1]	43; 39	57; 13	18
13	12; 19	1; 45	10; 11	26; 57	44; 12	57; 31 [+1]	17
14	11; 47	2; 8	10; 20	27; 33 [+1]	44; 44	57; 48 [+1]	16
15	11; 16	2; 31	10; 29	28; 8 [+1]	45; 17	58; 3 [−1]	15
16	10; 44	2; 53	10; 37	28; 43	45; 49	58; 19	14
17	10; 13	3; 15	10; 45	29; 18	46; 21 [+1]	58; 33	13
18	9; 42	3; 37	10; 52	29;53[e]	46; 52	58; 46	12
19	9; 11	3; 58	10; 59 [+1]	30; 28	47; 23	58; 59	11
20	8; 41	4; 19	11; 4	31; 3	47; 54	59; 10	10
21	8; 10	4; 40	11; 10	31; 38	48; 25 [+1]	59; 20	9
22	7; 40	5; 0	11; 14	32; 13	48; 55	59; 29 [−1]	8
23	7; 11	5; 19	11; 19	32; 48	49; 25 [+1]	59; 38	7
24	6; 41	5; 38[c]	11; 22	33; 23	49; 54	59; 45	6
25	6; 12	5; 57	11; 25	33; 58	50; 23	59; 51	5
26	5; 43 [+1]	6; 15	11; 28	34; 33	50; 51	59; 56	4
27	5; 14[a]	6; 33	11; 30	35; 8	51; 20	60; 0	3
28	4; 45	6; 50	11; 31	35; 43	51; 47	60; 3	2
29	4; 17	7; 7	11; 32	36; 18	52; 14	60; 5	1
30	+3; 49	−7; 23	−11; 33	+36; 52	+52; 41	+60; 5	0
	♍	♌	♋	♓	♒	♑	

[a] MS L: ♈ 27°: 14 and ♍ 3°: 44 [*sic*]
[b] MS L: ♉ 1°: 22 and ♌ 29°: 18 [*sic*]
[c] MS L: ♉ 24°: 38 and ♌ 6°: 33 [*sic*]
[d] MS O: ♎ 9°: 38 and ♓ 21°: 33 [*sic*]
[e] MS O: ♎ 18°: 53 and ♓ 12°: 58 [*sic*]

2.4. On a Non-Linear Interpolation Scheme Described by Ibn Yūnus

We know very little about the techniques used by medieval Islamic astronomers to compile mathematical and astronomical tables: the sources generally contain no information on such techniques. In the instructions to the *Very Useful Tables* no clues are given which would indicate how the entries were computed.

Considering that the *Very Useful Tables* contain over 30,000 entries, it is reasonable to assume that *Ibn Yūnus* used some kind of non-linear interpolation scheme to compute intermediate entries between certain entries computed independently. In this connection, it is perhaps worth noting that in Chapter 10 of the *Ḥākimī Zīj Ibn Yūnus* describes a non-linear interpolation scheme to be used for computing values of the Sine function.[20] His scheme, described in words in the Leiden manuscript, may be represented in algebraic notation as follows.

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[20] *Cf. King* [1], 82–85, and *Schoy* [4], 390–391, for a translation of the relevant passage. Other non-linear schemes were suggested by *al-Bīrūnī* (ca. 1025) and *al-Kāshī* (ca. 1420). *Cf. Schoy* [5], 40–41, and *Kennedy* [3], respectively. *Ibn Yūnus'* method is similar to a scheme used in earlier Chinese sources. *Cf. Kennedy* [4], 438–439 and *Juschkewitsch* [1], 84–86.

It is required to find the values of $\mathrm{Sin}\,(p;\,q^\circ)$ where

$$0 \leqq p \leqq 89 \quad \text{and} \quad 1 \leqq q \leqq 59$$

from a table giving the values of the Sine function for each 0; 30° of argument. Let

$$\mathrm{Sin}\,p = a; \quad \mathrm{Sin}\,(p;\,30) = b; \quad \text{and} \quad \mathrm{Sin}\,(p+1) = c.$$

Now find an approximation for $\mathrm{Sin}\,(p;\,30)$, say d, by interpolating linearly between the values of $\mathrm{Sin}\,p$ and $\mathrm{Sin}\,(p+1)$; thus

$$d = \tfrac{1}{2}(a+c).$$

Next define an "interpolation base" (*aṣl al-taʿdīl*) thus:

$$\Delta = b - d.$$

For given q, compute an approximate value of $\mathrm{Sin}\,(p;\,q)$ by linear interpolation and then add a correction Δ_q which is a certain fraction of Δ; thus

$$\mathrm{Sin}\,(p;\,q) \approx a + \frac{q}{60}(c-a) + \Delta_q$$

where

$$\Delta_q = \frac{4q(60-q)}{3600}\,\Delta.$$

(Note that Δ_q assumes the value zero at $q = 0$ and 60, and the maximum value Δ at $q = 30$.)

This procedure defines a second-order interpolation scheme which is easily modified so that it can be used to interpolate between values of a function computed independently for, say, each 5° of argument.[21] I have, however, been unable to establish that *Ibn Yūnus* used such a scheme.

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[21] Suppose that for a certain function f,

$$f(5n) = a, \quad f(5n+5) = b, \quad \text{and} \quad f(5n+10) = c,$$

where n is an integral number of degrees, and

$$\Delta = b - \tfrac{1}{2}(a+c).$$

Then, for $1 \leqq r \leqq 4$ and $6 \leqq r \leqq 9$, we have

$$f(5n+r) = a + \frac{r}{10}(c-a) + c_r\Delta,$$

where

$$c_r = c_{10-r} = \frac{4r(10-r)}{100}.$$

Thus

$$c_1 = c_9 = 0;\,21{,}36 \approx 0;\,22,$$
$$c_2 = c_8 = 0;\,38{,}24 \approx 0;\,38,$$
$$c_3 = c_7 = 0;\,50{,}24 \approx 0;\,50,$$

and

$$c_4 = c_6 = 0;\,57{,}36 \approx 0;\,58.$$

By use of coefficients c_r rounded to one significant digit the intermediate values can be generated with great facility, especially if one has a sexagesimal multiplication table (on such tables in the Islamic sources see *Irani* [2] and *King* [2], Appendix). This procedure, when applied to the entries for each 5° of argument in Tables X1 and X2, yields two sets of values which are not identical with either set of *Ibn Yūnus'* values, and which are slightly less accurate.

There is as yet no evidence that *Ibn Yūnus* did not compute most of the tables himself. However, the possibility that the services of trained calculators were available to him cannot be ruled out.

2.5. On Certain Relevant Formulae of Modern Spherical Astronomy

For the convenience of the reader I list the following basic formulae of spherical astronomy.[22]

The solar declination δ is related to the solar longitude λ and the obliquity of the ecliptic ε by the formula

$$\delta = \arcsin\,(\sin\varepsilon\,\sin\lambda). \qquad (\mathrm{X}1)$$

The modern formula for the hour-angle $t(h, \delta, \varphi)$ is

$$t = \arccos\left\{\frac{\sin h - \sin\delta\,\sin\varphi}{\cos\delta\,\cos\varphi}\right\}. \qquad (\mathrm{X}2)$$

When $h = 0°$ the hour-angle measures the length of half daylight, D. Thus

$$\begin{aligned} D &= \arccos\,(-\tan\delta\,\tan\varphi) \\ &= 90° + \arcsin\,(\tan\delta\,\tan\varphi) = 90° + f, \end{aligned} \qquad (\mathrm{X}3)$$

where f is the excess of half daylight over 90°. The time elapsed since sunrise, T, is therefore

$$T = D - t = f + \arcsin\left\{\frac{\sin h - \sin\delta\,\sin\varphi}{\cos\delta\,\cos\varphi}\right\}. \qquad (\mathrm{X}4)$$

The modern formula for the solar azimuth $a(h, \delta, \varphi)$, measured from the prime vertical and considered positive when the azimuth is southerly, is

$$a = \arcsin\left\{\frac{\sin h\,\sin\varphi - \sin\delta}{\cos h\,\cos\varphi}\right\}. \qquad (\mathrm{X}5)$$

When $h = 0°$, the azimuth measures the rising amplitude, $\psi(\lambda)$. With the convention that $\psi \gtrless 0$ as $\delta \gtrless 0$, the formula for ψ is

$$\psi = \arcsin\left\{\frac{\sin\delta}{\cos\varphi}\right\}. \qquad (\mathrm{X}6)$$

It is not difficult to show the equivalence of the rules outlined in the *Zīj* for determining t, D, T, f, a, and ψ to these modern formulae. The reader is referred to my analysis of the spherical astronomy in the *Ḥākimī Zīj* for a discussion of procedures other than those of spherical trigonometry by which *Ibn Yūnus'* formulae can be derived.

Part 3: Analysis

The *Very Useful Tables* are based on *Ibn Yūnus'* values of 23; 35° for the obliquity of the ecliptic and 30; 0° for the latitude of Cairo-Fustat (Arabic, *Miṣr*), where these parameters are involved. All functions are tabulated to two sexagesimal digits unless otherwise indicated.

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[22] For a modern discussion of spherical astronomy the reader is referred to *Smart* [1] or *Woolard & Clemence* [1].

3.1. Auxiliary Tables of Spherical Astronomical Functions

Several of the tables have their counterparts in the *Ḥākimī Zīj*, and these have been analyzed in my study of the spherical astronomy of *Ibn Yūnus*. Of the tables described in this Section, (a), (b), (k), and (l) give values for each degree of λ'; (c), (d), …, (h) give values for each degree of $\lambda'\,(\delta \gtrless 0)$; and (i) and (j) give values for each degree of λ. The formulae given for the functions tabulated are taken from the *Ḥākimī Zīj*.

(a) Tables of solar declination, $\delta(\lambda)$, with values to seconds (MS D) as in Chapter 11 of the *Zīj*, or rounded to minutes (MS HA). The solar declination is determined by

$$\delta(\lambda) = \text{arc Sin}\left\{\frac{\text{Sin}\,\varepsilon\,\text{Sin}\,\lambda}{R}\right\} \tag{1}$$

and in Chapter 11 of the *Zīj Ibn Yūnus* tabulates

$$\frac{n\cdot\text{Sin}\,\varepsilon}{R} \qquad n = 1, 2, \ldots, 60$$

to facilitate the computation of $\delta(\lambda)$. In an independent work entitled the *Kitāb al-Mayl*, *Ibn Yūnus* tabulated the solar declination to four sexagesimal digits for each minute of λ', also giving differences to be added for each second.[23]

(b) Table of the half excess of daylight, $f(\lambda)$, as in Chapter 15 of the *Zīj*, but with values rounded to minutes. This function is defined by

$$f(\lambda) = \text{arc Sin}\left\{\frac{\text{Tan}\,\delta(\lambda)\,\text{Tan}\,\varphi}{R}\right\} \tag{2}$$

and in Chapter 13 of the *Zīj Ibn Yūnus* tabulates Tan $\delta(\lambda)$.

(c) Table of the solar meridian altitude, $h_n(\lambda)$. This is easily computed by means of the formula

$$h_n(\lambda) = \bar{\varphi} + \delta(\lambda). \tag{3}$$

Cf. the graph of $h_n(\lambda)$ in Fig. 4 ($a = 90°$).

(d) Table of the half diurnal arc, $D(\lambda)$, based on the formula

$$D(\lambda) = 90° + f(\lambda) \qquad (f \gtrless 0 \text{ as } \delta \gtrless 0). \tag{4}$$

(e) Table of the nocturnal arc, $2N(\lambda)$, based on the formula

$$2N(\lambda) = 360° - 2D(\lambda). \tag{5}$$

The two tables in MS D were computed from separate tables of $D(\lambda)$, since the entries differ by as much as 0; 4° in isolated places.

(f) Table of the length of the seasonal day hours in equatorial degrees, $\tilde{h}(\lambda)$, as in Chapter 15 of the *Zīj*, with values rounded to minutes.

(g) Table of factors for converting equatorial degrees to seasonal day hours, with values in minutes and seconds. This function is the reciprocal of $\tilde{h}(\lambda)$.

(h) Table of the length of daylight in equinoctial hours, computed by dividing the diurnal arc, $2D$, by 15.

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[23] See *King* [1], 96–99 on these tables.

(i) Table of normed right ascensions (measured from Capricorn 0°), as in Chapter 13 of the *Zīj*. The function tabulated is $\alpha(\lambda) + 90°$, where

$$\alpha(\lambda) = \text{arc Sin}\left\{\frac{\text{Tan}\,\delta(\lambda)\cdot R}{\text{Tan}\,\varepsilon}\right\} \quad \text{for } \lambda \leqq 90°. \tag{6}$$

(j) Table of oblique ascensions, $\alpha_\varphi(\lambda)$, for $\varphi = 30°$, as in Chapter 14 of the *Zīj*. The function tabulated is calculated by means of the formula

$$\alpha_\varphi(\lambda) = \alpha(\lambda) - f(\lambda). \tag{7}$$

(k) Table of the solar rising amplitude, $\psi(\lambda)$. MS D.84r has values to seconds as in Chapter 18 of the *Zīj*. The following equivalent expressions for ψ are given by Ibn Yūnus in the *Zīj*.

$$\psi = \text{arc Sin}\left\{\frac{\text{Sin}\,\delta\cdot R}{\text{Cos}\,\varphi}\right\} = \text{arc Sin}\left\{\frac{\dfrac{\text{Sin}\,\varepsilon\,\text{Sin}\,\lambda}{R}\cdot R}{\text{Cos}\,\varphi}\right\} \tag{8}$$
$$= \text{arc Sin}\left\{\frac{\text{Sin}\,(\max\psi)\,\text{Sin}\,\lambda}{R}\right\}.$$

Fig. 3 shows how ψ varies with λ ($h = 0°$).

(l) Table of the solar altitude in the prime vertical, $h_0(\lambda)$ ($\delta > 0$ only). MS D.85v has values to seconds as in Chapter 18 of the *Zīj*, but the tables in the other sources with values to minutes contain numerous copyists' errors. The following equivalent expressions for h_0 are given in *Ibn Yūnus' Zīj*;

$$h_0 = \text{arc Sin}\left\{\frac{\text{Sin}\,\delta\cdot R}{\text{Sin}\,\varphi}\right\} = \text{arc Sin}\left\{\frac{\dfrac{\text{Sin}\,\varepsilon\,\text{Sin}\,\lambda}{R}\cdot R}{\text{Sin}\,\varphi}\right\} \tag{9}$$
$$= \text{arc Sin}\left\{\frac{\text{Sin}\,(\max h_0)\,\text{Sin}\,\lambda}{R}\right\}.$$

Fig. 4 shows how h_0 varies with λ ($a = 0°$).

(m) Table of arc Sines, with arguments 1, 2, …, 60 and corresponding arcs given in degrees and minutes. There is no such table in the *Ḥākimī Zīj*, but a corresponding table in the Yemeni *Mukhtār Zīj*, MS LA:169v, a work which contains many of *Ibn Yūnus*' tables, has values to two sexagesimal digits for each 0; 30 of argument. In an independent work entitled the *Kitāb al-Jayb*, *Ibn Yūnus* tabulated the Sine function to five sexagesimal digits for each minute of arc, also giving differences to be added for each second of arc.[24]

3.2. Tables of Time Since Sunrise and the Hour-Angle[25]

These tables give $T(h, \lambda)$ and $t(h, \lambda)$ for each integral degree of solar altitude up to the maximum 83° and each degree of solar longitude, $\lambda'\,(\delta \gtrless 0)$. There are no such tables in the *Ḥākimī Zīj*. In MS D the tables of both functions for a given altitude generally occupy two facing pages of the manuscript. In MS C only t is tabulated, and in MS B the functions are tabulated separately. In MS G triplets of entries for T and t, as well as $a(h, \lambda)$, the solar azimuth, are given.

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[24] See *King* [1], 85–89. On *Ibn Yūnus*' extensive tables for the tangent function see footnote 41 below.

[25] The Arabic term *dāʾir (min al-falak)* literally means "that part (of the celestial sphere) which has revolved", and it is measured in equatorial degrees. I have rendered it by the phrase "time since sunrise". Further, I use the modern expression "hour-angle" for the Arabic *faḍl al-dāʾir*, literally "remainder of the *dāʾir*", which is also measured in equatorial degrees. *Ibn Yūnus* thought of both functions as arcs of the celestial equator.

When the sun does not reach a given altitude for certain solar longitudes, no entries are given in the table for those longitudes. Thus, for altitudes of about 60° and above, the entries for more than one altitude can be tabulated on a given pair of facing pages.

The tables of $T(h, \lambda)$ might have been computed by a method implicit in Chapter 15 of the *Ḥākimī Zīj*.[26] I suspect that *Ibn Yūnus* tabulated the function

$$g(\lambda) = \frac{R^2}{\operatorname{Cos}\delta(\lambda)\operatorname{Cos}\varphi} \tag{10}$$

called by him simply the ratio (*nisba*), and actually the ratio of Vers $D(\lambda)$ to Sin $h_n(\lambda)$. For each integral value of h we then form the product

$$p(h, \lambda) = \operatorname{Sin} h \cdot g(\lambda) \tag{11}$$

and then, if f is the half-excess of daylight, T can be found by means of the formula

$$T(h, \lambda) = f(\lambda) + \operatorname{arc}\operatorname{Sin}\{p(h, \lambda) - \operatorname{Sin} f(\lambda)\}. \tag{12}$$

The operations are readily carried out since both $f(\lambda)$ and Sin $f(\lambda)$ are tabulated in Chapter 15 of the *Ḥākimī Zīj*.[27] Formula (12) is trivially equivalent to formula (X4). A formula for $t(h, \lambda)$ not stated in the *Ḥākimī Zīj*, but outlined in an anonymous *Zīj* related to that of *Ibn Yūnus*, MS PB.62v, is the following:

$$\begin{aligned} \operatorname{Vers} t &= \operatorname{Vers} D(\lambda) - g(\lambda) \cdot \operatorname{Sin} h \\ &= \operatorname{Vers} D(\lambda) - p(h, \lambda). \end{aligned} \tag{13}$$

Note that since

$$\operatorname{Vers} t = R - \operatorname{Cos} t$$

and

$$\operatorname{Vers} D = R - \operatorname{Cos} D = R - \operatorname{Cos}(90° + f) = R + \operatorname{Sin} f,$$

formula (13) is trivially equivalent to formula (X2).

For a given value of h, the values of T or t can thus be found by means of very simple operations. Clearly, if either T or t is already computed, the other can be found without difficulty since their sum $D(\lambda)$ is already tabulated (*cf.* Section 3.1 (d)).

It is often stated that *Ibn Yūnus* was the first to propound the so-called prosthaphaeresis formula

$$\cos\varphi\cos\delta = \tfrac{1}{2}\{\cos(\varphi - \delta) + \cos(\varphi + \delta)\}$$

(to facilitate the computation of the hour-angle from the solar altitude). This assertion is incorrect and can be traced to *Delambre*'s misunderstanding of the material in Chapter 15 of the *Ḥākimī Zīj*.[28]

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[26] The reverse procedure for finding h from T is outlined in the *Ḥākimī Zīj*, and there is some evidence that *Ibn Yūnus* tabulated the function $g(\lambda)$. See further *King* [1], 31–32, 132–133, and 151–152.

[27] See *King* [1], 144 and 146.

[28] See *King* [1], 7 and 149 for a further discussion of this.

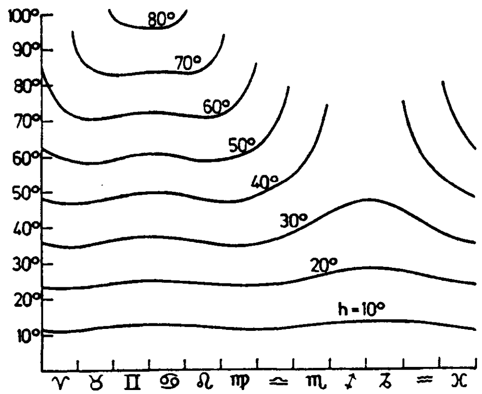

Fig. 1. Time since sunrise as a function of solar longitude, for altitudes 10°, 20°, …, 80°.

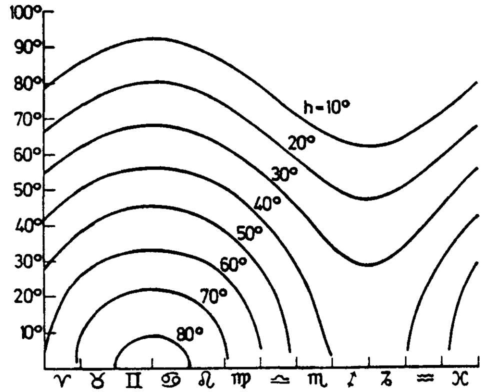

Fig. 2. Hour-angle as a function of solar longitude, for altitudes 10°, 20°, …, 80°.

Figs. 1 and 2 show the behaviour of the functions $T(h, \lambda)$ and $t(h, \lambda)$ for 10° intervals of h and all solar longitudes.

Tables for reckoning time from solar altitude were fairly common in the Islamic Middle Ages, but in most of the tables known to me the arguments are the instantaneous solar altitude and the meridian altitude. The only tables known to me in which the arguments are solar altitude and solar longitude are those of *al-Khalīlī* (ca. 1375) for the latitude of Damascus,[29] an anonymous Yemeni astronomer for Taʿizz, Yemen,[30] and *Ṣāliḥ Effendī* (ca. 1750) for Istanbul.[31] The tables of *al-Maqdisī* (ca. 1300) for Cairo were apparently plagiarized from those of *Ibn Yūnus*.[32]

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[29] These are contained in MSS CQ, PF, and DB. On *al-Khalīlī* see also *King* [2] and [3].

3.3. Tables of Solar Azimuth

These tables give $a(h, \lambda)$, measured from the prime vertical, for each integral degree of solar altitude up to the maximum 83° and each degree of solar longitude λ' ($\delta \gtrless 0$). In the *Ḥākimī Zīj* there are tables of $a(\lambda)$ for altitudes 30° and 35°: no reason is given for choosing these two altitudes.[33] The two tables in the *Zīj* are slightly more accurately computed than the corresponding tables in the other sources. See further Section 2.3.

In the margin of the tables in MSS D and E for altitudes 1° to 53° it is stated for which solar longitudes the azimuth changes from north to south for that particular altitude. In MS B the change in direction is more crudely indicated. In the margin of the tables for altitude 54° in MS D we find the comment that "there are no more azimuths in the north for this altitude and those above it." When, for certain longitudes, the sun does not attain a given altitude, the entries in the table are left blank. For altitudes of about 60° and above, the entries for more than one altitude can be tabulated on one folio. In MS E, for a given altitude, the meridian altitude is given for the longitude whose azimuth is closest to 90°. Thus, in the tables for altitude 37°, the last significant entry in the right-hand side of the table is an azimuth of 87; 3° for Sagittarius 17° and Capricorn 13°. The next and final entry is given as 0; 0°, which does not mean due east or west, but that the sun does not attain the altitude 37° for Sagittarius 18° or Capricorn 12°. The meridian altitude is accurately stated as 37; 3, 22° for the solar longitudes having azimuth 87; 3°. The purpose of giving this detail is not clear to me.

In MS D the entries for altitudes 81°, 82°, and 83° are garbled. This manuscript also has entries for altitudes 84° to 88° (MS K has entries up to 86°), which are meaningless since the maximum altitude is 83; 35° for *Ibn Yūnus*' parameters. I suspect that the last page of *Ibn Yūnus*' original azimuth tables was either not completed or lost, and that these additional entries were added by some incompetent astronomer later. In MS G the entries for altitudes 81°, 82°, and 83° have been corrected by an individual named *al-Rashīdī*. See further the descriptions of MSS G and K in Appendix B.

It seems reasonable that *Ibn Yūnus* would have calculated the values of $a(h, \lambda)$, which are very accurate and seldom in error by more than ± 1 in the second digit, using a method described in Chapter 20 of the *Ḥākimī Zīj*, and already known to his predecessors *Ḥabash* and *al-Battānī*. This involves the formula:[34]

$$a(h, \lambda) = \text{arc Sin}\left\{\frac{[k(h) - \text{Sin}\,\psi(\lambda)]\cdot R}{\text{Cos}\,h}\right\} \qquad (14)$$

where k is a function widely used in Islamic azimuth calculations, referred to here as the horizontal variation (*Ibn Yūnus* uses the expression *ikhtilāf al-ufq*),[35] and ψ is the solar rising amplitude. These two functions are defined by:

$$k(h) = \frac{\text{Sin}\,h\ \text{Sin}\,\varphi}{\text{Cos}\,\varphi} \qquad (15)$$

and

$$\text{Sin}\,\psi(\lambda) = \frac{\text{Sin}\,\delta\cdot R}{\text{Cos}\,\varphi}. \qquad (16)$$

Both $k(h)$ and $\text{Sin}\,\psi(\lambda)$ are tabulated for the latitudes of Cairo-Fustat and Baghdad and for each degree of both arguments in Chapters 18 and 19 of the *Ḥākimī Zīj*.[36] For a given solar altitude h and $R/\text{Cos}\,h$ are constant: thus the computation of a involves only three operations, an addition or subtraction, a multiplication, and taking an inverse Sine. Formula (14) is trivially equivalent to formula (X 5).

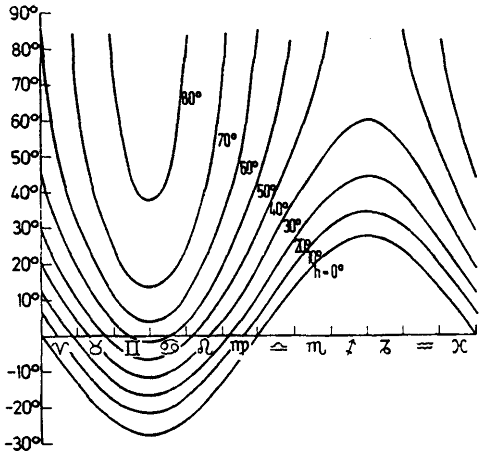

Fig. 3. Solar azimuth as a function of solar longitude, for altitudes 10°, 20°, ..., 80°. The curve for altitude 0° represents the solar rising amplitude.

The behaviour of the function $a(h, \lambda)$ for 10° intervals of h and all values of λ is shown in Fig. 3. The curve for $h = 0°$ represents the solar rising amplitude $\psi(\lambda)$.

On *Ibn Yūnus*' tables of the solar azimuth for certain altitudes at the equinoxes and solstices, located as this paper was in press, see the description of MS H in

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[30] These are contained in MS MB.

[31] These are contained in MS ZD, one of the largest sets of astronomical tables from the medieval period, with over 80,000 entries.

[32] On *al-Maqdisī* see *Suter* [1], no. 383. I have examined MS CO, which contains *Ibn Yūnus*' tables of T, t, and a as functions of h and λ, and also the instructions to the *Very Useful Tables*, introduced in *al-Maqdisī*'s name. See also the description of MS G in Appendix B.

[33] See *King* [1], 199–200.

[34] See *King* [1], 186–187.

[35] The function $k(h)$ measures the component along the horizontal north-south axis of the distance traversed by a celestial body since rising. (The radius of the celestial sphere is taken as R.) See further *King* [3], note 25.

[36] See *King* [1], 169–171 and 179–180, and also [3], notes 25 and 26.

Appendix B. The "azimuth tables" attributed in MS CY to the Egyptian astronomer *Ibn al-Majdī*[37] (d. 1447) are basically *Ibn Yūnus'* tables of $a(h)$ at the equinoxes and solstices. Tables of $a(h, \lambda)$ similar to those of *Ibn Yūnus* were computed for the latitude of Damascus by *al-Ḥalabī*[38] (d. 1455), but the azimuth tables of *al-Maqdisī* of Cairo were plagiarized from those of *Ibn Yūnus.*[39]

3.4. Tables of Solar Altitude in Certain Azimuths

It is possible to find $h(a, \lambda)$ from the azimuth tables by interpolation, but *Ibn Yūnus* compiled a set of tables for obtaining the solar altitude directly, and indicated as well a method for computing them. In Chapters 23 and 24 of the *Ḥākimī Zīj* he presents tables of $h(a, \lambda)$ for ten different values of a, and instructions for using them to determine the meridian.[40] MS D of the *Very Useful Tables* contains two of these tables, for azimuths 30° and 60° *S*. These tables are arranged symmetrically, like the time and azimuth tables, whereas those in the *Zīj* give values of h for each degree of λ in 12 columns of 30 entries.

Ibn Yūnus' method for finding $h(a, \lambda)$, outlined in his *Zīj*, is as follows. For a given azimuth a, first compute the altitude of the sun at an equinox, h_e, when it has this azimuth; thus

$$h_e = \text{arc Sin}\left\{\frac{\text{Sin}\, a \cdot R}{\sqrt{\left[\frac{\text{Sin}\, \varphi \cdot R}{\text{Cos}\, \varphi}\right]^2 + \text{Sin}^2 a}}\right\}. \tag{17}$$

Ibn Yūnus does not suggest the simpler formula[41]

$$h_e = \text{arc Tan}\left\{\frac{\text{Sin}\, a \cdot R}{\text{Tan}\, \varphi}\right\}. \tag{18}$$

Next compute a "correction arc" for the solsticial points thus:

$$C = \text{arc Sin}\left\{\frac{\text{Cos}\, h_e\ \text{Sin}\, \varepsilon}{\text{Sin}\, \varphi}\right\}. \tag{19}$$

A correction arc for general solar longitude λ is then found by

$$c(\lambda) = \text{arc Sin}\left\{\frac{\text{Sin}\, C \cdot \text{Sin}\, \lambda}{R}\right\} \qquad (c \gtrless 0 \text{ as } \delta \gtrless 0), \tag{20}$$

and with this the solar altitude for the longitude in question is found by

$$h(a, \lambda) = h_e(a) + c(\lambda). \tag{21}$$

The various cases $a \gtrless 0$ and $\delta \gtrless 0$ are carefully distinguished in the text.

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[37] On *Ibn al-Majdī* see *Suter* [1], no. 432, and *King* [1], 41, 45, and 143. The numerous works of this scholar deserve investigation.

[38] On *al-Halabī* see *Suter* [1], no. 434. I have examined MS CQ, which contains his azimuth tables, tabulated with the functions $t(h, \lambda)$ and $T(h, \lambda)$ computed by *al-Khalīlī* (see note 29 above). The format of these tables is as in MS G (*cf.* Section 3.2).

[39] *Cf.* footnote 32 above.

[40] On the tables and their use see *King* [1], 217–222, and for an analysis of the methods of *Ibn Yūnus* and his late contemporary *al-Bīrūnī* for computing $h(a, \lambda)$, see *King* [1], 212–217.

[41] *Ibn Yūnus* did not appreciate the full use of the tables of the tangent function which he laboriously computed for each minute of argument. *Cf. King* [1], 10 and 65.

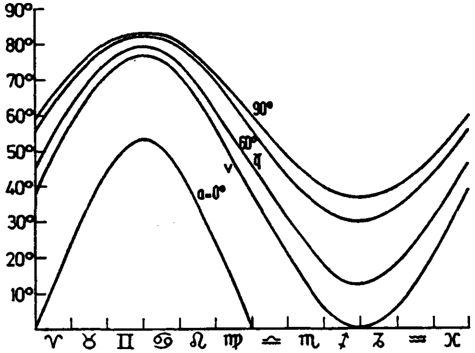

Fig. 4. Solar altitude as a function of solar longitude, for azimuths 0° (altitude in the prime vertical), 27; 30° (altitude in the azimuth of the ventilator), 37° (altitude in the azimuth of the qibla), 60° (for determining the meridian), and 90° (meridian altitude).

Thus, for each azimuth one needs first to calculate h_e and Sin C, and with these the values of $h(a, \lambda)$ can be found by three simple operations.

The tables of solar altitude in the azimuths of the qibla (see Section 3.6), and the ventilator (see Section 3.9), are of the same type. Fig. 4 shows the behaviour of the function $h(a, \lambda)$ for various azimuths selected by *Ibn Yūnus*. When the azimuth is 0° or 90°, the sun is in the prime vertical or the meridian. The corresponding altitudes h_0 and h_n (see Section 3.1 (l) and (c)) are also shown in Fig. 4.

MS CX[42] consists of an anonymous set of tables of the solar altitude $h(D, T)$, computed for each integral degree of D between 104° and 76° and each integral degree of T such that $1° \leqq T \leqq 2D$. The values of φ and ε underlying the tables are those of *Ibn Yūnus*, and the function is also tabulated for the extremal values of D for these parameters, that is 104; 36° and 75; 24°. For each value of D the associated value of T_a (see Section 3.7) is given at the head of each table. I suspect that the tables, which contain over 5,500 entries, were computed by *Ibn Yūnus*.

Apart from tables of the solar altitude in the azimuth of the qibla which occur in later prayer-tables,[43] no "altitude tables" like those of *Ibn Yūnus* are attested yet in any other known Islamic sources.

3.5. Tables of the Duration of Morning and Evening Twilight and of Darkness

Islamic astronomers adopted various criteria for defining twilight and adopted various values for the angle of depression of the sun below the horizon when twilight begins in the morning or ends in the evening. Some also differentiated between the values for morning and evening, holding that morning twilight lasts a little longer than evening twilight.[44] *Ibn Yūnus*' tables for twilight are associated with phenomena described as "the disappearance of the red twilight glow" (*mughīb al-shafaq al-aḥmar*) and "the first appearance of the true morning twilight glow (as opposed to the false morning twilight glow or zodiacal light)" (*ṭulūʿ al-fajr al-ṣādiq*). In the morning the "true dawn" may be preceded by the zodiacal light, called in Arabic the "false dawn" (*al-fajr al-kādhib*); it is the true dawn which marks the beginning of the time for the morning prayer.[45] *Ibn Yūnus* does not discuss twilight phenomena in his *Zīj* but does describe how the length of twilight can be computed if one associates a particular value for the angle of solar depression with the first and last appearances of twilight in the morning and evening sky.

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[42] *Cf. Suter* [1], no. 411 for the references to the manuscript, and the description in Appendix B.

[43] These are attested in several sets of medieval prayer-tables currently being analyzed.

In Chapter 16 of his *Zīj Ibn Yūnus* suggests calculating the time from daybreak to sunrise for a given solar longitude by calculating the time from sunrise to the moment at which the solar altitude is 18° above the horizon when the sun is at the opposite point of the ecliptic.[46] In the instructions to the *Very Useful Tables* he states that to find the duration of twilight one should enter with the longitude of the point on the ecliptic opposite the sun in the tables of time since sunrise for the appropriate angle of depression taken as altitude. It is a little strange that he does not then refer to the special tables for twilight which are also contained in the work. These tables are simply the corresponding tables of time since sunrise with the northern and southern signs interchanged.[47] In these twilight tables *Ibn Yūnus* uses 19° for morning twilight and 17° for evening twilight. There are also tables based on 20° for morning and 16° for evening, and the marginal comments "weak" in these two tables (MS D.10r and 10v) suggest that someone had used them and found them to be unsatisfactory.

The parameter 18° was probably used both by *Ḥabash* (*ca.* 850) and *al-Nayrīzī* (*ca.* 900),[48] and the works of these scholars were known to *Ibn Yūnus*. *al-Battānī*, on the other hand, did not discuss twilight at all. *al-Bīrūnī* (*ca.* 1025) suggested 18° for both morning and evening twilight, but also mentioned that some people used 17°.[49] Amongst these was an individual named *al-Qāyinī*, probably a contemporary of *al-Bīrūnī*: *al-Qāyinī* prepared tables for the duration of twilight based on this parameter which have been analyzed by *E. S. Kennedy* & *M.-L.*

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[44] For a discussion of the Islamic definitions of twilight see *Wiedemann* & *Frank* [1]. For a detailed description of the stages and phenomena of twilight consult *Minnaert* [1], 268–273, and on quantitative measurements of the angle of solar depression at daybreak and nightfall see *Hellmann* [1].

[45] On the false dawn in the Islamic sources see *Wiedemann* & *Frank* [1], 31–32, and *Wiedemann* [3]. See also *Minnaert* [1], 290–295, for a modern discussion.

[46] *Cf. King* [1], 159–161. See also *Schoy* [3] for an analysis of the methods of both *Ibn Yūnus* and *Abū ʿAlī al-Marrākushī* (d. *ca.* 1260).

[47] See p. 379. At the head of the table of the time since sunrise for 17° in MS D. 67v we find the title "table of the duration of evening twilight for the opposite degree of the ecliptic (*naẓīr*)". On the corresponding table for 19° in MS D we find the incomplete title "table of the duration of morning twilight".

[48] In MS Istanbul Yeni Cami 784,2, fol. 153r and MS Escorial ar. 961,6, respectively. The attribution of this material to the two authors is not certain.

[49] *al-Bīrūnī* [1], I, 948–950, translated in *Wiedemann* [2], 46.

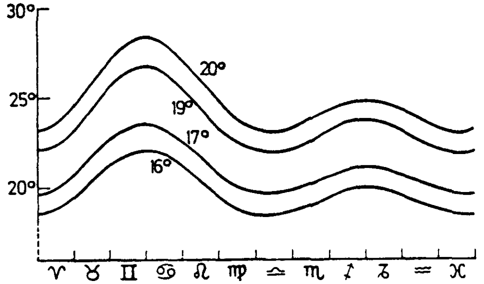

Fig. 5. Duration of twilight as a function of solar longitude, for angles of solar depression 16° and 17° (evening) and 19° and 20° (morning).

Davidian.[50] *A. Sabra* has recently shown that the *liber de crepusculis*, traditionally associated with *Ibn al-Haytham*, is due to the Andalusian *Ibn Muʿādh*, who lived in the second half of the eleventh century; in this work the parameter used is 18°.[51] In the thirteenth century *al-Marrākushī* used 20° and 16° for morning and evening twilight,[52] and in the fourteenth century *Ibn al-Shāṭir* used 19° and 17°.[53] His colleague *al-Khalīlī* based his tables of twilight and darkness on these two parameters,[54] which the fifteenth century scholar *Sibṭ al-Māridīnī* stated were the parameters used by all the professional *muwaqqits* of his day.[55] All of the Ottoman twilight tables which I have examined are based on these same parameters.[56]

Ibn Yūnus' tables for the duration of twilight and darkness are the earliest known tables of their kind. The purpose in tabulating these times was to regulate the dawn and evening prayers.

(a)–(b) The functions labelled "duration of twilight," and here denoted by r and s for morning and evening, respectively, are referred to by *Ibn Yūnus* as "arguments" (*ḥiṣṣa*). These arguments, being parts of the nocturnal arc, are simply measures of time in equatorial degrees. As stated above, the tables for twilight are simply the corresponding tables of time since sunrise with the northern and southern zodiacal signs reversed. Denoting the angles of depression for morning and evening by h_r and h_s, respectively, we have

$$r(\lambda) = T(h_r, \lambda^*) \quad \text{and} \quad s(\lambda) = T(h_s, \lambda^*), \tag{22}$$

where $\lambda^* = \lambda + 180°$.

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[50] See *Kennedy* & *Davidian* [1].

[51] See *Sabra* [1].

[52] *Wiedemann* & *Frank* [1], 13.

[53] *Ibn al-Shāṭir* mentions the values 19° and 17° in Chapter 38 of his *Zīj*, but also appears to have used 20° for morning twilight (*Wiedemann* & *Frank* [1], 13). Furthermore, a manuscript in Damascus of a work by *Ibn al-Shāṭir* gives the values 17° for evening and 16° for morning (*cf. Khoury* [1], 190).

[54] An analysis of these tables (contained in MS PF) is in preparation.

[55] *Wiedemann* & *Frank* [1], 13. On *Sibṭ al-Māridīnī* see *Suter* [1], no. 445.

[56] As, *e.g.*, in *Ṣālih Effendī*'s prayer-tables (*cf.* footnote 31), and the very popular Ottoman almanac published in *Navoni* [1] and *D'Ohsson* [1], I, facing p. 192.

Fig. 5 shows how the duration of twilight varies with solar longitude according to the various parameters used by *Ibn Yūnus*.

(c) The table of the duration of darkness, $n(\lambda)$, was compiled by subtracting the entries in the tables for morning and evening twilight from the appropriate entries in the table of the nocturnal arc, $2N(\lambda)$ (*cf.* Section 3.1 (e)). Thus

$$n(\lambda) = 2N(\lambda) - s(\lambda) - r(\lambda) \qquad (h_s = 17^\circ \text{ and } h_r = 19^\circ). \tag{23}$$

The length of darkness, as indicated by the title of the table in MS D, is the time from the beginning of the night prayer to the "true dawn."

3.6. Tables for Determining the Qibla by Means of the Sun

In Chapter 28 of the *Ḥākimī Zīj Ibn Yūnus* computes the azimuth of the qibla for Cairo-Fustat, here denoted by q, to be about 53° measured from the meridian. His procedure for determining the qibla is mathematically sound and has been analyzed by *C. Schoy* and more recently by myself. The discussion in the *Zīj* concludes with a table of the solar altitude in the azimuth of the qibla, $h_q(\lambda)$, based on the parameter $q = 52^\circ$, computed "some time ago". The value 53° is based on a different value of the longitude difference between Cairo and Mecca, but no further information is provided about the way in which this difference of longitude was measured.[57]

(a) The table in MS D.86v of $h_q(\lambda)$ is based on the parameter $q = 52^\circ$, and is compiled for each degree of longitude, that is, in twelve columns of thirty entries. There are numerous corrupt readings, as can be seen by checking the symmetry or by comparing the table with the more accurately computed one in Chapter 28 of the *Zīj*, which has been published by *C. Schoy*.[58] MS D.14r contains a table computed for $q = 53^\circ$, symmetrically arranged like the time and azimuth tables.

These tables were no doubt computed in the same way as those analyzed in Section 3.4 above, and *Ibn Yūnus* mentions in the *Zīj* that the table there could be used for determining the qibla or, if that is known, for determining the meridian. Fig. 4 shows how h_q varies with solar longitude ($q = 53^\circ$).

(b) The table in MS D for the hour-angle from the time the sun is in the direction of Mecca up to midday, t_q, is also symmetrically arranged. It is based on $q = 53^\circ$, and so the appropriate solar altitudes for this azimuth are given in MS D.14r. Using these values and *Ibn Yūnus*' tables of $t(h, \lambda)$, I obtain more accurate values of t_q than those of *Ibn Yūnus*. The table of the time, T_q, from sunrise to the time when the sun is in the azimuth of the qibla is easily derived from tables of $t_q(\lambda)$ and $D(\lambda)$; this table is found only in MS C.

(c)–(d) In MS G there are no such tables as in (a) or (b) above. Rather, there are corresponding tables for the solar altitude and the hour-angle when the sun is in the direction perpendicular to the qibla. The tables are based on $q = 53^\circ$.

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[57] *Cf. Schoy* [1], 33–43, and *King* [1], 256–268, especially 266, and also 327, note 11.

[58] *Schoy* [1], 42.

3.7. Tables for Determining the Beginning of the Time for the Afternoon Prayer

Ibn Yūnus' tables are based on the assumption that the time for the afternoon prayer (*ʿaṣr*) commences when the gnomon shadow is longer than its midday shadow by the length of the gnomon and ends at sunset.[59] Thus, if the length of the gnomon is $R = 60$ units, the altitude h_a at the beginning of the time for prayer is defined by

$$\operatorname{Cot} h_a = \operatorname{Cot} h_n + R,$$

so that

$$h_a = \operatorname{arc\,Cot}(\operatorname{Cot} h_n + R), \tag{24}$$

or in modern notation

$$h_a = \operatorname{arc\,cot}(\cot h_n + 1).$$

The determination of the earliest permitted time for the afternoon prayer is not mentioned in the *Ḥākimī Zīj*.[60]

(a) *Ibn Yūnus* tabulates the solar altitude, $h_a(\lambda)$, at the beginning of the interval for the prayer, for each degree of λ' ($\delta \gtrless 0$). Similar tables computed for other latitudes, or of the function $h_a(h_n)$, occur in numerous *zījes*. MS C also contains tables of the functions

$$\operatorname{Cot}_{60} h_a(\lambda) \quad \text{and} \quad \operatorname{Cot}_{12} h_a(\lambda)$$

not attested in the other sources of the *Very Useful Tables*.

(b) Given the values $h_a(\lambda)$ it is not difficult to compute the values of the corresponding hour-angle t_a by using the tables of $t(h, \lambda)$. The few values I have calculated by hand, using interpolation in the tables of $t(h, \lambda)$, are closer to the accurate values that those contained in the sources. For example, the altitude for the sun in Gemini 1° is 40; 34°, according to *Ibn Yūnus* (accurately, 40; 35°). The hour-angles for altitudes 40° and 41° and this longitude are respectively 54; 47° and 53; 37°. The difference is 1; 10° for 1° of altitude, and hence 0; 40° for 0; 34° of altitude. Thus, the required time from midday to the beginning of the afternoon prayer is

$$54;47^\circ - 0;40^\circ = 54;7^\circ.$$

The value in *Ibn Yūnus*' tables is 54; 5°, and the accurately computed value is 54; 7°. The table of $t_a(\lambda)$ in MS B has been published by *C. Schoy*.[61]

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[59] On the determination of the earliest permitted time for the afternoon prayer see *Wiedemann & Frank* [1], 7–10, and *Schoy* [1], 43–53.

[60] The subject is treated in the Yemenī *Mukhtār Zīj*, MS LA. 31v, a work based mainly on *Ibn Yūnus* (*cf. King* [1], 46–47). Here the "end of the afternoon prayer" is before sunset, namely, when the solar altitude is

$$h_b = \operatorname{arc\,Cot}(\operatorname{Cot} h_n + 2R)$$

or in modern notation

$$h_b = \operatorname{arc\,cot}(\cot h_n + 2).$$

This altitude, particularly in the Hanafī legal school, also defines the beginning of the afternoon prayer or of a second afternoon prayer. *Cf. Wiedemann & Frank* [1], 8–10.

[61] *Schoy* [1], 53.

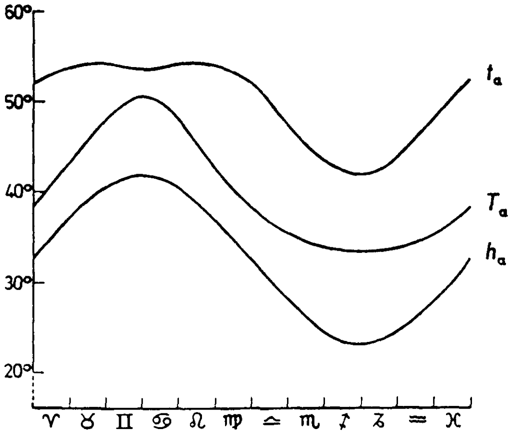

Fig. 6. Solar altitude at the beginning of the time for the afternoon prayer (h_a), and the corresponding hour-angle (t_a), and time before sunset (T_a).

(c) The tables of $T_a(\lambda)$, the time from the beginning of the afternoon prayer to sunset, are easily derived from those of $t_a(\lambda)$ and $D(\lambda)$.

Fig. 6 shows how h_a, t_a and T_a vary with solar longitude for latitude 30°.

3.8. Tables Relating to Institutions Associated with Prayer-Times

A group of tables occurring in MS G, as well as in the *Natījat al-afkār* of *al-Lādhiqī*, are associated with two activities called *salām* and *ṭafy*. It was the custom in medieval times to light candles (*qanādīl*) on the minarets of mosques during the nights of Ramaḍān. The term *ṭafy* refers to the extinction of these candles 20 minutes before daybreak, which may have been the signal to the faithful that the day's fasting should begin. In Egypt today the beginning of the fast is called the *imsāk*, and is also 20 minutes before daybreak. However, in MS DC.53r, at the beginning of a set of prayer-tables for latitude 36°, the times of the *imsāk* and *ṭafy al-qanādīl* are different. The *salām* was a special call of the muezzin, given a few minutes before daybreak, invoking blessings on the Prophet *Muḥammad*. It is clear from the following analysis that the later *muwaqqits* in Cairo were not of one mind regarding the precise times for the *salām* and *ṭafy*.

(a) The table in MS G and MS HA of *al-Lādhiqī*'s *Natījat al-afkār*, entitled "Time from sunset to the *salām*, two (equatorial) degrees before daybreak", is based on the function

$$T_\sigma(\lambda) = 2N(\lambda) - r(\lambda) - 2^\circ \qquad (h_r = 19^\circ). \tag{25}$$

In MS PH of *al-Lādhiqī*'s tables the corresponding entries are 1° more so that the *salām* is assumed to be 1° before daybreak. In a later anonymous set of tables of this function, MS CL, computed to three digits for each 0; 6° of solar longitude by linear interpolation, the entries are based on those in MS PH. Whilst this means the *salām* is 1° before daybreak, the title of the tables states that it is 2° before. On the other hand, a note by the title states that the *shaykh* who compiled the tables was in error because the time difference is 1° not 2°.

(b) Another table in this category is entitled "Time (from sunset) to the *ṭafy* in Ramaḍān on the basis that there is a difference of 5 (equatorial) degrees between this and daybreak." Assuming that the *ṭafy* takes place 5° before daybreak, it might be expected that the function tabulated would be

$$T_\tau(\lambda) = 2N(\lambda) - r(\lambda) - 5^\circ \qquad (h_r = 19^\circ). \tag{26}$$

This, however, is not the case. The table occurs in MS G, which also contains tables of $r(\lambda)$ and $s(\lambda)$ based on

$$h_r = 17^\circ \quad \text{and} \quad h_s = 19^\circ.$$

Nevertheless, if the above expression is indeed that upon which the table is based, the value of h_r must be 20; 15°. This parameter produces close but not consistent agreement of text and recomputation. It seems unlikely, however, that such a parameter would have been used.

(c) Another table is found only in manuscripts of *al-Lādhiqī*'s tables. It is entitled "(Oblique) ascensions of (the ascendant at the time of) the *salām*, two (equatorial) degrees before daybreak." The title also states that if one subtracts three degrees from the values, the result is the ascension of (the ascendant at the time of) the *ṭafy*. An entry is given for each degree of λ in 12 columns of 30 entries.

If the ascendant at daybreak is H, then

$$\alpha_\varphi(\lambda_{\mathrm{H}}) = \alpha_\varphi(\lambda) - r(\lambda). \tag{27}$$

The table is based on the function

$$\alpha_\sigma(\lambda) = \alpha_\varphi(\lambda) - r(\lambda) - 2^\circ \qquad (h_r = 19^\circ,\ \varphi = 30^\circ) \tag{28}$$

computed with *Ibn Yūnus'* tables.

In later tables of this function based on those described above, values are given for each 0; 3° or 0; 6° of solar longitude, computed by linear interpolation. In MSS CM and CN, however, the entries are larger than those in MS HA by ½° and 1° respectively. From these few tables investigated thus far, it is apparent that the time of the *salām* was variously defined as being 1°, 1; 30°, or 2° (that is 4, 6, or 8 minutes) before daybreak.

(d) Another table in this category is entitled "(Oblique) ascensions of (the ascendant at) nightfall". It has the same format as the table in (c) above, and is based on the function

$$\alpha_s(\lambda) = 360^\circ - \{\alpha_\varphi(180^\circ - \lambda) - s(\lambda)\} \qquad (h_s = 17^\circ,\ \varphi = 30^\circ) \tag{29}$$

computed with *Ibn Yūnus'* tables. That this expression is correct is easily demonstrated. MS C contains a similar table for daybreak.

3.9. Table for Orienting Ventilators

Ibn Yūnus' Very Useful Tables contain a table giving the solar altitude, h_v, in the azimuth of the ventilator (*al-bādahanj*), for each degree of λ' ($\delta \gtrless 0$). The table is based on an azimuth of 27; 30° (*S* of *E*), that is, the azimuth of the rising sun at the winter solstice.[62] It is understood that the back of the ventilator should face this direction, and that the front should be exposed to the winds from the north-west. The azimuth of the rising sun at the winter solstice is a very nice direction, long popular amongst the Egyptians, but it is far from the direction which would afford optimal orientation for a ventilator in Cairo.[63] According to the fifteenth century historian of Egypt *al-Maqrīzī*, the same azimuth was used to define the qibla in several early Egyptian mosques.[64]

The Iraqī scholar *ʿAbd al-Laṭīf al-Baghdādī* (1162–1231), who visited Egypt about the year 1200, made this observation on the ventilators of Egyptian houses:[65]

> (The Egyptians) make the openings of their houses exposed to the agreeable winds from the north. One sees hardly any houses without ventilators. These ventilators are tall and wide, and open to every action of the wind; they are erected carefully and with much skill. One can pay between one hundred and five hundred dinars for a single ventilator, but small ones for ordinary houses cost no more than one dinar each.

Further, in a collection of short treatises by *Ibn Yūnus*,[66] MS M.110v: 5–11, we find the following instructions:

> To mark the direction of the ventilator, first establish the four cardinal directions, and then count from the east-point southwards by the amount of the rising amplitude of (the sun at the first point of) Capricorn. Next extend a line (from the center) in this direction, and this will be the direction for the ventilator. Form a rectangle with another line, and set up the *maḥalla* (?) on this perpendicular. A good procedure is to divide the front in ten parts and make the side five and one-half, according too the technical convention of the craftsmen. God, may He be exalted, grants success.

The short treatise, *Tuḥfat al-aḥbāb fī l-bādahanj wa-l miḥrāb* of *Ibn al-Majdī*, is devoted to methods of marking the azimuth of the ventilator and the qibla on a plane surface and shows that the orientation of ventilators in this way was still of some concern in fifteenth century Egypt.[67]

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[62] In Chapter 18 of his *Zīj*, *Ibn Yūnus* accurately computes the solisticial rising amplitude for his parameters to be 27; 30,53°. *Cf. King* [1], 169. The recomputed values used in Table 9 are based on this value.

[63] Northerly winds are more numerous in Cairo at all seasons. From a study of the wind directions there over an 18-year period (I.D. 1117, 252–256), I calculate that the "resultant", which would seem to correspond to an optimum orientation, is about 75° S of E.

[64] *Cf. al-Maqrīzī* [1], IV, 21ff, on the divergence of the qiblas in early Egyptian mosques, summarized in the article *Masdjid* in EI_1, III, 337b–338a.

[65] *Cf. Zand* [1], 178–179. The translation is my own.

[66] On this manuscript see *King* [1], 10 and 348. The second procedure outlined in the passage translated is approximate and yields an azimuth of arc tan (0.55) or 28;48° south of east.

[67] I have examined MS EB of this work by *Ibn al-Majdī*. See also *Wiedemann* [1], I, 546–547 and 589. On *Ibn al-Majdī* see footnote 37 above.

For illustrations of ventilators still in use in Dubai and Sharjah, Union of Arab Emirates, see *Azzi* [1].

The entries in *Ibn Yūnus*' table of $h_v(\lambda)$ are not as accurately computed as those in similar tables of $h(a, \lambda)$ (see Section 3.4). Many of the entries are in error by about −0; 5°, which would suggest that the value used for $h_e(a)$ was in error by this amount. More curious is the fact that in the table in MS D the entries in the sixth column decrease to 0; 41° rather than to 0; 0°, as one would expect, since the altitude in this azimuth at the winter solstice is 0°. This is also the case in the corresponding table in MS CY.7v, and it may be that the correction to 0; 41° is an attempt to account for the displacement of the visible horizon from the true horizon. (See further Section 3.10.) The corresponding table in MS CE of *al-Lādhiqī*'s *Natīјat al-afkār* has entries which decrease to 0; 0°. The differences would hardly affect the efficiency of a ventilator oriented in the direction of the rising sun at the winter solstice.

Fig. 4 shows how h_v varies with solar longitude.

3.10. Table of Corrections for Horizontal Refraction

In MS G there is a curious table entitled "difference minutes" (*daqāʾiq al-ikhtilāf*), here denoted by $\Delta D(\lambda)$, tabulated for each degree of λ' ($\delta \gtrless 0$). The table is not contained in any of the other known manuscripts of the *Very Useful Tables*, and neither in MS G nor in any other known work of *Ibn Yūnus* is there any explanation of the nature or purpose of this function. For reasons stated below I believe that the table in the form in which it appears in MS G is *not* due to *Ibn Yūnus*.

In some manuscripts of *al-Lādhiqī*'s *Natījat al-afkār* (*e.g.*, MS CE), values of the same function are given for each zodiacal sign, and it is stated that the function is intended to measure the time in equatorial minutes taken by the sun to move between the true horizon (*al-ufq al-ḥaqīqī*) and the visible horizon (*al-ufq al-marʾiy*). Thus the table is of considerable interest, since hitherto it has been thought that the quantitative determination of horizontal refraction was not discussed by Islamic scientists. The theory of the qualitative effects of atmospheric refraction is outlined already in *Ptolemy*'s *Optics*.

The function tabulated in MS G occurs also in the prayer-tables of *Muḥammad al-Manūfī* (MS CH, but not MS CI), a *muwaqqit* associated with the Ghūrīya Mosque in Cairo, who lived about the year 1500. *al-Manūfī* modified the prayer-tables of *Ibn Yūnus*, without reference to his source, to account for the effect of the difference minutes in the tables associated with horizon phenomena.[68]

---

[68] It may be that *al-Manūfī* is to be identified with *Shams al-Dīn Muḥammad b. Abī l-Fatḥ al-Ṣūfī al-Miṣrī* (*Suter* [1], no. 447), who died about 1495. *Suter* lists fifteen works by this individual, including prayer-tables and a commentary to the *Zīj* of *Ulugh Beg*, in which he adapted the planetary tables for the longitude of Cairo. A manuscript in Damascus of another work due to *Shams al-Dīn al-Ṣūfī*, which supports the identification with *al-Manūfī*, contains a table of the "difference minutes" and a table of the half-excess for latitude 33; 30° (Damascus), corrected for these minutes. *Cf. Khoury* [1], 205–206.

Muḥammad al-Manūfī's tables in MSS CH and CI also contain instructions for taking the solar radius into account in time-keeping by the sun. Another manuscript in Damascus contains four pages of tables entitled "the difference minutes of the solar radius to be added to the semi diurnal arc for latitude 33; 30° (Damascus)", by the Egyptian *Muḥammad al-Tandatāʾī al-Azharī*. *Cf. Khoury* [1], 61.

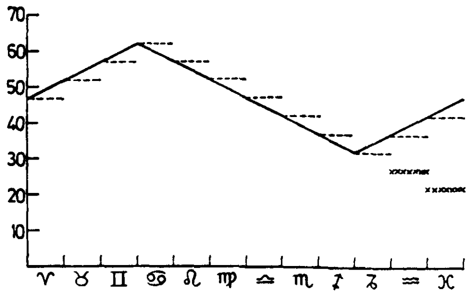

Fig. 7. Variation of the "difference minutes" with solar longitude according to *Ibn Yūnus* and *al-Manūfī* (——), and *al-Lādhiqī* (MS PH.7v: ----; MS HA.3: ××××).

His son *ʿAbd al-Qādir al-Manūfī* wrote a treatise on the difference minutes, and the Damascus astronomer *Ibn al-Shāṭir* (ca. 1350) also discussed them: manuscripts of their works are extant in Cairo.[69]

The tables in MSS G and CH give a value of $\Delta D(\lambda)$ to minutes and seconds for each degree of λ' ($\delta \gtrless 0$), calculated by linear interpolation between the values at the equinoxes and solstices. The significant entries, in equatorial minutes, are as follows:

Equinoxes	47
Summer solstice	62
Winter solstice	32.

In several of the manuscripts of *al-Lādhiqī*'s tables which I have examined, a value of $\Delta D(\lambda)$ to one digit is given for each zodiacal sign, and in one source (MS HA.3) the difference minutes decrease uniformly from 47 at the autumnal equinox to 17 at the vernal equinox! The various tables encountered thus far are represented graphically in Fig. 7.

The relation of this function to *Ibn Yūnus* is claimed by *al-Manūfī*. In the introduction to his prayer-tables (MS CH.7v) he states that *Ibn Yūnus* found the difference minutes to be 47 at the equinoxes, increasing by 5 for each sign to the summer solstice and decreasing by 5 for each sign to the winter solstice.

Now, only part of the theory described by *al-Manūfī* and represented in the tables makes sense. It seems reasonable to assume that *Ibn Yūnus* would have considered the visible horizon to be at a fixed angle of depression below the true horizon. If this is so, the above equinoctial and solsticial values are easily shown to be mutually inconsistent. On the other hand if the three values are the results of observations made at the equinoxes and solstices, then only one of them is a good result and the other two are considerably in error. Of course, we have absolutely no control over the results of observations when we have no information concerning the kind of measurements *Ibn Yūnus* might have made to derive values for the difference minutes. So let us investigate the three values on which the tables are based.

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[69] See *Suter* [1], no. 479, especially p. 194, 11. 2–4 (*Suter's sehr geringe Abweichungen* are the *daqāʾiq al-ikhtilāf*), and *Brockelmann* [1], II, 156, no. 4, entry 11.

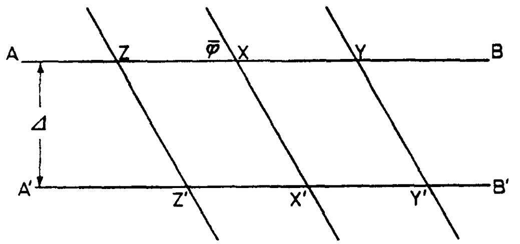

Fig. 8. Plane projection of the visible and true horizons (AB and A'B'), and the solar day-circles at the equinoxes (XX'), summer solstice (YY'), and winter solstice (ZZ'). Not to scale.

Consider Fig. 8, which represents a portion of the western sky about the west-point in plane cross-section. The sun sets across the horizon at an angle $\bar{\varphi}$. AB represents the true horizon and A'B' the visible horizon at an angle of depression Δh below the true horizon. The problem is to find to a first approximation how long the sun takes to move between these two horizons. XX' represents the path of the center of the sun at the equinoxes and YY' and ZZ' its path at the summer and winter solstices respectively. Clearly:

$$XX' = YY' = ZZ' = \Delta h \sec \varphi.$$

Now the time taken by the sun to move from X to X' is measured by the arc XX' of the celestial equator. Thus the difference minutes at the equinoxes, ΔD_0, equal $\Delta h \sec \varphi$. Since *Ibn Yūnus*, according to *al-Manūfī*, takes this as 0; 47° for latitude 30°, we have

$$\Delta D_0 = \Delta h \sec 30° = 0;\, 47°$$

whence

$$\Delta h \approx 0;\, 40° = 2/3°.$$

This then is the vertical displacement of the horizon due to refraction corresponding to the value for ΔD_0 attributed to *Ibn Yūnus*. The modern value for this displacement is 0; 35°.[70]

The solsticial values attributed to *Ibn Yūnus* are, in the light of modern knowledge, less reasonable. A distance of 0; 47° on either of the solisticial day-circles corresponds approximately to an arc of

$$0;\, 47° \sec \varepsilon = 0;\, 51°$$

on the celestial equator. It is clear that *the time taken by the sun to move between the two horizons is, to a first approximation, the same at both solstices.* This time is some 0; 4° more than the corresponding time at the equinoxes, according to the above calculation.

Ibn Yūnus was a master of spherical astronomy, and I consider it unlikely that he would make as elementary an error as to give radically different values of ΔD at the solstices. After all, he was able to compute accurately the time from sunrise to the time at which the solar altitude is h, that is, $T(h, \lambda)$, for integral values of h from 1° to 83°. Also, his tables for the duration of twilight prove that he could also tabulate $T(h, \lambda)$ accurately for solar positions below the horizon. From his table of $T(h, \lambda)$ for altitude 1° *Ibn Yūnus* could have read off *immediately* that the time taken by the sun to move 1° *below* the horizon was, in equatorial minutes, as follows:[71]

---

[70] *Cf., e.g., Smart* [1], 69, and *Woolard & Clemence* [1], 79.

Equinox	70
Summer solstice	79
Winter solstice	79.

From these values we can find by linear interpolation the corresponding values for an angle of depression of 2/3°, namely:[72]

Equinox	47
Summer solstice	53
Winter solstice	53.

It is then quite reasonable to interpolate linearly between the equinoctial and solsticial values of ΔD for general solar longitudes, because the variation in ΔD is relatively small.[73]

I suspect that *Ibn Yūnus*, in a work other than the *Ḥākimī Zīj* and the *Very Useful Tables*, stated that *the difference minutes were 47 at the equinoxes and increased about 5 to maxima at the solstices*, which would be virtually correct. The assertion of *al-Manūfī* that *Ibn Yūnus* had stated that *the difference minutes were 47 at the equinoxes and changed by 5 for each sign with a maximum at the summer solstice and a minimum at the winter solstice* is then simply garbled. My hypothesis has no substantiation in the manuscripts examined thus far.

Part 4: Ibn Yūnus' Instructions on the Use of the Tables

The instructions accompanying the tables in MSS D and G contain virtually no information about the way the tables were intended to be used in practice, nor do they reveal how the tables were computed. They begin with pious statements about God as the Creator of the Universe. The Qur'ānic verse 10.5/5

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[71] Accurately computed for $\varphi = 30°$ and $\varepsilon = 23;35°$, these values are respectively 69, 78, and 78.

[72] Accurately computed: 46, 52, and 52 respectively.

[73] It is perhaps worth noting the occurrence of another linear zigzag function in a short treatise attributed to *Ibn Yūnus*. The treatise describes a candle-clock consisting of twelve oil-candles and has been discussed in *Wiedemann & Hauser* [1], 18, and more fully in *Kennedy & Ukashah* [1]. It is required to fill the candles in such a way that if they are all lit at sunset, one candle goes out at the end of each seasonal hour of the night. Since the seasonal night hours vary in length throughout the year, the amount of oil required in each candle also varies. The author of the treatise gives the amounts of oil to be used in the first candle for each zodiacal sign. These quantities define a linear zigzag function, reminiscent of those used in Babylonian astronomy. Indeed the parameter underlying the table is the ratio 3:2 for maximum to minimum daylight, a basic Babylonian parameter.

> (God) made the sun as a shining light and made the moon a luminary and measured out its mansions

is continued thus (MS D.83r: 5):

> for perfecting prognostications about what He orders and what He effects.

The Qur'ānic verse 37.6/5

> (God) adorned the lower heavens with the stars

is continued (MS D.83r: 5–6):

> and the times are known by what rises and sets.

Since, states *Ibn Yūnus*, time-keeping (*ʿilm al-mawāqit*) was ordained by God, he thought he would prepare these tables which would be the "limit of usefulness" (*ghāyat al-intifāʿ*).

The remainder of *Ibn Yūnus*' instructions on using the tables (MS D.83r–83v) is translated in full:

> To determine the time from sunrise to any instant you wish, enter the solar longitude in the table of the instantaneous altitude, and you will find the required time, if the solar altitude is in the east. If it is in the west, subtract the value in the table from the diurnal arc, and the remainder will be the time up to that instant. God—may He be exalted—knows best.
>
> Example: Suppose the sun is in 21° of Taurus, and the instantaneous altitude is 30°. We find 35;38°[a] in the table: this is the time from sunrise to the instant in question. If the altitude is in the west, subtract 35;38° from the diurnal arc, which is 201;46°[b]. The remainder is 166;8°, and this is the required time.
>
> To determine how much of the day has passed, and how many hours of daylight are left, divide the time since sunrise by the number of degrees corresponding to a seasonal day hour for the day in question, or by 15 if you want equinoctial hours. The result is the number of hours of daylight that have passed. Subtract the seasonal hours from 12, and the equinoctial hours from the number of hours of daylight on the day in question: the remainder will be the number of hours of daylight left.
>
> Example: Suppose the sun is in 11° of Aries, and the instantaneous altitude is 45° in the east. We find the time since sunrise for this altitude as before: it is 53;19°[c]. We divide this by the number of degrees corresponding to a seasonal day hour, which is 15;24°,[d] and the result is [3][e];27. These are the hours of daylight that have passed. Subtract them from 12, and the remainder, which is the number of hours of daylight left, is 8;33. If we divide

---

[a] As in MS D.54v.

[b] As in MS D.12r.

[c] As in MS D.39v.

[d] MS D.13r has the more accurate value 15;25°.

[e] Text: 4 (*sic*).

the time since sunrise by 15, the quotient is [3; 33][f], and this is the number of equinoctial hours which have passed. We subtract these from the number of equinoctial hours of daylight on the day in question, which is 12; 20, and the remainder is 8; 47. This is the number of hours of daylight left. I have included a table at the end of the book in which the entry corresponding to the solar longitude is multiplied by the time since sunrise to give the seasonal day hours passed since sunrise.

To determine (the altitude of the sun for) the seasonal and equinoctial hours, enter the solar longitude in the tables of the time since sunrise for each altitude and turn through table after table until you find a time as large as the degrees corresponding to the seasonal or equinoctial hours. Interpolate if necessary, and the result will be the required altitude.

Example: Suppose the sun is in the last degree of the sign of Gemini, and we want the altitude after the first seasonal hour. We enter the solar longitude in the tables, turning through table after table, and find opposite the longitude in the table for 13° a time of 16; 24°[g]. We subtract this from the number of degrees in one seasonal hour, which is 17; 26°,[h] and the remainder is 1; 2°. We divide this by the difference between the time [i][in the tables for altitude 13° and for altitude 14°, which difference is 1; 15°. The ratio of one to the other is 0; 50, which we add to the first altitude][i] and so the required altitude is 13; 50°. If we want the altitude after one equatorial hour, we enter the solar longitude in the tables and we find opposite it 13; 56°. We subtract this from 15°, and the remainder is 1; 4°, which we divide by the difference between the time for altitude 11° and that for 12°, which is 1; 15°. The ratio of the one to the other is 0; 51, which we add to the first altitude, obtaining 11; 51° for the required altitude.

To determine the ascendant at the eastern horizon, add the time since sunrise to the oblique ascension of the solar longitude, and the result is (the oblique ascension of) the ascendant at that time.[74]

Example: Suppose the sun is in 18° of the sign of Scorpio and the instantaneous altitude is 25°. We take the time since sunrise for this altitude and find it to be 33; 20°.[j] We add this to the oblique ascension of the sun, namely, 235; 52°,[k] and the sum is 269; 12°. We take the longitude corresponding to this: 16; 18° results, being the degree of the sign of Sagittarius which is rising.

To determine the hour-angle at the beginning of the afternoon prayer, you should know that there is a table for this included at the end of the book. You enter the solar longitude as argument and read off the required value.

To determine the half diurnal arc, you enter the solar longitude in the appropriate table and read the required value.

To determine the duration of morning twilight, which is the remainder of the nocturnal arc to sunrise, you enter the longitude directly opposite the solar longitude on the ecliptic in the table of the time since sunrise for altitude 19°, and the value you find will be the required time.[75]

To determine the time from sunrise to the time when the sun is in the azimuth of the qibla, enter the solar longitude in the appropriate table and read the required value.

To determine the azimuth, proceed as with the tables for time. The direction of the azimuth is south if the sun is in the southern zodiacal signs, or if it is in the northern signs and the altitude is greater than the altitude in the prime vertical; otherwise, it is north. God knows best.

Certain interesting facts will be revealed to anyone who examines these tables closely.[76] For, example, if the solar altitude equals the declination, then both the azimuth and the time since sunrise are equal to the complement of the other. Further, if you take the declination as altitude, and you know the complement of the time since sunrise for the solar longitude, then it will be the half excess of daylight, if the latitude is 30°. Numerous other facts and points of interest relating to night and day also become evident. God—may He be exalted—knows best.

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[f] Text: 4; 13 (*sic*).
[g] As in MS D.71v.
[h] As in MS D.13r.
[i] Missing in MS D.
[j] As in MS D.59v.
[k] This is accurate, and is the value given in the *Ḥākimī Zīj*. MS HA.21 has 235; 51°.

[74] In modern notation

$$\alpha_\varphi(\lambda_{\mathrm{H}}) = \alpha_\varphi(\lambda) + T(h, \lambda).$$

[75] *Cf.* footnote 47 above.

[76] The two statements which follow are easily verified. Firstly, since

$$t = \arc \sin \left\{ \frac{\cos h \cos a}{\cos \delta} \right\}$$

(*cf. King* [1], 150), we have, when $h = \delta$,

$$a + t = 90°.$$

Secondly, since

$$T(h, \lambda) = f(\lambda) + \arc \sin [p(h, \lambda) - \sin f(\lambda)]$$

where

$$p(h, \lambda) = \frac{\sin h}{\cos \delta(\lambda) \cos \varphi}$$

(*cf.* formula (12)), we have, when $h = \delta$ and $\varphi = 30°$, that

$$p = \frac{\tan \delta}{\cos 30°} = \frac{(\tan \delta \tan 30°)}{\sin 30°} = 2 \sin f(\lambda)$$

(*cf.* formula (2)). Therefore,

$$\begin{aligned} T(h, \lambda) &= f(\lambda) + \arc \sin [2 \sin f(\lambda) - \sin f(\lambda)] \\ &= 2f(\lambda), \end{aligned}$$

so that

$$\begin{aligned} \bar{t} &= 90° - (D - T) = T - (D - 90°) \\ &= T - f = f. \end{aligned}$$

Part 5: Sample Entries from the Tables

Tables 1 to 10 below contain sample entries from the corresponding tables in Sections 1 to 10 in Part 3. The entries are adequate to convey some idea of the accuracy of *Ibn Yūnus'* computation and to identify any new material which may come to light. The entries shown are, unless otherwise indicated, taken from the first, middle, and last rows of each of the six columns in the original table, that is, for longitudes

1°	31	61	181	211	241
15	45	75	195	225	255
30	60	90	210	240	270

The values agree with recomputation except where the error in the second digit is shown in square brackets, computed according to the convention

error = (value in text) — (recomputed value).

The values quoted are usually taken from the tables in MSS D, G, and HA. Variants between the manuscripts are, in general, attributable to copyists' errors, though, for example, see Tables X1 and X2 in Section 2.3.

Table 1. *Spherical astronomical functions not tabulated in the Ḥākimi Zīj*

(c) Solar meridian altitude — MS D.14v

60; 24°	71; 53	80; 29	59; 36	48; 7	39; 31
65; 58 [+1]	76; 26	82; 44	54; 2 [−1]	43; 34	37; 16
71; 32	80; 16	83; 35	48; 28	39; 44	36; 25

(d) Half diurnal arc — MS D.12r

90; 14°	96; 59	102; 27	89; 46	83; 1	77; 33
93; 27	99; 49 [+1]	104; 0	86; 33	80; 11 [−1]	76; 0
96; 46	102; 19	104; 36	83; 14	77; 41	75; 24

(e) Nocturnal arc — MS D.6v

179; 32°	166; 2	155; 6 [+1]	180; 28	193; 58	204; 54 [−1]
173; 6	160; 24 [+1]	152; 0	186; 54	199; 36 [−1]	208; 0
166; 28	155; 22	150; 48	193; 32	204; 38	209; [12][a]

(g) Conversion factor — MS D.3r

[0]; 3, 59	; 3, 43	; 3, 31	; 4, 0 [−1]	; 4, 21 [+1]	; 4, 38 [−1]
; 3, 51	; 3, 36	; 3, 27 [−1]	; 4, 10	; 4, 29	; 4, 45 [+1]
; 3, [44][b] [+1]	; 3, 31	; 3, 26 [−1]	; 4, 19 [−1]	; 4, 38	; 4, 46

(h) Length of daylight in equinoctial hours — MS G.75v

12; 2	12; 56	13; 40	11; 58	11; 4	10; 20
12; 28	13; 19 [+1]	13; 52	11; 32	10; 41 [−1]	10; 8
12; 54	13; 39	13; 57	11; 6	10; 21	10; 3

---

[a] Illegible in MS D.

[b] MS D: 46 (*sic*).

Table 2. *Time since sunrise and hour-angle*

(a) Time since sunrise for altitude 30° — MS D.54r

35; 13°	34; 55	36; 5 [+1]	35; 20 [+1]	38; 23	43; 50 [−1]
34; 19 [+1]	35; 23	36; 34 [−1]	36; 24 [+1]	40; 44 [−1]	46; 12 [−1]
34; 53 [−1]	36; 2	36; 49 [+1]	38; 14	43; 39	47; 17

Time since sunrise for altitude 45° — MS D.39r

54; 35° [+1]	52; 27	53; 28 [+1]	54; 56 [+1]	65; 59 [+2]	
53; 1 [+2]	52; 44	54; 2 [−1]	58; 17 [−2]		
52; 28 [+1]	53; 25 [+1]	54; 19 [+1]	65; 15 [−1]		

Time since sunrise for altitude 60° — MS D.24r

85; 1° [−5]	71; 11	70; 54 [+2]			
74; 31 [+7]	70; 33 [+1]	71; 22 [−1]			
71; 20 [+2]	70; 52 [+2]	71; 39			

(b) Hour-angle for altitude 30° — MS D.53v

55; 1°	62; 4	66; 22 [−1]	54; 26 [−1]	44; 38	33; 43 [+1]
58; 38 [−1]	64; 26	67; 26 [+1]	50; 9 [−1]	39; 27	29; 48 [+1]
61; 51 [−2]	66; 17	67; 47 [−1]	45; 0	34; 2	28; 7

Hour-angle for altitude 45° — MS D.38v

35; 39° [−1]	44; 32	48; 59 [−2]	34; 50 [−2]	17; 2 [−2]	
40; 26 [−2]	47; 5	49; 58 [+1]	28; 16 [+2]		
44; 18 [−1]	48; 54 [−1]	50; 18	17; 59 [+1]		

Hour-angle for altitude 60° — MS D.24r

5; 13° [+5]	25; 48	31; 33 [−2]			
18; 56 [−6]	29; 15 [−1]	32; 38 [+1]			
25; 26 [−3]	31; 27 [−2]	32; 57			

Table 3. *Solar azimuth (northern azimuths are shown negative)*

Solar azimuth for altitude 30° — MSS D.106r and 0.124v–125v

18; 54°	3; 22	− 7; 40[c] [−1]	20; 3[e] [+1]	37; 27	53; 8[h] [+1]
11; 16	−2; 31	−10; 29	28; 7[f]	45; 17	58; [3][i] [−1]
3; 50[a] [+1]	−7; 24[b] [−1]	−11; 32[d] [+1]	36; 5[3][g] [+1]	52; 41	60; 5

Solar azimuth for altitude 45° — MS D.99v

34; 2[8]°[j]	13; 56	0; 20	36; 5 [+1]	66; 3	
24; 6	6; 34 [−4]	−3; 5	48; 17		
14; 32 [+1]	0; 40	−4; 21	64; 40 [−1]		

Solar azimuth for altitude 60° — MS D.91r

79; 42°	31; 37	11; 4			
49; 32	20; 17	6; 10			
32; 33	11; 32	4; 22			

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[a] MS O: 49 (accurate).

[b] MS O: 23 (accurate).

[c] MS O: 39 (accurate).

[d] MS O: 33 (accurate).

[e] MS O: 1.

[f] MS O: 8.

[g] MS D: 56; MS O: 52.

[h] MS O: 7 (accurate).

[i] MS D: 8 (*sic*); MS O: 3.

[j] MS D: 23 (*sic*).

Table 4. *Solar altitude for various azimuths*

Solar altitude for azimuth 30° MSS D.9r and 0.27v–28v

41; 30°	59; 3	72; 50	40; 17	22; 45	8; 57
49; 54	66; 13	76; 39	31; 53	15; 34	5; 9
58; 30	72; 29	78; 7	23; 17	9; 18	3; 41 [+1]

Solar altitude for azimuth 60° MSS D.9v and 0.36r–37r

56; 45°	69; 32 [+1]	79; 9	55; 52	43; 6	33; 28
62; 55 [+1]	74; 36	81; 42	49; 43[b]	38; 1	30; 56 [+1]
69; 8[a]	78; 55	82; 40	43; 30 [+1]	33; 43 [+1]	29; 58

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[a] MS O: 8; MS D: 20 (*sic*).

[b] MS O: 43; MS D has no entry for the second digit.

Table 5. *Duration of twilight and darkness*

(a) Duration of morning twilight $(h_{\mathrm{r}} = 19^\circ)$ MS D.7v

22; 7° [+1]	23; 24	25; 38	22; 3 [−1]	22; 12	23; 10 [+1]
22; 32	24; 24	26; 32 [+2]	20; 0 [+1]	22; 37	23; 33 [+1]
23; 20	25; 33 [−1]	26; 52	22; 10 [−1]	23; 6 [−1]	23; 42

Duration of morning twilight $(h_{\mathrm{r}} = 20^\circ)$ MS D.10v

23; 18° [+1]	24; 42	27; 7 [−1]	23; 15	23; 22	24; 20
23; 45[a] [−1]	25; 48 [+1]	28; 5[b]	23; 8	23; 47	24; 45 [+1]
24; 38	27; 3	28; 28 [−1]	23; 21 [+1]	24; 18	24; 54

(b) Duration of evening twilight $(h_{\mathrm{s}} = 16^\circ)$ MS D.10r

18; 35° [+1]	19; 34 [+1]	21; 15 [−2]	18; 33	18; 44	19; 34 [−1]
18; 52 [−2]	20; 20	21; 56 [−1]	18; 29 [−2]	19; 6	19; 55
19; 30	21; 12 [−2]	22; 13	18; 43	19; 32 [−1]	20; 4

Duration of evening twilight $(h_{\mathrm{s}} = 17^\circ)$ MS D.7r

19; 45°	20; 49 [−1]	22; 42 [−1]	19; 43	19; 52 [−2]	20; 46 [−1]
20; 6	21; 39 [−2]	23; 26 [−1]	19; 40	20; 15 [−1]	21; 8
20; 46 [−1]	22; 38 [−1]	23; 44 [−1]	19; 51 [−1]	20; 44 [−1]	21; 16 [−1]

(c) Duration of darkness $(h_{\mathrm{r}} = 19^\circ$ and $h_{\mathrm{s}} = 17^\circ)$ MS D.6r

137; 40° [−1]	121; 49 [+1]	106; 46 [+2]	138; 41	151; 54 [+2]	160; 58 [−1]
130; 28	114; 21 [+3]	102; 2 [−1]	145; 14 [−1]	156; 44	163; 19 [−1]
122; 22 [+1]	107; 11 [+2]	100; 12 [+1]	151; 30 [+1]	160; 47 [+1]	164; 14 [+1]

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[a] MS D.5r: 45; MS D.10v: 43 (*sic*).

[b] MS D.5r: 8 (*sic*).

Table 6. *Tables for determining the qibla*

(a) Solar altitude in the azimuth of the qibla $(q = 52^\circ)$ MSS D.86v and 0.119r–120r

47; 23°	63; 13	75; 27 [+1]	46; 18	30; 28	18; 15[a] [+1]
54; 59	69; 37	78; 45	38; 42	24; 4	14; 55
62; 43	75; 8	80; 2 [+1]	30; 57	18; 33	13; 39

Solar altitude in the azimuth of the qibla $(q = 53^\circ)$ MS D.14r

46; [4]3° [−2]	62; 44 [−2]	75; 10	45; 40 [+2]	29; 35 [−2]	17; 11 [−2]
54; 26	69; 13 [−2]	78; 31 [−1]	37; 54 [−3]	23; 8	13; 50
62; 15 [−1]	74; 52 [+1]	79; 48 [−2]	30; 4 [−3]	17; 30 [−1]	12; 32 [−1]

(b) Hour-angle when the sun is in the azimuth of the qibla $(q = 53^\circ)$ MS D.8v

33; 12° [+1]	22; 0 [+4]	12; 35 [−1]	33; 57	45; [1]2	54; 3[2][b] [+1]
27; 50 [−1]	17; 12 [+2]	9; 57 [+2]	39; 19 [+2]	49; 57 [−1]	57; 13
22; 21 [+4]	12; 48 [−3]	8; 51	44; 51 [+1]	54; 18 [+1]	58; 18 [+1]

(c) Solar altitude in the azimuth perpendicular to the azimuth of the qibla $(q = 53^\circ)$ MS G.71v

54; 36°	68; 7 [+1]	78; 20	53; 40	40; 9 [−1]	29; 56
61; 6	73; 28 [−2]	81; 2 [−2]	47; 10	34; 48 [+1]	27; 14 [+1]
67; 42	78; 6 [+1]	82; 6 [+1]	40; 34 [−1]	30; 10 [−1]	26; 10 [−1]

(d) Hour-angle when the sun is in the azimuth perpendicular to the azimuth of the qibla $(q = 53^\circ)$ MS G.77r

20; 25° [+1]	13; 13 [−2]	7; 11 [−17]	20; 51 [−2]	28; 2	33; 50
16; 52 [−8]	10; 10 [−6]	5; 48 [−1]	24; 19 [+2]	31; 0 [−1]	35; 29 [+1]
13; 24 [−5]	7; 22 [−15]	5; 1 [−10]	27; 49 [+1]	33; 43 [+2]	36; 7 [+1]

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[a] MS O: 14.

[b] MS D: 37.

Table 7. *Tables for determining the beginning of the time for the afternoon prayer*

(a) Solar altitude MS D.11v

32; 33° [+1]	3[6]; 59 [−1]	40; 34 [−1]	32; 13	27; 48	24; 20 [+1]
34; 38 [−2]	38; 51	41; 35 [+1]	30; 6	25; 59	23; 24 [+2]
36; 50 [−2]	40; 28 [−1]	41; 58 [+1]	27; 56	24; 26 [+1]	23; 0

(b) Hour-angle MS D.15v

51; 54° [−1]	53; 56 [−1]	54; 5 [−2]	51; 42	47; 45 [−2]	43; 41 [+1]
53; 6 [−1]	54; 12	53; 52 [−4]	50; 2	45; 45	42; 23 [−1]
53; 55	54; 6 [−2]	53; 48 [−2]	47; 57 [+1]	43; 45 [−2]	41; 53 [−1]

(c) Time before sunset MS D.13v

38; 20° [+1]	43; 3 [+1]	48; 22 [+2]	38; 4	35; 16 [+2]	33; 52 [−1]
40; 21 [+2]	45; 37 [+1]	50; 4	36; 31	34; 26 [−1]	33; 37 [+1]
42; 51 [−1]	48; 13 [+2]	50; 48 [+2]	35; 17 [−1]	33; 56 [+2]	33; 31

Table 8. *Tables of functions relating to institutions associated with the prayer times*
(*N.B.* These tables have not been recomputed)

a) Time from sunset to the *salām* MSS G.74r and HA.16

155; 25°	140; 38	127; 28	156; 24	169; 46	179; 44
148; 34	134; 0	123; 23	162; 52	174; 59	182; 27
141; 8	127; 49	121; 56	169; 21	179; 31	183; 30

(b) Time from sunset to the *ṭafy* in Ramaḍān MS G.76v

150; 51°	135; 59	122; 39	151; 50	165; 19	175; 25
143; 57	129; 15	118; 36	158; 23	170; 36	178; 12
136; 27	123; 1	116; 58	164; 55	175; 12	179; 20

(c) Oblique ascensions of the ascendant at the time of the *salām* MS HA.25–26
(values are taken from the head of each column)

$\lambda =$ 1°	336; 34°	$\lambda =$ 31°	356; 27	$\lambda =$ 61°	18; 45
91	47; 38	121	83; 37	151	121; 15
181	157; 4	211	191; 37	241	226; 7
271	259; 59	301	290; 20	331	315; 27

(d) Oblique ascensions of the ascendant at nightfall MS HA.23–24
(values are taken from the head of each column)

$\lambda =$ 1°	200; 54°	$\lambda =$ 31°	236; 38	$\lambda =$ 61°	273; 59
91	309; 25	121	337; 59	151	0; 20
181	20; 24	211	41; 43	241	67; 9
271	97; 47	301	131; 47	331	166; 21

Table 9. *Table for orienting ventilators by the sun*

Solar altitude in the azimuth of the ventilator MS D.84v

39; 14° [−3]	57; 23 [−3]	71; 47	37; 59 [−3]	19; 49 [−5]	5; 28 [−5]
47; 54 [−4]	64; 49 [−4]	75; 42 [−5]	29; 41 [+19]	12; 22 [−5]	1; 29 [−4]
56; 49 [−3]	71; 23 [−2]	77; 16 [−4]	20; 22 [−6]	5; 49 [−6]	0; 41[a]

[a] MS CE: 0; 0. *Cf.* Section 3.9.

Table 10. *Table of corrections for horizontal refraction*

"Difference minutes" MS G.76v

47; 10	52; 10	57; 10	46; 50	41; 50	36; 50
49; 30	54; 30	59; 30	44; 30	39; 30	34; 30
52; 0	57; 0	62; 0	42; 0	37; 0	32; 0

Appendix A: Index to the Tables in the Manuscript Sources

Nota bene: References are restricted to MSS D and G, which are fairly complete; to MSS B, E, and H which contain only a few tables; and to MSS HA and PH of *al-Lādhiqī*'s *Natījat al-afkār*.

Introduction: D.83r–83v and G.1v–3v

Tables:

Category	Function tabulated	Sources
1(a)	$\delta(\lambda)$	D.11r, 4r HA.8; PH.8v
(b)	$f(\lambda)$	D.14v, 4v HA.9; PH.8v
(c)	$h_n(\lambda)$	D.14v, 4v; G.68r HA.9; PH.9v
(d)	$D(\lambda)$	D.12r; G.68v HA.10; PH.10r
(e)	$2N(\lambda)$	D.6v, 5v; G.72r HA.17; PH.12v
(f)	$\tilde{h}(\lambda)$	D.13r; G.76r
(g)	$1/\tilde{h}(\lambda)$ (factor for converting equatorial degrees to seasonal day hours)	D.3r; G.75r; B.21r
(h)	$2D(\lambda)/15$ (length of daylight in equinoctial hours)	G.75v
(i)	$\alpha'(\lambda)$	HA.19–20 PH.14v–15r
(j)	$\alpha_{30}(\lambda)$	HA.21–22 PH.15v–16r
(k)	$\psi(\lambda)$	D.12v, 3v, 84r HA.7; PH.9r
(l)	$h_0(\lambda)$	D.12v, 3v, 85v HA.7; PH.9r
(m)	arc Sin (x)	D.8r
2(a)	$T(h, \lambda)$	D.85v–15r ($h = 1^\circ$–83°) G.4v–65v ($h = 1^\circ$–83°) B.12r–20r ($h = 52^\circ$–83°)
(b)	$t(h, \lambda)$	D.85v–15r ($h = 1^\circ$–83°) G.4v–65v ($h = 1^\circ$–83°) B.11r–11v ($h = 8^\circ, 9^\circ$) H.1v–35r ($h = 1^\circ$–83°)
3	$a(h, \lambda)$	D.86r–12v ($h = 1^\circ$–88°) G.4v–65v ($h = 1^\circ$–83°)

Category	Function tabulated	Sources
		B.1 v–10 v $(h = 1^\circ\text{–}19^\circ)$
		E.30 v–63 v $(h = 1^\circ\text{–}78^\circ)$
		H.36 v–70 v $(h = 1^\circ\text{–}83^\circ)$
		K $(h = 1^\circ\text{–}86^\circ)$
	$a(h)$ $(\lambda' = 0^\circ, 90^\circ)$	H.71 r; CY.1 v
4	$h(a, \lambda)$	D.9r $(a = 30^\circ)$
		D.9v $(a = 60^\circ)$
5 (a)	$r(\lambda)$	D.10v, 5r $(h_r = 20^\circ)$
		D.7v $(h_r = 19^\circ)$
		G.73 v $(h_r = 19^\circ)$
		HA.18 $(h_r = 19^\circ)$
		PH.14r $(h_r = 19^\circ)$
(b)	$s(\lambda)$	D.10r $(h_s = 16^\circ)$
		D.7r $(h_s = 17^\circ)$
		G.73r $(h_s = 17^\circ)$
		HA.14 $(h_s = 17^\circ)$
		PH.12r $(h_s = 17^\circ)$
(c)	$n(\lambda)$	D.6r $(h_r = 19^\circ, h_s = 17^\circ)$
		G.72v $(h_r = 19^\circ, h_s = 17^\circ)$
		HA.15 $(h_r = 19^\circ, h_s = 17^\circ)$
		PH.13r $(h_r = 19^\circ, h_s = 17^\circ)$
6 (a)	$h_q(\lambda)$	D.86v $(q = 52^\circ)$
		D.14r $(q = 53^\circ)$
		G.70v $(q = 53^\circ)$
(b)	$t_q(\lambda)$	D.8v $(q = 53^\circ)$
		G.71r $(q = 53^\circ)$
(c)	$h_{\bar{q}}(\lambda)$	G.71v $(q = 53^\circ)$
(d)	$t_{\bar{q}}(\lambda)$	G.77r $(q = 53^\circ)$
7 (a)	$h_a(\lambda)$	D.11v; G.69v
		HA.11; PH.10v
(b)	$t_a(\lambda)$	D.15v; G.69r; B.20v
		HA.12; PH.11r
(c)	$T_a(\lambda)$	D.13v; G.70r
		HA.13; PH.11v
8 (a)	$D_\sigma(\lambda)$	G.74r
		HA.16; PH.13v
(b)	$D_\tau(\lambda)$	G.76v
(c)	$\alpha_\sigma(\lambda)$	HA.25–26
		PH.17v–18r
(d)	$\alpha_s(\lambda)$	HA.23–24
		PH.16v–17r
	$\alpha_r(\lambda)$	C
9	$h_v(\lambda)$	D.84v
		CE
10	$\Delta D(\lambda)$	G.74v

Appendix B: Description of Manuscripts Consulted

Nota bene: Only MSS B, C, D, E, G, H, K, and CZ are described here. The numerous related sources will be discussed in a subsequent paper. Manuscripts of the *Ḥākimī Zīj* are described in *King* [1], Appendix C.

B: MS Berlin Ahlwardt 5753 (= Landberg 574) (Foliated 1r–21r)

This is a fragment of the *Very Useful Tables.* The title on fol. 1r reads: *Azimuth Tables by Ibn Yūnus, the Calculator.* There are several notices of possession on the title folio, one dated 840 Hijra (=1436).

The tables on fols. 1v–20r are in considerable confusion. Each table is headed only by the degree of solar altitude. For altitudes 1° through 19° (fols. 1v–10v) they are azimuth tables. These are followed by hour-angle tables for altitudes 8° and 9° (fols. 11r–11v) and tables of the time since sunrise for altitudes 52° through 83° (fols. 12v–20r). The format of all the tables is the same as that in MS D. On fol. 20v there is a table of $t_a(\lambda)$, which has been published in *Schoy* [1], 53, and on fol. 21r, a table of $1/\tilde{h}(\lambda)$. An additional folio at the end of the MS contains an illegible title and some instructions on a calculation in spherical astronomy.

C: MS Cairo Tīmūrīya, riyāḍīyāt 191 (Paginated 1–90)

This is a copy of part of the *Very Useful Tables.* The title reads *Hour-Angle Tables,* and most of the work consists of a complete set of these. There are also tables of

$$h_n,\ r,\ s,\ 2N,\ T_q,\ t_q, \text{ and } \alpha_{30}$$

as well as for the time of the *salām*, and the oblique ascensions of the ascendant at daybreak and nightfall. The MS contains tables of the functions

$$\mathrm{Cot}_{60}\, h_a(\lambda) \quad \text{and} \quad \mathrm{Cot}_{12}\, h_n(\lambda),$$

not found in the other sources, and also a table of equatorial coordinates of 60 stars, not due to *Ibn Yūnus.*

D: MS Chester Beatty 3673 (Foliated 121r–1v, as for a Latin MS)

This is a virtually complete copy of the *Very Useful Tables,* incorrectly cataloged as the *Ḥākimī Zīj.* The title on fol. 121r reads: *Ibn Yūnus' Azimuth Tables,* but this refers only to the first part of the tables. This folio also contains some notices of possession, a floriated *ṭughrā* signature, and the date of copying, 772 Hijra (=1371). The azimuth tables (fols. 121v–96r), instructions (fols. 83r–83v), and tables for timekeeping (fols. 82v–3v) are written in three different hands. All of the tables, however, are carefully and beautifully copied. This manuscript has been the major source for the present study, and all of the tables are analyzed in Part 3 and indexed in Appendix A.

The azimuth tables for altitudes 81°, 82°, and 83°, are garbled and some meaningless entries for altitudes 84° to 88° have been included. (*Cf.* the description of MS K below.) The tables of time since sunrise and hour-angle (fols. 82v–15r) are complete, and each pair of facing pages for lower altitudes has values of both functions tabulated for a given altitude.

Fol. 82r has been filled with odd items such as notices of possession, verses of poetry, and a list of Graeco-Coptic numerals, given with their Arabic *abjad* equivalents (*cf.*, for example, *Ritter* [1]). Some astrological writings (fols. 2r–1v) have been bound at the end of the MS.

E: MS Escorial ar. 924,7 (Foliated 30r–63v)

This is an incomplete copy of *Ibn Yūnus'* azimuth tables. The title on fol. 30r reads: *Azimuth Tables of Ibn Yūnus, and Cotangent Tables, Computed for Each Minute of Argument.* Below this is the curious note: *fi ʿilm al-rumūz li-fiqh al-majmaʿ*, "on the science of secret signs for the understanding of the community (?)." Fols. 30v–63v contain azimuth tables for solar altitudes 1° through 78°. The format is the same as that in MS D. The remainder of the azimuth tables, and the Cotangent tables, are missing. *Ibn Yūnus'* extensive Sine and solar declination tables, computed for each minute of argument, and with differences for the seconds, are discussed in *King* [1], 85–89 and 96–99.

G: MS Cairo Dār al-Kutub, mīqāt 108 (Foliated 1r–77v)

This is a fairly complete copy of the *Very Useful Tables.* The copyist was *ʿAbd al-Bārī' Naṣr al-ʿAshḥāwī* (?) *al-Rifāʿī al-Shūnī*, and the date of copying 1218 Hijra (=1803!).

The title folio (fol. 1r) gives the full title of the work, and the author as *Ibn Yūnus*, "the Egyptian observer for latitude 30°". The same folio contains a table of *notae* (*madākhil*) for the Muslim and Coptic months, in two sets of 12×7 entries. To use these tables one must know the *notae* of Muḥarram 1 or Tūt 1. These can be found from tables which occur later in the MS, giving, for a particular Coptic year, the *nota* of Tūt 1 (*ʿalāma*), the corresponding Hijra date, and the *nota* of Muḥarram 1 for that Muslim year. The range of the table is 1521–1680 Diocletian, or 1219–1383 Hijra, that is, 1804–1963. Calendrical tables of this sort are not found in the other MSS of *Ibn Yūnus'* works.

The instructions (fols. 1v–3v) are similar to those in MS D (see Part 4 above), but the angles of solar depression to be used for morning and evening twilight are stated as 20° and 16° (nevertheless the twilight tables in MS G are based on 19° and 17°), and the last paragraph of the introduction in MS D is missing. The instructions end with the statement "this is the end of the treatise of *al-mqsy* (*al-Maqdisī*)" (*cf.* footnote 32), and are followed by some additional notes (fols. 3v:8–4r:13) by *Aḥmad b. Muḥammad b. Aḥmad al-Azharī*, known as *al-Bakhāniqī*. He states that he organized this book so that the three functions T, t, and a were tabulated together. He further states that the table of T is due to *al-mqsy* (*al-Maqdisī*); that the table of t may be different from one which had been calculated previously (referring to discrepancies between the tables of *Ibn Yūnus* and *al-Maqdisī*?); and that the copyists' errors in the table of a computed by *Abū l-Husayn* (*sic*) *b. Yūnus* had been corrected by *Shams al-Dīn Abū ʿAbd Allāh Muḥammad b. Ibrāhīm b. ʿAbd Allāh al-Rashīdī* (apparently referring to the garbled tables for altitudes 81°, 82°, and 83°—see Section 3.2 above). He continues with the remark that *Ibn Yūnus* had tabulated ψ and h_0 together on one page, and also the functions h_v, h_q, $h_{\bar{q}}$, and $a(h)$ at the equinoxes and solstices. His next statement is that the tables of t_q, h_n, h_a, Cot h_a, t_a, T_a, $\tilde{h}$, r, and s, are due to *al-mqsy* (*al-Maqdisī*). The question of the relationship between the tables of *Ibn Yūnus* and of *al-Maqdisī* clearly requires further treatment. *al-Bakhāniqī* also refers to various tables which are not on the microfilm that I have examined, giving Cotangents, and the functions α, α', and α_φ for Cairo. Another table not contained on the microfilm but mentioned by *al-Bakhāniqī* in the introduction, is referred to as *al-dā'ir al-āfāqī*, "universal hour-angle table." He remarks that this is for computing the hour-angle (the MS has "solar longitude"!) according to a method of *Ibn Yūnus*, which was the shortest method he knew. (A table in MS CW.80v bears the title "table for finding the hour-angle with ease,' and is simply Sec ϑ tabulated to two significant digits for each degree of ϑ. Clearly, a table of $g(\lambda)$ provides a simpler means of computing the hour-angle from the solar altitude—see Section 3.2—but I have not yet located such a table in the sources.)

In MS G, the time since sunrise, hour-angle, and solar azimuth are tabulated in triplets for each integral degree of altitude. Thus, each double-opening has 6 columns of 30 triplets (for lower altitudes). This part of the MS was completed on Jumādā I 20, 1218 Hijra (=1803). Following the calendrical tables described above are *Ibn Yūnus'* tables for general time-keeping purposes (fols. 68r–77r). Only the tables of $h_{\bar{q}}(\lambda)$ and $t_{\bar{q}}(\lambda)$, and of $\Delta D(\lambda)$ are not contained in the other MSS consulted. This part of the MS was completed on Jumādā II 8, 1218 Hijra, some two weeks after the completion of the first part. A note in a different hand on the last folio (fol. 77v) describes the calculation of time since sunrise from solar altitude.

Added in proof:

H: MS Cairo Azhar, falak 4382 (Foliated 1-71)

This is a carefully copied MS of *Ibn Yūnus'* tables of $t(h, \lambda)$ (fols. 1v–35r) and $a(h, \lambda)$ (fols. 36v–70v), and also of $\psi(\lambda)$ and $h_0(\lambda)$ (fol. 71v). Of particular interest is a table, not contained in the other sources, of the solar azimuth for certain altitudes at the equinoxes and solstices (fol. 71r). That this table is original is attested by a remark in the *Ḥākimī Zīj* to the effect that if one has a table of $a(h)$ computed for the equinoxes, then the half-excess of daylight for a general solar longitude can be found using the relation:

$$f(\lambda) = a\{\delta(\lambda)\}.$$

Cf. King [1], 142–143, and also the description of MS G above.

For the equinoxes, *Ibn Yūnus* tabulates $a(h)$ for each degree of solar altitude from 1° to 60°. For the winter and summer solstices he tabulates $a(h)$ for each 0; 5° of solar altitude between 36; 5° and 36; 25°, and between 83; 5° and 83; 35° as well as for the value 83; 33°, respectively. Sample entries are shown in the following table:

Equinoxes	
Altitude	Azimuth
1°	0; 35°
2	1; 9
15	8; 54
30	19; 28
45	35; 16
59	73; 55
60	90; 0

Winter Solstice	
Altitude	Azimuth
36; 5°	82; 53°
36; 10	83; 43
36; 15	85; 2
36; 20	86; 24
36; 25	90; 0

Summer Solstice	
Altitude	Azimuth
83; 5°	67; 29°
83; 10	69; 23
83; 30	80; 47
83; 33	84; 48
83; 35	90; 0

K: MS Cairo Dār al-Kutub, mīqāt 137M

This undated manuscript contains *Ibn Yūnus*' complete azimuth tables. The altitude argument runs beyond 83° to 86° (cf. Section 3.3 above), and in MS D it runs to 88°. The two sets of extra entries are clearly related, and are extracted below. The only explanation I can offer for these nonsense values is that they were added in order to make the tables more appealing to someone from, say, Medina. Assuming the latitude of that city to be 25° (as used, for example, in the prayer-tables in MS Damascus Ẓāhirīya 27), the maximum solar altitude would be 88; 35°.

	84°		85°		86°		87°	88°	
	MS D	MS K	MS D	MS K	MS D	MS K	MS D		
♊ 17°	78; 0	78; 12							13°
18	78; 52	—							12
19	75; 52	78; 52	78; 52	78; 12					11
20	73; 44	73; 44	75; 44	78; 52	74; 44	74; 44			10
21	71; 27	71; 24	73; 25	75; 40	73; 25	73; 25	73; 25		9
22	70; 31	70; 31	71; 32	73; 25	71; 32	71; 30	71; 32	71; 32	8
23	68; 30	68; 30	70; 1	71; 32	70; 1	70; 1	70; 1	70; 1	7
24	67; 1	68; 1	68; 45	70; 1	68; 45	68; 45	68; 15	68; 15	6
25	67; 46	67; 46	67; 48	68; 45	67; 48	67; 48	67; 48	67; 48	5
26	66; 49	67; 49	66; 2	67; 48	66; 2	67; 2	66; 2	66; 2	4
27	66; 27	66; 27	66; 27	67; 2	66; 27	66; 27	66; 27	66; 27	3
28	66; 3	66; 3	66; 4	67; 27	66; 4	66; 4	66; 4	66; 4	2
29	65; 48	65; 18	65; 48	66; 4	65; 48	65; 48	65; 48	65; 48	1
30	65; 42	65; 42	65; 41	65; 48	65; 41	65; 41	65; 41	65; 41	0 ♋

(In MS K the following meridian altitudes are given for the extra columns:

84°: 84; 40, 30° at ♊ 17°
85°: 85; 49, 1° at ♊ 19°
86°: 86; 30, 0° at ♊ 20°.)

CX: MS Cairo Dār al-Kutub, mīqāt 72M

This MS consists of a complete set of tables of $h(D, T)$. The tables are beautifully copied in the hand of *Muḥammad b. Muḥammad b. ʿAbd al-Qawīy al-Qurashī*, known as *Ibn al-Kitānī al-Ālātī*, the instrument maker and calculator (*cf. Suter* [1], no. 411), and are dated Cairo, 747 H (=1346/7). The beginning of the introduction is missing, but the part which survives mentions twilight tables, and gives the above information about the copyist. Only the altitude tables are contained in the MS, and these are briefly discussed in Section 3.4.

Sigla of Manuscripts Consulted

On MSS B, C, D, E, G, H, and K see p. 347. Other MSS cited are the following:

L	MS Leiden Or. 143 (Chapters 1 to 22 of the *Ḥākimī Zīj*)
M	MS Milan Ambrosiana 281e (Treatises due to *Ibn Yūnus*)
O	MS Oxford Bod. Hunt. 331 (Chapters 21 to 44 of the *Ḥākimī Zīj*)
CE	MS Cairo Dār al-Kutub, *miqāt* 190 (*Natījat al-afkār* by *al-Lādhiqī*)
CH	MS Cairo Dār al-Kutub, *miqāt* 107
CI	MS Cairo Dār al-Kutub, *miqāt* 177 (Prayer-tables of *al-Manūfī*)
CL	MS Cairo Dār al-Kutub, *miqāt* 57
CM	MS Cairo Dār al-Kutub, *miqāt* 157
CN	MS Cairo Dār al-Kutub, *miqāt* 158 (Anonymous prayer-tables based on those of *Ibn Yūnus*)
CO	MS Cairo Dār al-Kutub, *miqāt* 53 (Tables for timekeeping attributed to *al-Maqdisī*)
CQ	MS Cairo Dār al-Kutub, *miqāt* 71 (Tables due to *al-Khalīlī* and *al-Ḥalabī*)
CW	MS Cairo Dār al-Kutub, *miqāt* 204M (contains anonymous spherical astronomical tables for Cairo)
CX	MS Cairo Dār al-Kutub, *miqāt* 72M (Altitude tables, perhaps by *Ibn Yūnus*)
CY	MS Cairo Dār al-Kutub, *miqāt* 58M ("Azimuth tables" attributed to *Ibn al-Majdī*)
DB	MS Damascus Ẓāhirīya 3116 (contains *al-Khalīlī*'s tables for time-keeping)
DC	MS Damascus Ẓāhirīya 8868M (Anonymous prayer-tables for latitude 36°)
EB	MS Escorial ar. 970,10 (*Tuḥfat al-aḥbāb* by *Ibn al-Majdī*)

HA	MS Hartford Theological Seminary 621 (*Natījat al-afkār* by *al Lādhiqī*)
LA	MS British Museum 768 (Or. 3624) (The *Mukhtār Zīj* of *Abū l-ʿUqūl*)
MB	MS Milan Ambrosiana C 84 (Anonymous Yemeni tables for time-keeping)
PB	MS Paris Bibliothèque Nationale ar. 2513 (Anonymous *Zīj* based on *Ibn Yūnus*)
PF	MS Paris Bibliothèque Nationale ar. 2558 (Prayer-tables of *al-Khalīlī*)
PH	MS Paris Bibliothèque Nationale ar. 2553 (*Natījat al-afkār* by *al-Lādhiqī*)
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Smithsonian Institution
Project in Medieval Islamic Astronomy
American Research Center in Egypt
2, Midan Kasr el-Doubara
Garden City, Cairo

X

Astronomical Timekeeping in Fourteenth-Century Syria

Introduction

An important aspect of medieval Islamic astronomy was *ʿilm al-miqāt*, astronomical timekeeping and the determination of the astronomically-defined times of prayer.[2] The astronomers who specialized in this branch of astronomy were known as *muwaqqits*, and the main astronomers who bore this name were associated with the Umayyad Mosque in Damascus in the fourteenth century and with the Azhar Mosque in Cairo in the fifteenth century.

Until a few years ago it was generally thought amongst historians of Islamic science that the *muwaqqits* of medieval Islam had not contributed very much to the development of astronomy. One reason for this opinion is that *ʿilm al-miqāt* is a fairly simple discipline; thus, for example, all of the standard problems can be solved with an astrolabe. Another reason was that the scientific content of the known works of these *muwaqqits*, that is, treatises on the determination of the prayer times, treatises on the use of astrolabes, almucantar and sinical quadrants, and sundials, is indeed rather meager. Another reason is that the most significant works on astronomical timekeeping by Muslim astronomers had never been studied in modern times before 1970.

My paper is devoted to a brief description of some of these works from

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(2) For a brief survey of the origins of *ʿilm al-mīqāt* in the *ḥadīth* and *sunna* of the Prophet Muḥammad see the article "*Mīkāt*" by A. J. Wensinck in the *Encyclopoedia of Islam*, lst. ed. (A medieval poem on the prayer-times is presented in the Arabic version of this article.) For a survey of the definitions of the prayer-times in medieval astronomical sources see E. Wiedemann and J. Frank, "Die Gebetszeiten im Islam", *Sitzungsberichte der phys.- med. Sozietät zu Erlangen*, 58 (1926), pp. 1-32, reprinted in E. Wiedemann, *Aufsätze zur arabischen Wissenschaftsgeschichte*, (Hildesheim, 1970), vol. II, pp. 757-788. For a survey of medieval tables for timekeeping see my forthcoming *Studies in Astronomical Timekeeping in Medieval Islam*: this includes a detailed analysis of all of the Syrian material mentioned in this article.

fourteenth-century Damascus. The works that I shall discuss are tables for astronomical timekeeping. Nowadays the times of prayer are regulated by tables: these tables have a history of over a thousand years, and I find it rather strange that no medieval examples had ever been studied until a few years ago. The medieval tables for timekeeping represent a tradition of sophisticated mathematical astronomy that started in Abbasid Baghdad but more especially in Fatimid Cairo, reached its zenith at the Umayyad Mosque in Damascus during the time of Ibn al-Shāṭir, and continued all over the Ottoman Empire thereafter.

The basic problems of Islamic astronomical timekeeping

In order to appreciate the tables described below it is necessary to have some idea about spherical astronomy. Two of the main problems of spherical astronomy are the determination of time from solar or stellar altitude, and the determination of solar or stellar azimuth from the solar or stellar altitude. The hour-angle t and closely related time since sunrise T, as well as the azimuth a, are functions of the instantaneous solar altitude h, the solar declination δ, and the local latitude φ. Mathematically correct solutions to the problems of the determination of $t\ (h, \delta, \varphi)$ and $a\ (h, \delta, \varphi)$ were available to the earliest Muslim astronomers from Indian sources, and all Islamic *zījes* contain the standard trigonometric solutions.[3] Most Islamic *zījes*, however, do not contain tables for timekeeping, which is one reason why these tables have remained unknown until recently. (Another reason is that most Arabic manuscripts containing astronomical tables are not properly cataloged.)

The most obvious practical application of astronomical timekeeping in medieval Islam was in the determination of the times of Muslim prayer. These are defined in terms of the apparent daily motion of the sun across the sky. The Muslim day begins at sunset with the *ṣalāt al-maghrib*, the *ṣalāt al-ʿishā'* begins at the disappearance of evening twilight, the *ṣalāt al-fajr* begins at the appearance of morning twilight, the *ṣalāt al-ẓuhr* begins when the sun starts to decline from the meridian, and the *ṣalāt al-ʿaṣr* begins when the shadow of any object has increased over its midday shadow by the length of the object. Two other topics treated in some medieval treatises on timekeeping are the determination of the *qibla* or direction of Mecca for a given locality and the determination of the visibility of the lunar crescent. Clearly the mathematical determination of the prayer times requires physical theories of twilight and the effect of refraction at the horizon, as well as the determination of time from solar altitude. Likewise, the determination of the *qibla* requires the application of sophisticated trigonometric formulae, and the computation of solar and lunar positions relative to each other and to the local horizon on an evening when visibility is anticipated also involves non-trivial computational techniques.

Ibn Yūnus and the Cairo corpus of tables for timekeeping

I do not propose to discuss the scanty evidence that we have of the determination of the prayer-times in early Islam or even in Abbasid Baghdad. Suffice it to say that we do have some very simple Abbasid tables displaying the length of the permitted intervals for the prayer times for each day of the year, computed for the latitude of Baghdad. We also have a highly sophisticated table for reckoning the time of day or night from solar or stellar altitudes from tenth-century Baghdad. But the use of such tables was not so widespread in the first few centuries of Islam that we find any reference whatsoever to them in the *Ifrād al-maqāl fī amr al-ẓilāl* of al-Bīrūnī, which is our most important source on early Islamic timekeeping.[4]

The celebrated late-tenth-century Egyptian astronomer Ibn Yūnus[5] can be said, according to our present knowledge, to have started the tradition of astronomical timekeeping which characterized the activities of the later *muwaqqits* of Cairo and Damascus. It seems that he was the first to compile sets of tables of functions useful in timekeeping of the kind that were used in later Cairo and Damascus. In his great work *al-Zīj al-Ḥākimī* dedicated to the Fatimid Caliph al-Ḥākim, he presented a set of tables displaying the altitude of the sun in various azimuths, notably the azimuth of the *qibla*, as well as the azimuth of the sun for various altitudes. These tables, computed very accurately for the latitude of Cairo, gave values in degrees and minutes for each degree of solar longitude. Each of these tables contains 180 entries. In another work Ibn Yūnus compiled a set of tables called *Kitāb al-samt* in which he tabulated the solar azimuth in degrees and minutes for each degree of solar altitude and each degree of solar longitude. The total number of entries in this table is about 10,000 and most of them are correct to the nearest minute. There is no doubt that these enormous tables inspired the later astronomers of Cairo and Damascus. Indeed the late-thirteenth-century Egyptian astronomer al-Maqsī compiled a similar table which he called *Kitāb al-dā'ir*, displaying the time since sunrise as a function of solar altitude and solar longitude.

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(3) On Islamic *zījes* see E. S. Kennedy, "A Survey of Islamic Astronomical Tables", *Transactions of the American Philosophical Society*, 42 (1956).

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(4) Published in Hyderabad, 1948 from a unique manuscript preserved in Bankipore. Translation and commentary by E. S. Kennedy published by the Institute for the History of Arabic Science in Aleppo, 1976.

(5) On Ibn Yūnus see the article in the *Dictionary of Scientific Biography* and on the Cairo corpus see D. A. King, "Ibn Yūnus' *Very Useful Tables* for Reckoning Time by the Sun", *Archive for the History of Exact Sciences*, 10 (1973), pp. 342-394.

In the mid-fourteenth century the Egyptian astronomer Ibn al-Kattānī prepared a set of hour-angle tables using al-Maqsī's tables. By the late fourteenth century there had been compiled a corpus of timekeeping tables for Cairo incorporating Ibn Yūnus' *Kitāb al-samt*, al-Maqsī's *Kitāb al-dā'ir*, and Ibn al-Kattānī's *Kitāb faḍl al-dā'ir*, as well as some thirty tables of simpler spherical astronomical functions and functions relating to the times of prayer. Altogether the tables contain over 35,000 entries, all computed to two sexagesimal figures. This corpus of tables was used in Cairo until the nineteenth century.

Two of the leading astronomers of fourteenth-century Damascus studied astronomy in Egypt, namely, al-Mizzī and Ibn al-Shāṭir. Both compiled tables for timekeeping, and no doubt both were inspired by the tables they saw in use in Egypt.

The tables for timekeeping of al-Mizzī, Ibn al-Shāṭir, and al-Karakī

Shams al-Dīn Abū ᶜAbd Allāh Muḥammad ibn Aḥmad ibn ᶜAbd al-Raḥīm al-Mizzī[6] was born in 690/1291, studied in Cairo, and later worked as a *muwaqqit* at the Umayyad Mosque in Damascus, where he died in 750/1349. He was a fine instrument maker and four quadrants made by him are preserved in various museums. He also wrote treatises on the use of the astrolabe, sine quadrant, and almucantar quadrant.

Al-Mizzī's tables for timekeeping are known to us from two sources. Firstly, his prayer-tables are preserved in the unique MS Cairo Dār al-Kutub *miqāt* 62M, copied about 900/1500. Secondly his hour-angle tables are described by the Jerusalem *muwaqqit* al-Karakī, who was a student of al-Mizzī. Al-Karakī mentions al-Mizzī's tables in his introduction to a corpus of timekeeping tables for Jerusalem, preserved in the unique MS Leipzig Universitätsbibliothek 808, copied in Nablus in 805/1402-03. However, al-Mizzī's hour-angle tables are not extant in the known manuscript sources.

Al-Mizzī's prayer-tables have the same format as those in the Cairo corpus (see Plate 2) and are computed for the parameters

$$\varphi = 33;27^\circ \quad \text{and} \quad \epsilon = 23;33^\circ,$$

which were also used by Ibn al-Shāṭir in some of his early works. These tables display the following functions, with values for each degree of solar longitude computed to two sexagesimal digits: the solar declination and right ascensions (both of which are independent of φ), the solar meridian altitude, the

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(6) On al-Mizzī see L. A. Mayer, *Islamic Astrolabists and their Works*, (Geneva, 1956), pp. 61-62, and the references there cited.

equinoctial degrees in each seasonal hour of daylight, the equation of half daylight, the semidiurnal arc, the number of equinoctial hours, the meridian shadow length (base 12), the solar altitude at the beginning of the ᶜaṣr, the nocturnal arc, the oblique ascensions, the solar rising amplitude, the solar altitude in the prime vertical, the intervals between midday and the beginning of the ᶜaṣr and sunset, the duration of evening twilight, and the duration of morning twilight. All of these functions were tabulated in the Cairo corpus. From al-Karakī's description it is clear that al-Mizzī's hour-angle tables displayed the hour-angle as a function of solar altitude and longitude for the same values of φ and ϵ.

Ibn al-Shāṭir's[7] tables for timekeeping are described in a short introduction that he prepared to them, which survives in a unique copy, MS Leiden Universiteitsbibliotheek Or. 1001, fols. 108r-113r, prepared in 1010/1601. Ibn al-Shāṭir mentions that his tables were computed for latitude 34° (locality unspecified), and that the values of the various functions were displayed side by side for each degree of solar longitude. The functions specifically mentioned (fol. 110v) are: the solar meridian altitude, the meridian shadow length (base 7), the semidiurnal arc, the solar altitude at the beginning of the ᶜ*aṣr* and the intervals between midday and the ᶜ*aṣr* and sunset, the nocturnal arc, the durations of morning and evening twilight, and the hour-angle when the sun is in the azimuth of the qibla. Now MS Cairo Dār al-Kutub *miqāt* 1170, fols. 11r-22v, copied about 1050/1640, contains an anonymous set of tables of precisely these functions, and two others which are trivially related, tabulated side by side (see Plate 3), and computed for the parameters

$$\varphi = 34;0^\circ \quad \text{and} \quad \epsilon = 23;31^\circ.$$

This value of the obliquity is the one derived by Ibn al-Shāṭir in 765/1363-64, and although the tables in this Cairo manuscript are accompanied by a star catalog for the year 1033/1623 attributed to a later astronomer, it seems that the prayer-tables are indeed those of Ibn al-Shāṭir.

Similar sets of prayer-tables for parameters

$$\varphi = 41;15^\circ \text{ (Istanbul)} \quad \text{and} \quad \epsilon = 23;31^\circ$$

and

$$\varphi = 36;0^\circ \quad \text{(Aleppo)} \quad \text{and} \quad \epsilon = 23;31^\circ$$

together with al-Khalīlī's prayer-tables for Damascus, are contained in MS Cairo Dār al-Kutub Ṭalᶜat *miqāt* 255, fols. 61v-80v, copied about 1100/1700.

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(7) On Ibn al-Shāṭir see E. S. Kennedy and I. Ghanem, eds., *The Life and Work of Ibn al-Shāṭir: an Arab Astronomer of the Fourteenth Century* (Aleppo, 1976), and the article in the *Dictionary of Scientific Biography*.

Another set for parameters

$$\varphi = 21;30^\circ \quad \text{(Mecca)} \quad \text{and } \epsilon = 23;31^\circ,$$

again together with al-Khalīlī's prayer-tables for Damascus, is contained in MS Cairo Dār al-Kutub *mīqāt* 184, fols. lv-61r, copied about 1250/1835. There is no indication of the identity of the compiler of these tables for Istanbul, Aleppo, or Mecca, but the value used for ϵ is characteristic of the Syrian tradition of Ibn al-Shāṭir and al-Khalīlī.

The only other Syrian sets of tables for timekeeping known to have been compiled in the fourteenth century are those of al-Karakī and al-Khalīlī. Zayn al-Dīn Abū Bakr ibn Muḥammad ibn Ayyūb al-Tamīmī al-Karakī was a *muwaqqit* in Jerusalem, and as noted above, he was a student of al-Mizzī. His timekeeping tables are computed for latitude 32°, that is, Jerusalem, and obliquity 23;35°. They display both the time since sunrise and the hour-angle side by side for each degree of altitude and for each degree of solar longitude. Thus the format differs from the tables of the Cairo corpus and al-Mizzī's tables. (See the extract in Plate 4.) It seems that al-Karakī simply altered the format of the earlier hour-angle tables for latitude 32° compiled by the Egyptian astronomer Ibn al-Rashīdī and inserted the corresponding value of the time since sunrise for each entry. An anonymous set of prayer-tables for latitude 32° is contained in MS Princeton Yahuda 861; these were probably computed by Ibn al-Rashīdī. There is no evidence that these tables for Jerusalem were widely used there. However, the Arabic scientific manuscripts preserved in Jerusalem have never been studied, and no doubt additional copies of the Jerusalem corpus may be discovered there when these manuscripts are again available for scholarly investigation.

al-Khalīlī and the Damascus corpus of tables for timekeeping

Shams al-Dīn Abū ᶜAbd Allāh Muḥammad ibn Muḥammad al-Khalīlī[8] was an astronomer associated first with the Yalbugha Mosque in Damascus and later with the Umayyad Mosque. We have no biographical information on him, save that he was a contemporary of Ibn al-Shāṭir. As far as we know, al-Khalīlī did not write on planetary astronomy. All of his known works deal with *ᶜilm al-mīqāt*. Indeed, his major work, a set of tables for astronomical timekeeping, represents the culmination of the medieval Islamic achievement in the mathematical solution of the problems of spherical astronomy. Some of these tables were used in Damascus until the nineteenth century, and those which were not computed specifically for the latitude of Damascus were also used in Cairo and Istanbul for several centuries. The main sets of tables (nos. 2-5 below) survive in about two dozen manuscripts, of which the finest is MS Paris B. N. ar. 2558, copied in Damascus in 811/1408. (See the extracts in Plates 5-9.)

Al-Khalīlī's tables can be subdivided as follows:

(1) tables of auxiliary mathematical functions for timekeeping by the sun for all latitudes;

(2) tables for reckoning time by the sun, for the latitude of Damascus;

(3) tables for regulating the times of Muslim prayer, for the latitude of Damascus;

(4) tables of auxiliary mathematical functions for solving the problems of spherical astronomy for all latitudes;

(5) a table displaying the *qibla*, that is, the direction of Mecca, as a function of terrestrial latitude and longitude; and

(6) tables for converting lunar ecliptic coordinates to equatorial coordinates.

The first set of tables compiled by al-Khalīlī consisted of auxiliary tables for timekeeping by the sun and also a table of the solar azimuth as a function of the solar meridian altitude and instantaneous altitude. The auxiliary tables, which contain over 9,000 entries, are intended specifically for facilitating the computation of the hour-angle for given solar altitude and solar longitude, and any terrestrial latitude. These tables in their original form survive in the unique MS Dublin Chester Beatty 4091, copied in 833/1429. They were not widely used, but they were plagiarized by later astronomers in both Cairo and Tunis.

The next two sets of tables correspond to those in the corpus of spherical astronomical tables computed for Cairo. They are computed for Ibn al-Shāṭir's parameters, namely

$$\varphi = 33;30^\circ \quad \text{and} \quad \epsilon = 23;31^\circ,$$

Although al-Khalīlī does not mention any of his Egyptian or Syrian predecessors, his tables were clearly intended to replace those of his elder colleague al-Mizzī. Al-Khalīlī's hour-angle tables and prayer-tables were used in Damascus until the nineteenth century. The Damascus *muwaqqit* Muḥammad ibn Muṣṭafa al-Ṭanṭāwī, who died in 1889, was one of the last to use them, and he also converted the entries in the hour-angle tables from equatorial degrees and minutes to equinoctial hours and minutes.

The fourth set of tables compiled by al-Khalīlī was designed to solve all the standard problems of spherical astronomy, and they are particularly useful

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(8) On al-Khalīlī see the article in the *Dictionary of Scientific Biography*. His auxiliary tables and *qibla* table are discussed in detail in D. A. King, "al-Khalīlī's Auxiliary Tables for Solving Problems of Spherical Astronomy," *Journal for the History of Astronomy*, 4 (1973), pp. 99-110, and "al-Khalīlī's *Qibla* Table", *Journal of Near Eastern Studies*, 34 (1975), pp. 81-122.

for those problems which in modern terms involve the use of the cosine rule for spherical triangles. It should be stressed that the underlying formulae were known to Muslim astronomers already in the ninth century. However, al-Khalīlī ingeniously tabulated three functions, and gave detailed instructions for their application. The functions are the following (the capital notation indicates that the medieval trigonometric functions are computed to base $R = 60$, thus $\mathrm{Sin}\,\vartheta = R \sin \vartheta$, etc.):

$$f_{\varphi}(\vartheta) = \frac{R\,\mathrm{Sin}\,\vartheta}{\mathrm{Cos}\,\varphi}\,, g_{\varphi}(\vartheta) = \frac{\mathrm{Sin}\,\vartheta\ \mathrm{Tan}\,\varphi}{R}\,,$$

and

$$G\,(x, y) = \mathrm{arc\ Cos}\left\{\frac{R\,x}{\mathrm{Cos}\,y}\right\},$$

computed for the domains

$$\vartheta = 1^{\circ}, 2^{\circ}, \ldots, 90^{\circ}$$

$$\varphi = 1^{\circ}, 2^{\circ}, \ldots, 55^{\circ}, \text{ as well as } 21;30^{\circ} \text{ (Mecca) and } 33;30^{\circ} \text{ (Damascus)}$$

$$x = 1, 2, \ldots, 59$$

$$y = 0^{\circ}, 1^{\circ}, \ldots, n\,(x)$$

where $n\,(x)$ is the largest integer such that

$$R\,x \leq \mathrm{Cos}\ n\,(x).$$

The entries in these tables, which number over 13,000, were computed to two sexagesimal digits and are invariably accurate. As an example of the use of these functions, we note the rule outlined by al-Khalīlī for finding the hour angle t for given solar or stellar altitude h, declination δ, and terrestrial latitude φ. This may be represented as follows:

$$t\,(h, \delta, \varphi) = G\,\left\{[f_{\varphi}(h) - g_{\varphi}(\delta)], \delta\right\},$$

and it is not difficult to show the equivalence of al-Khalīlī's rule to the modern formula

$$t = \arccos\left\{\frac{\sin h - \sin \delta \sin \varphi}{\cos \delta \cos \varphi}\right\}.$$

Likewise the corresponding azimuth a (here measured from the meridian) can be found using

$$a\,(h, \delta, \varphi) = G\left\{[\,g_{\varphi}(h) - f_{\varphi}(\delta)\,], h\right\}$$

which is equivalent to the modern formula

$$a = \arccos\left\{\frac{\sin h \sin \varphi - \sin \delta}{\cos h\ \cos \varphi}\right\}.$$

As an additional example of al-Khalīlī's ingenuity we may cite his rule for converting ecliptic coordinates (λ, β) to equatorial coordinates (α, Δ). This is equivalent for $0 < \lambda < 90^{\circ}$ to solving for x:

$$G\left\{x, \beta\right\} = 90^{\circ} - \lambda$$

and then solving for Δ:

$$f_{37}(\Delta) = \tfrac{1}{2}\left\{2\,g_{49}(\beta) + x\right\}$$

The corresponding normed right ascension $\alpha' (= \alpha + 90^{\circ})$ is then found using

$$\alpha' = \mathrm{G}\left\{[\,2\mathrm{f}_{37}(\beta) - 2g_{49}(\Delta)\,], \Delta\right\}$$

These rules are approximately equivalent to the modern formulae

$$\sin \Delta = \sin \beta \cos \epsilon + \cos \beta \sin \lambda \sin \epsilon$$

$$\sin \alpha \cos \Delta \sin \epsilon = \sin \Delta\ \cos \epsilon - \sin \beta$$

with $\epsilon \approx 23\tfrac{1}{2}^{\circ}$.

These auxiliary tables of al-Khalīlī were used for several centuries in Damascus, Cairo, and Istanbul, the three main centers of astronomical timekeeping in the Muslim world.

Al-Khalīlī's computational ability is best revealed by his *qibla* table. The determination of the *qibla* for a given locality is one of the most complicated problems of medieval Islamic trigonometry. If (L, φ) and (L_M, φ_M) represent the longitude and latitude of a given locality and of Mecca, respectively, and $\Delta L = |\ L - L_M\ |$, then the modern formula for $q(\varphi, L)$, the direction of Mecca for the locality, measured from the south, is:

$$q = \mathrm{arc\ cot}\left\{\frac{\sin \varphi \cos\ \Delta L - \cos \varphi \tan \varphi_M}{\sin\ \Delta L}\right\}.$$

Al-Khalīlī tabulated $q\,(\varphi, L)$ to two sexagesimal digits for the domains

$$\varphi = 10^{\circ}, 11^{\circ}, \ldots, 56^{\circ}, \text{ and also } 33;30^{\circ}$$

$$\text{and } \Delta L = 1^{\circ}, 2^{\circ}, \ldots, 60^{\circ},$$

and the vast majority of the 2,880 cntries are either accurately computed or in error by ± 0;1° or ± 0;2°. He states that he used the method for finding the *qibla* expounded by the late-thirteenth-century Cairo astronomer Abū ᶜAlī al-Marrākushī, and it seems that he used his universal auxiliary tables to compute the *qibla* values, although they are generally more accurate than can be derived from the auxiliary tables in their present form. Several other qibla tables based on approximate formulae are known from the medieval period. Al-Khalīlī's table does not appear to have been widely used by later Muslim astronomers, and indeed the geographical gazeteers found in most Islamic *zījes*, displaying the latitude, longitude, and *qibla* of the cities in the Islamic world are adequate for practical purposes.

The last set of tables known to have been compiled by al-Khalīlī is for converting lunar ecliptic coordinates to equatorial coordinates to facilitate computations relating to the visibility of the lunar crescent.

Conclusion

Al-Khalīlī's prayer-tables and hour-angle tables for Damascus were used there until the nineteenth century. As noted above they survive in about two dozen manuscript sources. His minor auxiliary tables were not widely used but were plagiarized by later Egyptian and Tunisian astronomers. His major universal auxiliary tables were copied in Damascus, Cairo, and Istanbul for several centuries. His *qibla* table, of which only three manuscript copies are known, does not appear to have been used much; I have the feeling that most later Muslim astronomers were unaware of its existence.

Various other later Syrian astronomers computed modest sets of prayer-tables for certain specific latitudes. Examples survive for Nablus, Tripoli, Lattakia, and Aleppo. The Syrian tradition in timekeeping also influenced astronomers in Tunis and Istanbul.

An interest in traditional Islamic astronomy continued in Syria until the turn of the present century.[9] The *Zījes* of Ibn al-Shāṭir and Ulugh Beg were modified and simplified. The European "*Zījes*" of Cassini and Lalande, available in Turkish and Arabic translation, did not succeed in replacing the modified versions of the *Zīj* of Ibn al-Shāṭir and the Syrian versions of the *Zīj* of Ulugh Beg. New treatises were written on the astrolabe, sine quadrant, and almucantar quadrant, as well as on the determination of the prayer times, but the only novel feature of these could be the number of chapters. The work of Ibn al-Shāṭir in planetary astronomy and of al-Khalīlī in astronomical timekeeping had marked the zenith of astronomy in medieval Syria.

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[9] Further information on this astronomical tradition is contained in a survey of mathematical astronomy in Egypt and Syria currently in preparation.

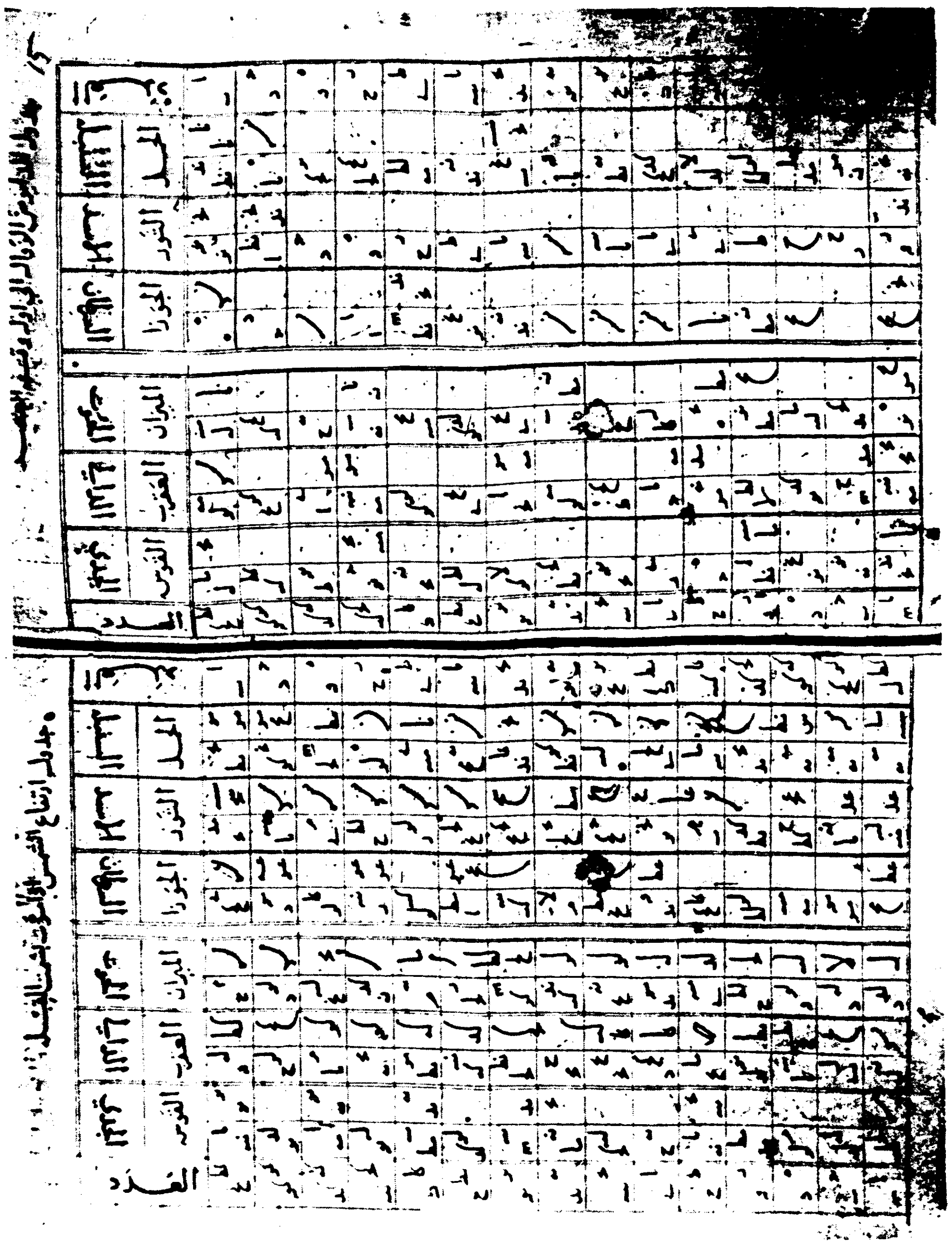

Plate 1. Two pages from the set of timekeeping tables for Cairo, from the Chester Beatty (Dublin) MS 3673. These two tables give the times between noon and the ᶜaṣr, and the solar altitude, at the instant that the sun passes through the azimuth of the *qibla*. One enters the table with the solar longitude, from the top of the page for longitudes from the first of Aries to the end of Gemini, and from the first of Libra to the end of Sagittarius; and from the bottom of the page for the remaining zodiacal signs. The set for Cairo contains about two hundred pages of tables.

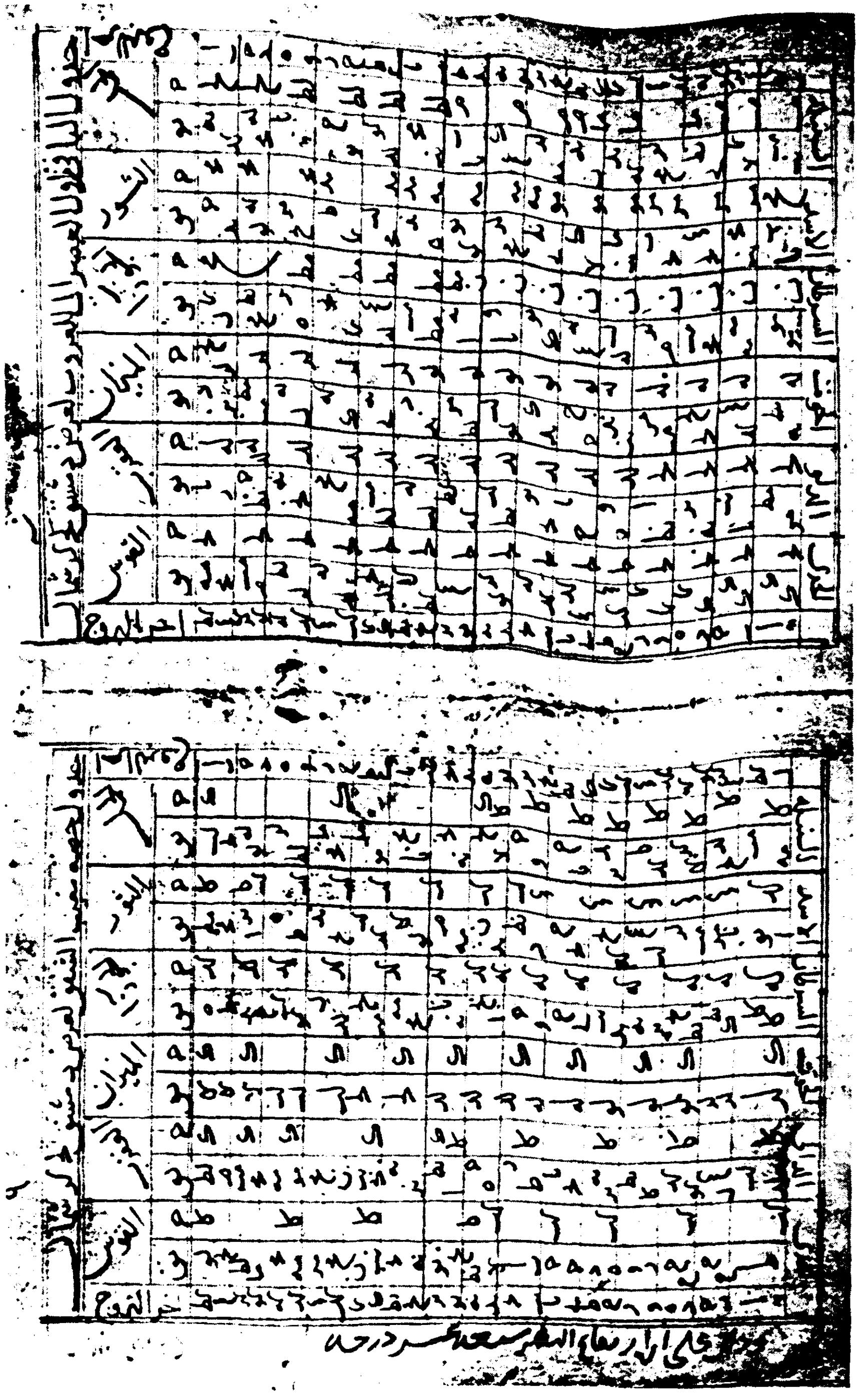

Plate 2. Pages from al-Mizzi's timekeeping tables, from Dār al-Kutub (Cairo) MS *mīqāt* 62, giving the times remaining from the beginning of the ᶜ*aṣr* to sunset, and the argument at the end of twilight. The set contains ten folios. Note that the arrangement of the table is similar to that of the Cairo set

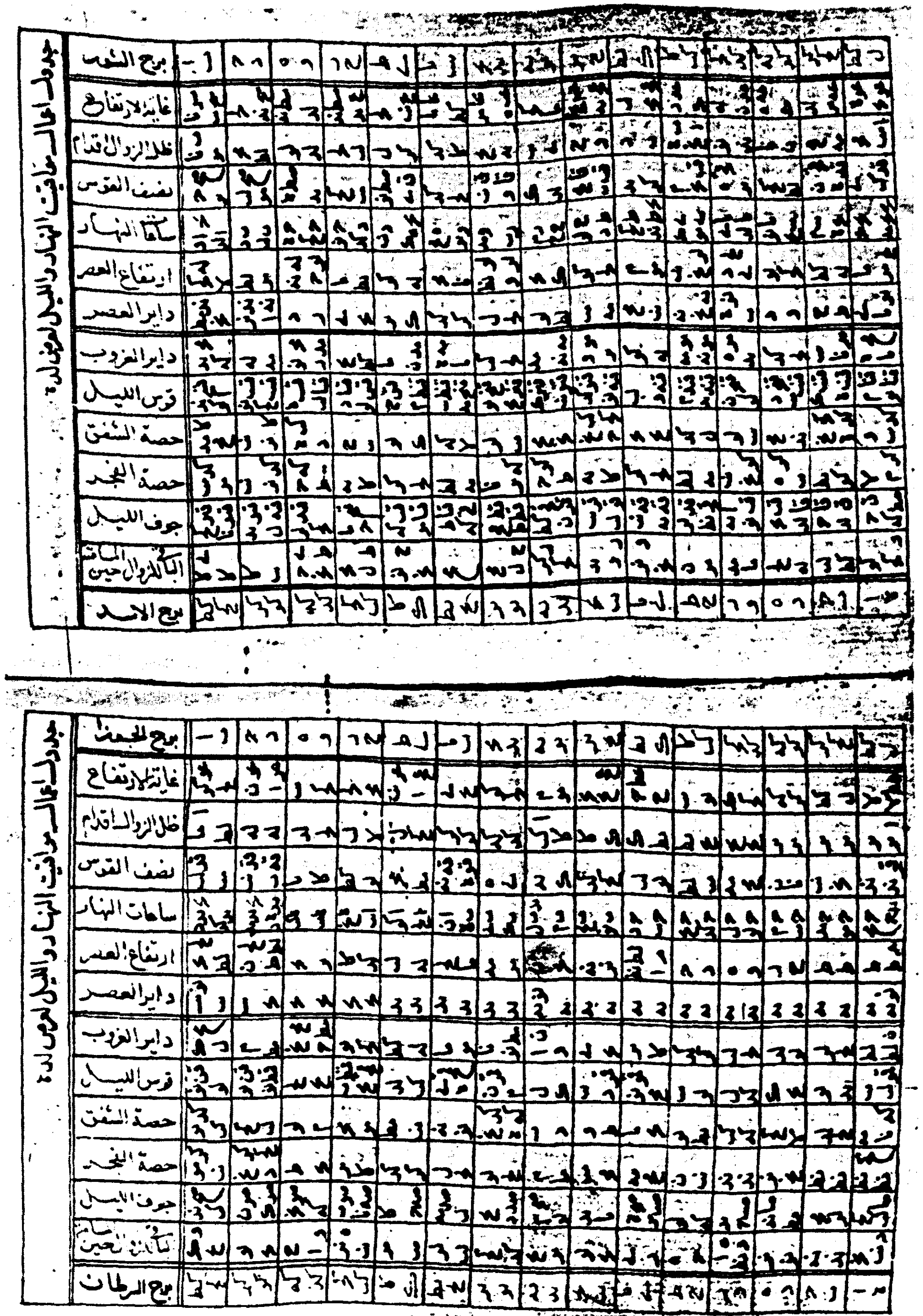

Plate 3. Pages from Ibn al-Shāṭir's timekeeping tables, calculated for latitude 34°, taken from Dār al-Kutub (Cairo) MS *mīqāt* 1170. Given are twelve functions side by side for the signs of Taurus

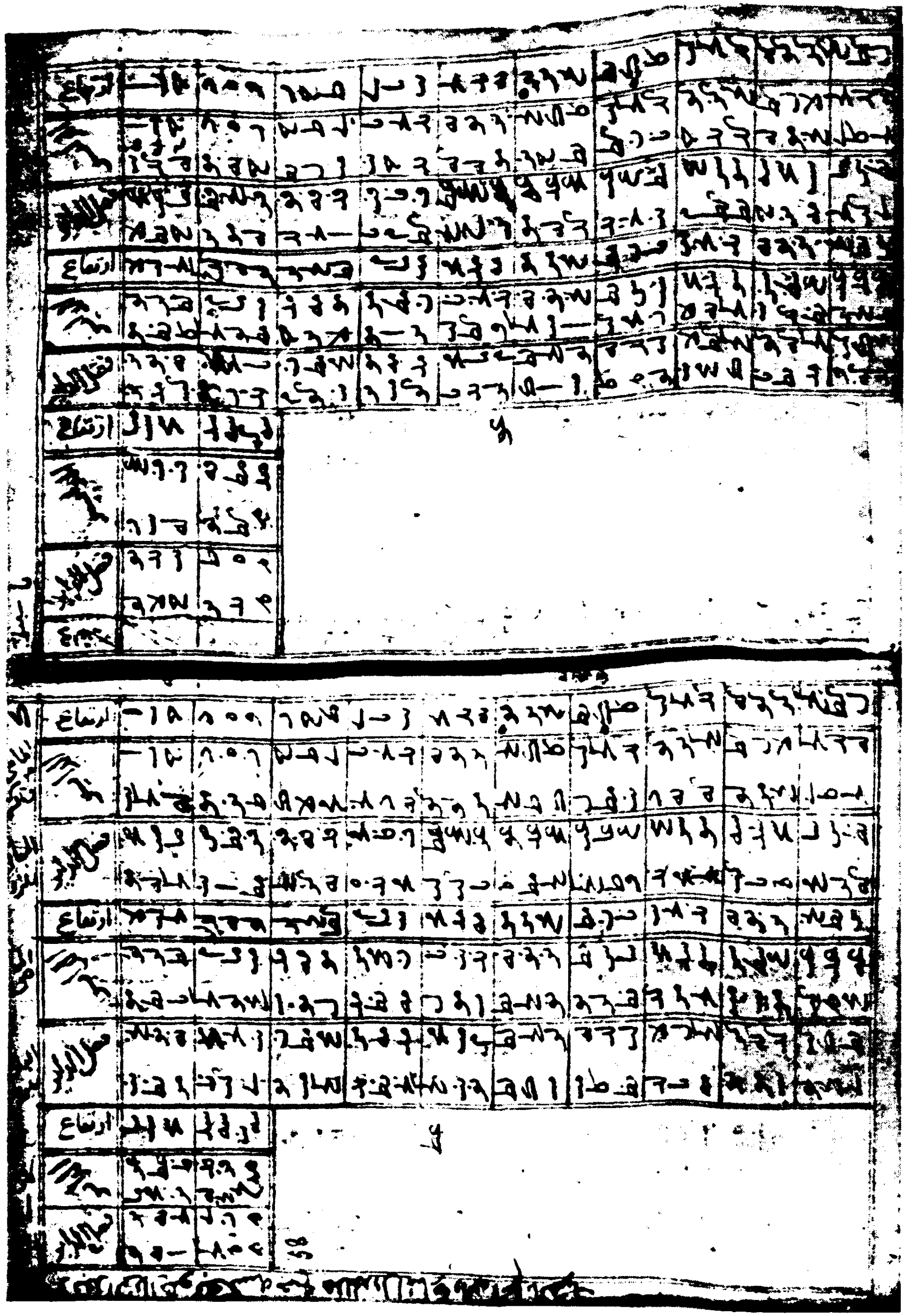

Plate 4. Pages from the tables of al-Karakī computed for the latitude of Jerusalem, from the Leipzig Universitätsbibliothek MS 808. They give the *dā'ir* and the *faḍl al-dā'ir*, i. e., the time elapsed from sunrise, and the time remaining until culmination for each degree of solar altitude up to its maximum, when the sun is in Aries 19° / Virgo 11° (on the right) and Aries 20 / Virgo 10° (on the left). The set consists of 180 pages of tables.

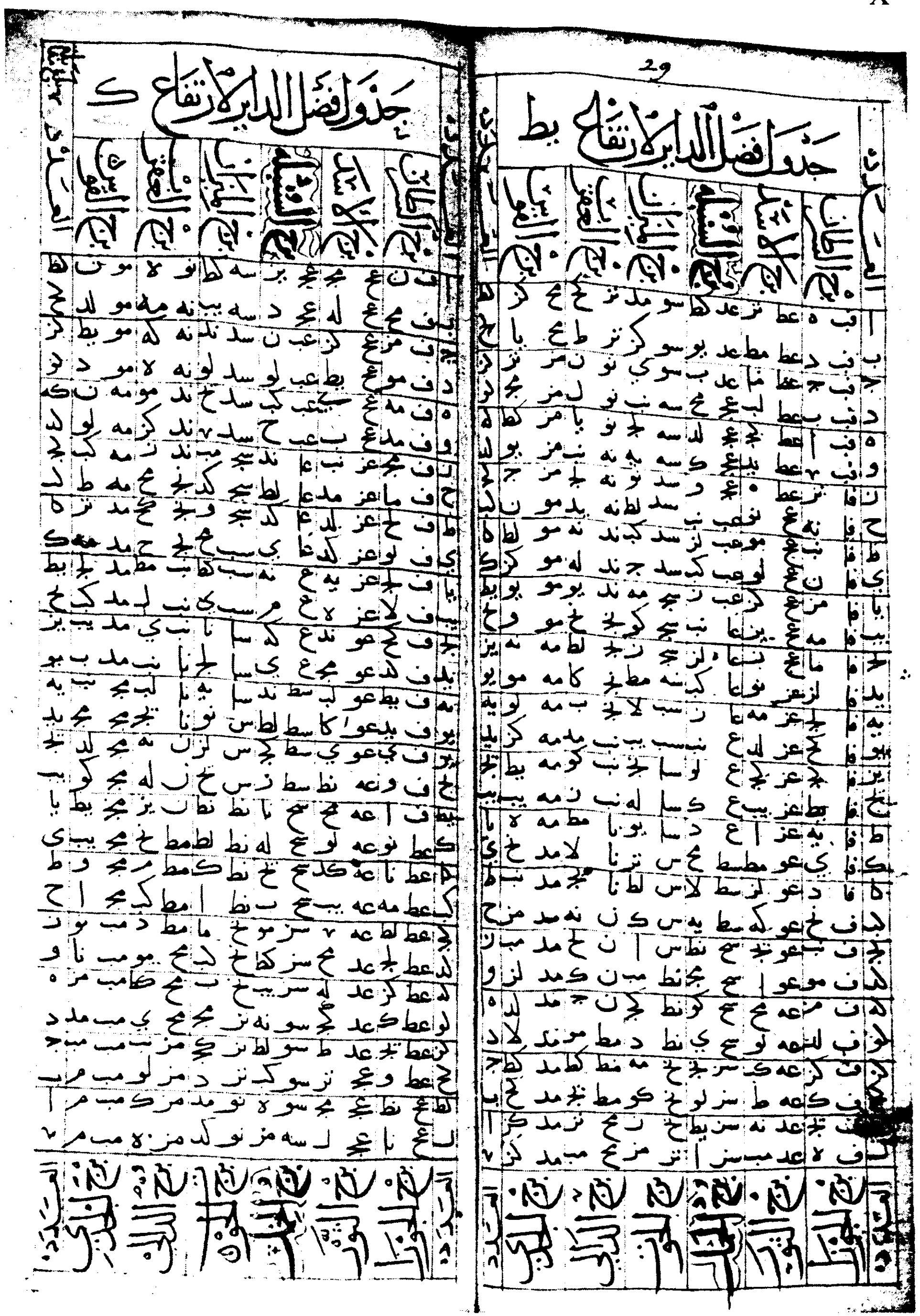

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Plate 5. A selection from the tables of al-Khalīlī for the solar hour-angle, from BN (Paris) MS ar. 2558. Values are given for solar altitudes of 19° and 20° for each degree of solar longitude. The three columns which follow the column of the argument are used for the northern signs, the other three for the southern signs. There are thirty *double pages like the above.*

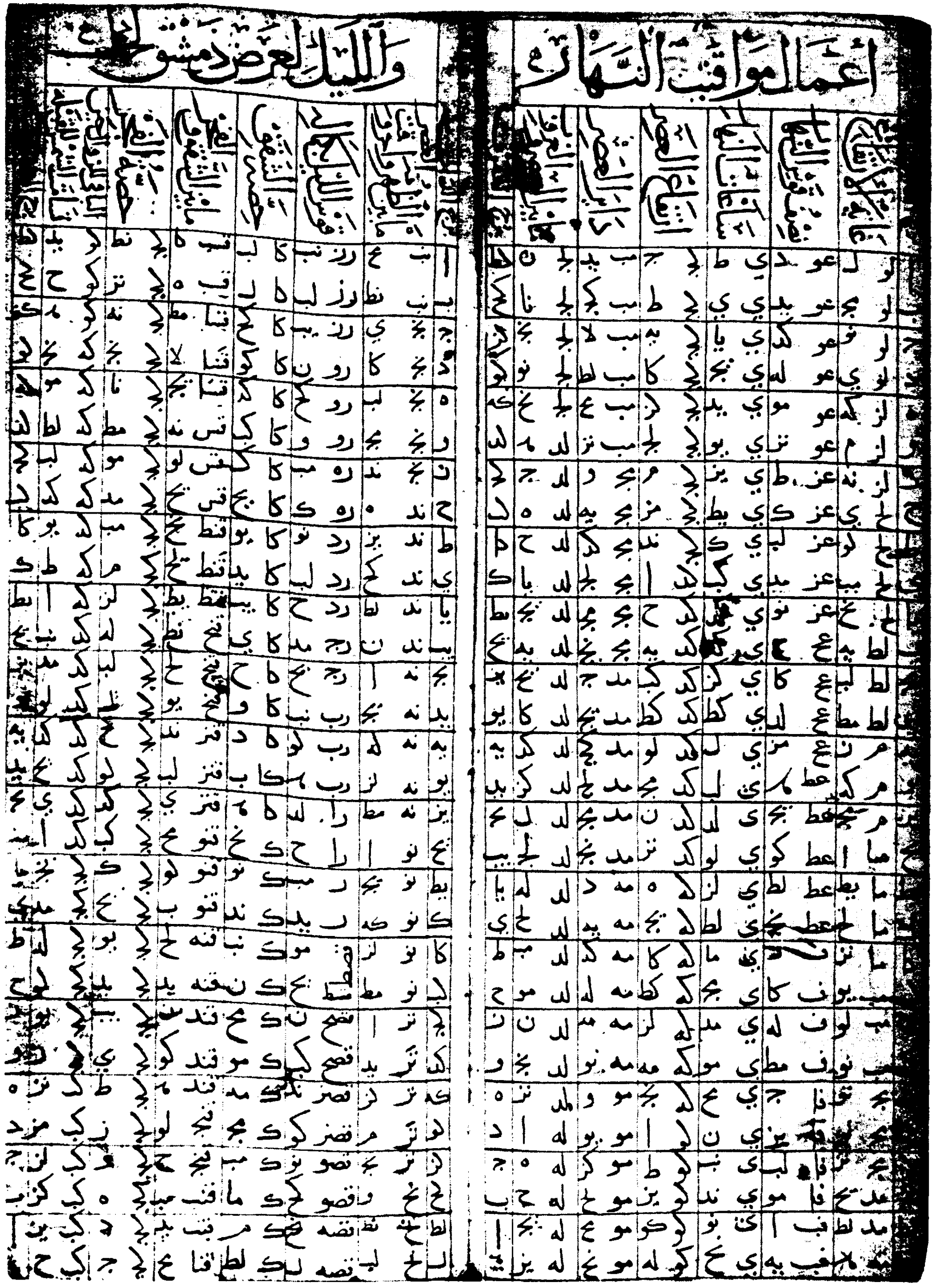

Plate 6. A selection from al-Khalīlī's timekeeping tables, from BN (Paris) MS ar. 2558. Entries are given for each of twelve functions, for each degree of solar longitude, for the signs of Aquarius and Scorpio. Both sides of the table contain columns of the argument. The tabulated functions are similar to those of Ibn al-Shāṭir (Cf.

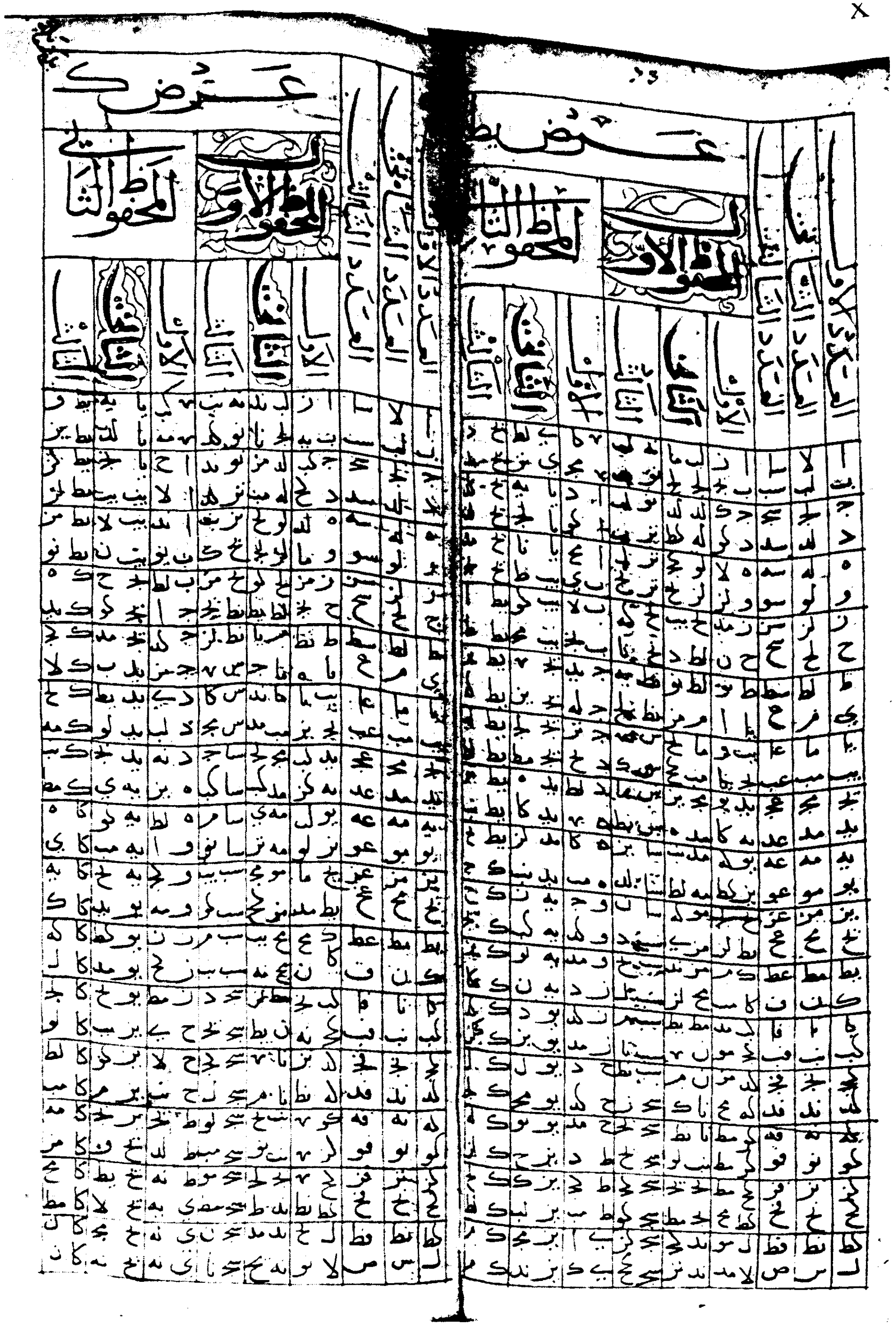

Plate 7. A selection from al-Khalīlī's *āfāqī* table, from BN (Paris) MS ar.2558. Given are the functions $f\varphi(\theta)$ and $g\varphi(\theta)$ called the "first and second retained", for $\varphi = 19°$ and $20°$, and for each degree

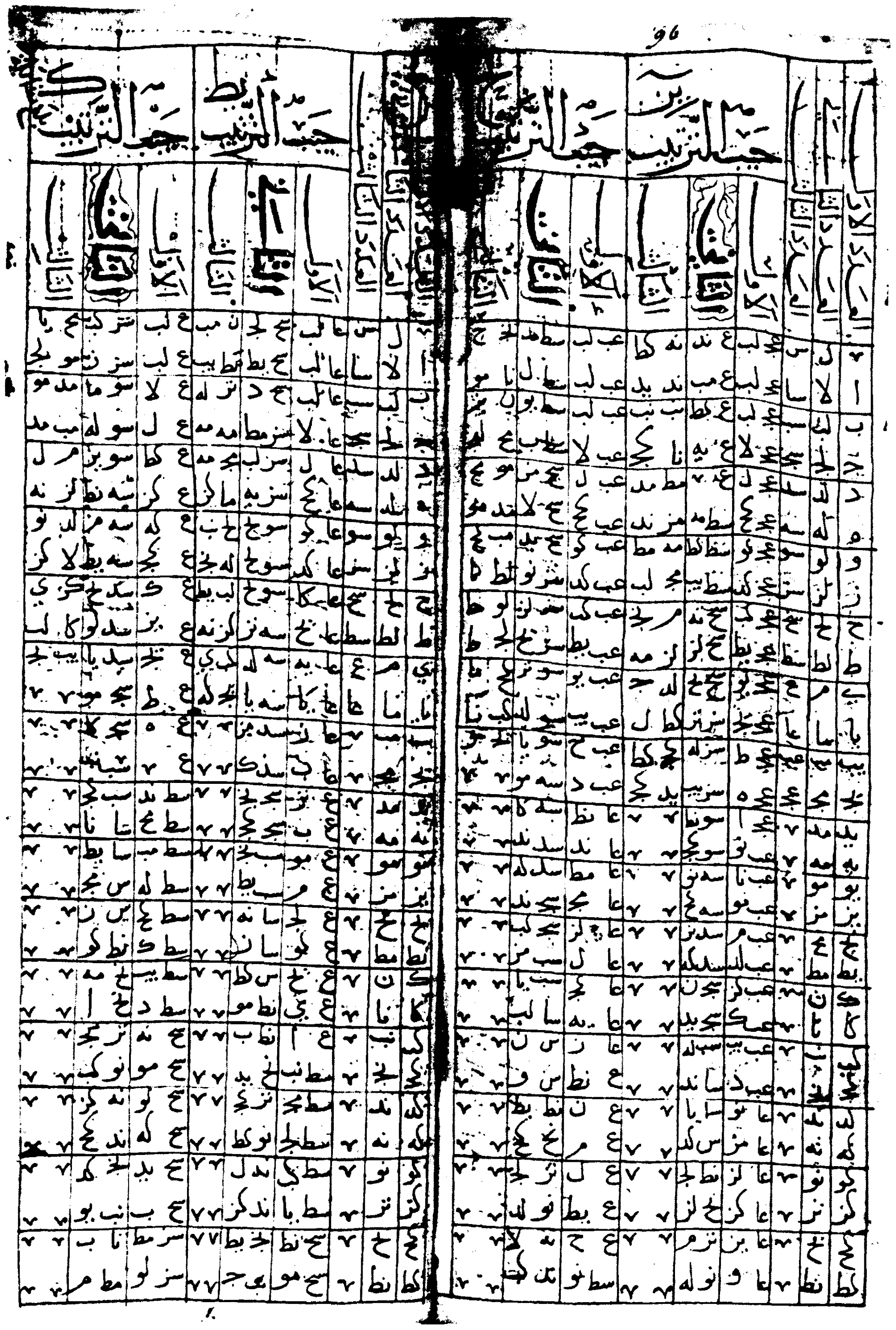

Plate 8. A selection from al-Khalīlī's *āfāqī* table, from BN (Paris) MS ar. 2558. This presents the function called *jayb al-tartīb* for values of one of the independent variables of 17°, 18°, 19°, and 20°, and for the other variable 1°, 2°, 3°, . . . , 90°. There are fourteen double-pages of tables of this function.

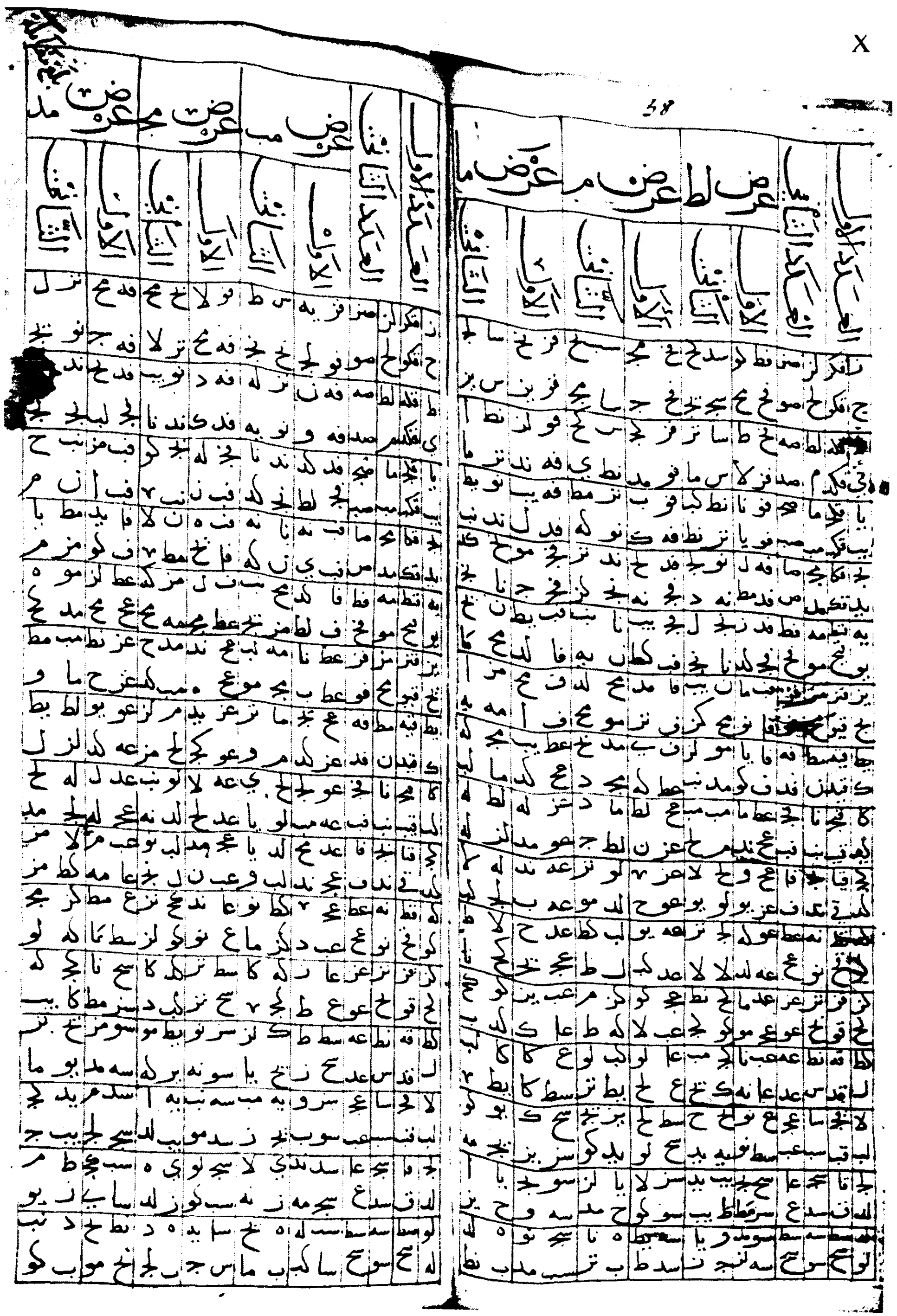

Plate 9. A selection from al-Khalīlī's *qibla* table, from BN (Paris) MS ar. 2558. The azimuth of the *qibla* is given for each degree of latitude from 39° through 44°, and for each degree of longitude from 7° through 127°. The set contains eight tables like this.

AL-KHALĪLĪ'S AUXILIARY TABLES FOR SOLVING PROBLEMS OF SPHERICAL ASTRONOMY

Early in the development of Islamic astronomy there were tabulated sets of functions which had no direct astronomical significance, but were of such a nature that ordered applications of them would lead to the solution of spherical astronomical problems. These auxiliary tables reveal considerable mathematical ingenuity on the part of their original compilers. Thus far the tables of three important Islamic astronomers have been studied, namely, those of Ḥabash al-Ḥāsib[1] (*ca* 850); Abū Naṣr Manṣūr[2] (*ca* 1000); and Ibn Yūnus[3] (*ca* 1000). The purpose of the present paper is to discuss the auxiliary tables of al-Khalīlī[4] (*ca* 1375), an individual hitherto unrecognized in the literature as one of the greatest calculators of the Middle Ages. al-Khalīlī was associated with the Umayyad Mosque in Damascus,[5] and was a colleague of the distinguished astronomer Ibn al-Shāṭir. Very little is known yet about the activities of these Damascus astronomers.[6]

The tables of Ḥabash and Abū Naṣr are comparable to each other in that they are essentially single-argument tables with a total of a few hundred entries.[7] The tables of Ibn Yūnus are scattered throughout his *Zīj*, a lengthy compendium on astronomy, and are similar to those of his predecessor Ḥabash and his contemporary Abū Naṣr. However, Ibn Yūnus, with his *Very useful tables* for timekeeping,[8] was the forerunner of an important development in Islamic astronomy, namely, the compilation of extensive tables for timekeeping by the Sun and for regulating the astronomically defined times of Muslim prayer. His spherical astronomical tables, many of which are double-argument tables, contain over 30,000 entries, calculated with remarkable accuracy. These tables, computed for the latitude of Cairo-Fusṭāṭ, 30;0°, and for obliquity 23;35°, served as a model not only to a host of individuals who plagiarized them,[9] but also to certain astronomers who developed them. One of the latter was al-Khalīlī, who tabulated virtually all the functions tabulated by Ibn Yūnus, but for the latitude of Damascus, namely, 33;30°, and for obliquity 23;31°.[10] al-Khalīlī, however, had the ingenuity to devise a set of auxiliary functions which would work for all latitudes, at the cost of computing over 13,000 entries! This did not exhaust his ingenuity (or his patience): he also computed a table giving the *qibla*, that is, the direction of Mecca, for all latitudes and longitudes within the world of Islam.[11] This table contains over 3,000 entries, whose accuracy is quite remarkable, considering the nature of the function tabulated.

The auxiliary tables of al-Khalīlī and instructions on their use are to be found in at least three manuscript sources:[12]

MS. Paris Bibliothèque Nationale, ar. 2558, fols 61v–104r;[13]
MS. Princeton Yahuda 861,2;[14]
MS. Escorial ar. 931, fols 171r–211v.[15]

The first of these is a beautiful manuscript, carefully copied in an elegant hand, and dated 811 Hijra (=1408). It is a complete copy of al-Khalīlī's prayer-tables and tables for timekeeping.[16] The second manuscript is a carelessly copied version of the prayer-tables[17] of al-Wafā'ī (*ca* 1450), similar to those of Ibn Yūnus, but computed for each degree of latitude from 21° (Mecca) to 41° (Istanbul), and containing over 20,000 entries. al-Khalīlī's auxiliary tables have been bound in the middle of these tables. The third manuscript is a volume comprising several astronomical works, in which al-Khalīlī's auxiliary tables have been inserted at the end. These tables were copied in 933 Hijra (=1526).

al-Khalīlī tabulates three auxiliary functions, each to two sexagesimal digits. For the first two, the horizontal argument is terrestrial latitude, and I denote the functions by f_ϕ and g_ϕ, where ϕ is the local latitude. The vertical argument is defined in the instructions, and the functions are called *al-maḥfūẓ al-awwal* and *al-maḥfūẓ al-thānī* respectively, "first and second functions". The third function, which I denote by G, has a quantity called *jayb al-tartīb*, perhaps best rendered as "the auxiliary Sine", for horizontal argument, and the vertical argument is defined in the instructions. The *jayb al-tartīb* is an auxiliary function widely used in late Islamic astronomy, and its use will become apparent in what follows.

The functions are not explained in the instructions, but are in fact:

$$f_\phi(\theta) = \frac{R \operatorname{Sin} \theta}{\operatorname{Cos} \phi},$$

$$g_\phi(\theta) = \frac{\operatorname{Sin} \theta \operatorname{Tan} \phi}{R},$$

$$G(x,y) = \operatorname{arc\,Cos} \left\{ \frac{xR}{\operatorname{Cos} y} \right\},$$

where the trigonometric functions are to base $R = 60$.[18] These three functions are tabulated for the domains:[19]

$$\begin{aligned} \theta &= 1^\circ, 2^\circ, \ldots, 90^\circ \\ \phi &= 1^\circ, 2^\circ, \ldots, 55^\circ, \text{ and } 33;30^\circ \text{ (Damascus)}, \\ x &= 1, 2, \ldots, 59, \\ y &= 0^\circ, 1^\circ, \ldots, n(x), \end{aligned}$$

where $n(x)$ is the largest integer such that

$$xR \leqslant \operatorname{Cos} n(x).$$

The values, written in standard Arabic sexagesimal notation,[20] are arranged in columns of thirty entries. On a given double opening one finds values of f_ϕ and g_ϕ for two consecutive values of ϕ, or values of G for four consecutive values of x. When the values of G are, in modern terms, non-real, no entry is given in the table. Columns which would be empty are omitted, and so, for example, the values of G for

$$x = 52, 53, \ldots, 59$$

fit on a double opening in MS. B.N. ar. 2558, fols 103v–104r. If the values of G were tabulated in a single table the resulting table would be trapezoidal in shape.

TABLE 1. Selected entries from the table of $f_\phi(\theta)$.

(Values are accurately computed, except where the error in the second digit is shown in square brackets.)

Argu-ment, θ	Latitude, ϕ											
	1°	2°	...	9°	...	19°	...	31°	...	49°	...	55°
1°	1; 3	1; 3		1; 4		1; 7[+1]		1;13		1;36		1;50
2°	2; 6	2; 6		2; 7		2;13		2;26[−1]		3;13[+1]		3;39
3°	3; 8	3; 8[−1]		3;10[−1]		3;20[+1]		3;39[−1]		4;49[+2]		5;29[+1]
⋮												
17°	17;32[−1]	17;33		17;45[−1]		18;33		20;29[+1]		26;45[+1]		30;36[+1]
⋮												
35°	34;25	34;26		34;50[+1]		36;23[−1]		40; 9		52;27		60; 0
⋮												
42°	40; 9	40;10		40;39		42;28		46;51[+1]		61;11[−1]		70; 0
⋮												
63°	53;28	53;30		54; 8		56;32		62;23[+1]		81;28[−1]		93;12
⋮												
78°	58;41[−1]	58;43		59;25		62; 4		68;28		89;26[−1]		102;19
⋮												
90°	60; 0[−1]	60; 2		60;44[−1]		63;28[+1]		70; 0		91;28[+1]		104;36

The entries in al-Khalīlī's tables were computed with remarkable precision. Most of the entries are accurate, and the error in the remainder is usually only ±1 in the second digit, and occasionally ±2. Errors in the table of G are slightly more frequent than in the tables of f_ϕ and g_ϕ, and larger isolated errors do occur. Tables 1–3 show random extracts from the three tables of al-Khalīlī, taken from MS. B.N. ar. 2558. The errors in the second digit are shown in square brackets, and are derived according to the convention

$$\text{error} = \text{text} - \text{recomputation}.$$

Table 4 shows a set of entries in the table of $G(x,y)$ for small values of x and large values of y, which are inaccurately computed. I am unable to explain how these errors might have arisen. In the Appendix to this paper I have indicated the kind of trigonometric and multiplication tables al-Khalīlī may have used to compile these auxiliary tables, which contain respectively 5,040, 5,040, and about 3,350 entries.

I shall now explain the method of using the tables, following al-Khalīlī's instructions. The rules which he outlines in words[21] are here expressed in modern mathematical notation and are numbered. I explain most of the rules by transforming the modern expression[22] for the function under discussion into a form which is equivalent to al-Khalīlī's rule, given the definitions of the functions f_ϕ, g_ϕ, and G. The arguments to be used in the tables are considered positive: in al-Khalīlī's introduction detailed instructions are given to cover all possible cases. Modern notation renders these superfluous in the commentary. The formulae on which his rules are based were well known in al-Khalīlī's time. Whilst some of them are not given in the *Zīj* of al-Battānī[23] (*ca* 900), all of them are contained in the *Ḥākimī Zīj* of Ibn Yūnus[24] (*ca* 1000).

TABLE 2. Selected entries from the table of $g_\phi(\theta)$.
(Values are accurately computed, except where the error in the second digit is shown in square brackets.)

Argument, θ	Latitude, ϕ 1°	2°	… 9°	… 18°	… 31°	… 49°	… 55°
1°	0; 1	0; 2	0;10	0;20	0;38	1;12	1;30
2°	0; 2	0; 4	0;20	0;41	1;16[+1]	2;25	3; 0[+1]
3°	0; 3	0; 6[−1]	0;30	1; 1	1;54[+1]	3;37	4;29
17°	0;18	0;36[−1]	2;46[−1]	5;42	10;32	20;10[−1]	25; 3
35°	0;36	1;12	5;27	11;11	20;41	39;36[+1]	49; 9
42°	0;42	1;25[+1]	6;21[−1]	13; 2[−1]	24; 6[−1]	46;10[−1]	57;20
63°	0;56	1;52	8;27[−1]	17;22	32; 7	61;31[+1]	76;21
78°	1: 2[+1]	2; 3	9;17[−1]	19; 4	35;16	67;30[−1]	83;48[−1]
90°	1: 3	2; 6	9;30	19;30	36; 3	69; 1	85;41

(i) To find the half diurnal arc, D, or half nocturnal arc, N, of a celestial body with declination δ,[25] given the terrestrial latitude ϕ, use:

$$\left.\begin{matrix} D \\ N \end{matrix}\right\} = G\ \{g_\phi(\delta), \delta\} \text{ for } \delta \underset{>}{\leq} 0, \text{ respectively.} \qquad (1)$$

In modern notation, for both $\delta \underset{<}{\geq} 0$:

$$D = 180° - N = 90° + \text{arc sin}\ \{\tan\delta \tan\phi\},$$

so that:

$$\left.\begin{matrix} D \\ N \end{matrix}\right\} = \text{arc cos}\left\{\frac{\sin\delta \tan\phi}{\cos\delta}\right\} \text{ for } \delta \underset{>}{\leq} 0, \text{ respectively.}$$

(ii) To find the hour-angle, t, given the solar altitude, h, for any declination and terrestrial latitude, use:[26]

$$t = G\ \{[f_\phi(h) - g_\phi(\delta)], \delta\}. \qquad (2)$$

The modern formula is:

$$t = \text{arc cos}\left\{\frac{\sin h - \sin\delta \sin\phi}{\cos\delta \cos\phi}\right\}$$

$$= \text{arc cos}\left\{\frac{\dfrac{\sin h}{\cos\phi} - \sin\delta \tan\phi}{\cos\delta}\right\}.$$

(iii) To find the lengths of evening and morning twilight, r and s, assuming angles of solar depression 17° and 19° at nightfall and daybreak respectively,[27] use:

$$s = N - G\ \{[f_\phi(17°) + g_\phi(\delta)], \delta\}, \qquad (3)$$

$$r = N - G\ \{[f_\phi(19°) + g_\phi(\delta)], \delta\}. \qquad (4)$$

The length of twilight for solar longitude λ and declination δ is easily found[28] by calculating the time for the Sun to attain an altitude of 17° or 19° when the solar longitude is $\lambda + 180°$ and the declination is $-\delta$. The values of G in (3) and (4) define the corresponding hour-angles (*cf.* (2) above). Note also that:

$$N(\lambda) = D\ (\lambda + 180°).$$

(iv) To find the solar rising or ortive amplitude, ψ, use:

$$\psi = 90° - G\ \{g_{45}(\delta), \phi\}, \qquad (5)$$

or solve:

$$g_{45}(\psi) = f_\phi(\delta). \qquad (6)$$

In modern notation:

$$\psi = \text{arc sin}\left\{\frac{\sin\delta}{\cos\phi}\right\},$$

so that:

$$\psi = 90° - \text{arc cos}\left\{\frac{\sin\delta \tan 45°}{\cos\phi}\right\},$$

and also:

$$\sin\psi \tan 45° = \frac{\sin\delta}{\cos\phi}.$$

Note that $g_{45}(\theta)$ is the Sine function.

TABLE 3. Selected entries from the table of $G(x,y)$.
(Values are accurately computed, except where the error in the second digit is shown in square brackets.)

Argument, y	Auxiliary Sine, x 1	2	… 11	… 20	… 32	… 46	… 59
0°	89; 3°	88; 6[+1]	79;26	70;32	57;45[−1]	39;56[−1]	10;28[−1]
1°	89; 3	88; 6[+1]	79;26	70;32	57;45[−1]	39;56	10;26
2°	89; 3	88; 6[+1]	79;26	70;31	57;44[−1]	39;54	10;17
13°	89; 1	88; 3[+1]	79; 9	70; 0	56;48[−1]	38; 6[−1]	—
32°	88;52	87;45	77;31	66;[51][b]	51; 2	25;18	—
47°	88;36	87;12	74;24	60;43[−1]	38;33	—	—
68°	87;26[−1]	84;54	60;42	27;10[+1]	—	—	—
74°	86;31[−1]	83; 2[−1]	48;18	—	—	—	—
89°	57;15[a]	—	—	—	—	—	—

[a] *Cf.* Table 4.
[b] Text: 41. The restoration is confirmed by the accuracy of the adjacent entries.

(v) To find the altitude of a celestial body in the prime vertical, h_0, use:

$$h_0 = 90^\circ - G\,\{g_{45}(\delta), \bar{\phi}\} \tag{7}$$

(MS. B.N. ar. 2558 has incorrectly:

$$h_0 = G\,\{g_{45}(\delta), \bar{\phi}\}),$$

where $\bar{\phi}$ is the complement of ϕ, or solve:

$$g_\phi(h_0) = f_\phi(\delta). \tag{8}$$

In modern notation:

$$h_0 = \arc\sin\left\{\frac{\sin\delta}{\sin\phi}\right\},$$

so that:

$$h_0 = 90^\circ - \arc\cos\left\{\frac{\sin\delta\tan 45^\circ}{\cos\bar{\phi}}\right\},$$

and also:

$$\sin h_0 \tan\phi = \frac{\sin\delta}{\cos\phi}.$$

(vi) To find the solar azimuth, a, measured from the meridian, use:

$$a = G\,\{[g_\phi(h) - f_\phi(\delta)], h\}. \tag{9}$$

The modern formula is:

$$a = \arc\cos\left\{\frac{\sin h\sin\phi - \sin\delta}{\cos h\cos\phi}\right\}$$

$$= \arc\cos\left\{\frac{\sin h\tan\phi - \dfrac{\sin\delta}{\cos\phi}}{\cos h}\right\}.$$

(vii) To find the solar declination from an observed pair of values (h,a), when the local latitude is known, first solve for x in:[29]

$$a = G\,\{x,h\}, \tag{10}$$

and then solve for δ in:

$$f_\phi(\delta) = g_\phi(h) - x. \tag{11}$$

This procedure is the reverse of (9) above.

(viii) To find both the solar declination and local latitude from two such pairs, subscripted 1 and 2, observed on the same day, first solve (10) for x_1 and x_2. Then define the quantity:

$$\Delta x = x_2 - x_1 \tag{12}$$

called the argument (*ḥiṣṣa*), and the quantity:

$$\Delta g = g_{45}(h_2) - g_{45}(h_1). \tag{13}$$

Next find the quantity ξ, called the correction (*ta'dīl*), defined by:

$$g_{45}(\xi) = \Delta g. \tag{14}$$

TABLE 4. Selected entries in the table of $G(x,y)$ containing anomalous errors.

Argument, y	Auxiliary Sine, x 1	2	3	4	5	6	7
						⋮	⋮
81°					⋮	44; 3[b]	41;45
82				⋮	53;12	34;[55][b,c]	33; 2
83			⋮	56;52	46;53	16;46[b]	16;55
84		⋮	61;24	50;20	37; 6	—	—
85	⋮	67;32	55; 1	40; 9	16;10	—	—
86	76;10	61;26	44;10	17; 1	—	—	—
87	71;23	50;20	16;46	—	—	—	—
88	61;34	17;45	—	—	—	—	—
89	57;15[a]	—	—	—	—	—	—

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[a] Whilst it is tempting to restore 57 to 17, it is also possible that al-Khalīlī computed R/Cos 89° as 57;15 (accurately, 57;18) and simply forgot to take the arc Cosine. MS Escorial ar. 931, fol. 196v also has 57;15.

[b] The entry for y = 60° has been omitted from this column and each of the entries is one space too high.

[c] Text: 15.

Corresponding recomputed values

	1	2	3	4	5	6	7
							⋮
81°						⋮	41;46
82				⋮	53;13	44; 4	33; 2
83			⋮	56;50	46;52	34;52	16;48
84		⋮	61;25	50;22	37; 8	16;56	—
85	⋮	67;31	55; 0	40; 6	17; 2	—	—
86	76;11	61;27	44;13	17; 7	—	—	—
87	71;26	50;26	17;11	—	—	—	—
88	61;28	17;14	—	—	—	—	—
89	17;15	—	—	—	—	—	—

Then solve for ϕ in:

$$g_\phi(\xi) = \Delta x. \tag{15}$$

Finally, the declination is to be found from (11) above. The procedures are easily verified by projecting the celestial sphere onto the meridian plane.[30]

(ix) To find the declination of a star, Δ, from its ecliptic coordinates (λ,β), first find λ'', the longitude difference between the star and the nearest solstice, and then solve for x in:

$$\lambda'' = G\,\{x,\beta\}. \tag{16}$$

The declination is then found by solving:[31]

$$f_{37}(\Delta) = \tfrac{1}{2}\,\{2g_{49}(\beta) + x\}. \tag{17}$$

To explain this procedure, let ϵ denote the obliquity of the ecliptic. The modern formula is then:

$$\Delta = \arc\sin\,\{\sin\beta\cos\epsilon + \cos\beta\sin\lambda\sin\epsilon\},$$

so that:

$$\sin\Delta = \sin\beta\cos\epsilon + \cos\beta\sin\lambda\sin\epsilon,$$

and:

$$\frac{\sin\Delta}{\sin\epsilon} = \frac{\sin\beta\cos\epsilon}{\sin\epsilon} + \cos\beta\sin\lambda.$$

In order to adapt this formula for use with al-Khalīlī's tables, we write:

$$\frac{\sin\Delta}{\cos\phi_1} = \tfrac{1}{2}\,\{2\sin\beta\tan\phi_2 + \cos\beta\sin\lambda\}.$$

It follows that the values ϕ_1 and ϕ_2 are defined by:

$$\tfrac{1}{2}\cos\phi_1 = \sin\epsilon \text{ and } 2\tan\phi_2 = \cot\epsilon.$$

Since al-Khalīlī, following Ibn al-Shāṭir, used $\epsilon = 23;31°$, the accurate values of ϕ_1 and ϕ_2 are:

$$\phi_1 = 37;4° \text{ and } \phi_2 = 48;58°.$$

al-Khalīlī used the approximations:

$$\phi_1 = 37° \text{ and } \phi_2 = 49°.$$

(x) To find the normed right ascension α' of a star,[32] use:

$$\alpha'' = G\,\{[2f_{37}(\beta) - 2g_{49}(\Delta)],\,\Delta\}, \tag{18}$$

where α'' is the normed right ascension measured from $\alpha' = 0°$ or $\alpha' = 180°$.

The modern formula for right ascension, α, may be written:

$$\begin{aligned}\alpha &= \text{arc sin}\left\{\frac{\sin\Delta\cos\epsilon - \sin\beta}{\cos\Delta\sin\epsilon}\right\}\\ &= \text{arc sin}\left\{\frac{\sin\Delta\cot\epsilon - \dfrac{\sin\beta}{\sin\epsilon}}{\cos\Delta}\right\},\end{aligned}$$

so that:

$$\begin{aligned}\alpha' &= \text{arc cos}\left\{\frac{-\left[2\sin\Delta\tan\phi_2 - \dfrac{2\sin\beta}{\cos\phi_1}\right]}{\cos\Delta}\right\}\\ &= \text{arc cos}\left\{\frac{\dfrac{2\sin\beta}{\cos\phi_1} - 2\sin\Delta\tan\phi_2}{\cos\Delta}\right\},\end{aligned}$$

where ϕ_1 and ϕ_2 are as defined in (ix) above.

In modern terms, it may be stated that al-Khalīlī's tables are auxiliary tables for solving problems of spherical astronomy which involve the cosine rule for spherical triangles. The formulae which define the functions treated by al-Khalīlī were well known to Islamic astronomers already in ʻAbbāsid times, but no Islamic treatise dealing with the spherical cosine rule in its generality is known from the medieval period. The formulae for the hour-angle and solar azimuth in terms of solar altitude, and for the declination and right ascension of a star in terms of its ecliptic coordinates can however be derived without difficulty by projection methods.

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1. R. Irani's thesis (Irani [1]) remains to be published. On Ḥabash, see Kennedy [1], 126–7 (nos. 15 and 16).
2. See Jensen [1].
3. See King [1], 29–33.
4. On Shams al-Dīn Abū ʻAbd Allāh Muḥammad b. Muḥammad al-Khalīlī, see Suter [1], no. 418.
5. Suter (*cf.* ref. 4) states that al-Khalīlī was associated with the Yalbaghā Mosque outside Damascus. The title folio of MS B.N. ar. 2558 states that he was a timekeeper (*muwaqqit*) at the Umayyad Mosque.
6. A. Sayili has already pointed to the need for further research in this area. *Cf.* Sayili [1], 245. On Ibn al-Shāṭir and his work, see Kennedy [1], 125 (no. 11); Wiedemann [1], ii, 729–38; and the recent studies listed by N. Swerdlow in *Journal for the history of astronomy*, iii (1972), 48, ref. 3.
7. C. Jensen has published a few entries from Abū Naṣr's table. The following table shows the corresponding recomputed values, to be compared with those in Jensen [1], 4.

I	II	III	IV	V	VI	VII
1°	2°	;24,16,10,41	;0,27,47,16	;1,54,54,58	;2, 5,38,17	;54,52,50,12
2	4	;24,16,50,37	;0,55,35,33	;3,49,45,39	;4,11, 7,24	;54,54, 0,38
3	6	;24,17,57,15	;1,23,25,52	;5,44,27,44	;6,16,18, 9	;54,55,57,36
4	8	;24,19,30,41	;1,51,19,15	;7,38,56,50	;8,21, 1,23	;54,58,40,27
44	88	;33,44, 1, 2	;25,37,20,18	;59,57,22,43	;59,57,48,25	;59,59,38,29
45	90	;34,19, 1,59	;26,31,57,38	;60, 0, 0, 0	;60, 0, 0, 0	;60, 0, 0, 0

It is clear that in numerous entries the third of the four digits is in error, and the fourth is meaningless. Furthermore, had Abū Naṣr used the value 23;35° for the obliquity, which was generally accepted in his time, rather than Ptolemy's value 23;51,20°, the entries in all but column VI (which are for the sine function) would generally differ even in the second digit from those given in his table.

It is perhaps worth remarking that al-Nayrīzī (*ca* 900) also tabulated some auxiliary functions. Two tables attributed to him, based on the *Khanḍa Khādyaka* value $R = 2{,}30$, are located in MS Berlin 5750, fols 83r–85r, of Ḥabash's *Zīj*. *Cf.* King [1], 29.

8. *Cf.* King [2] for a detailed analysis.
9. *Cf.* King [1], 43 and [2].
10. An analysis is in preparation.
11. This table is analyzed in detail in King [3]. It may be that al-Khalīlī computed the entries in the *qibla* table using his auxiliary tables.
12. I have not inspected MSS Berlin 5754–56, British Museum 977, 31, and Oxford I. 961, 1039,2, listed by Suter. *Cf.* ref. 4 above.

13. *Cf.* de Slane [1], 460–1.
14. I am indebted to Prof. R. Mach of Princeton University for kindly allowing me to consult his unpublished catalogue of the Yahuda Collection.
15. *Cf.* Renaud [1], 42–3. In this MS the tables are attributed to Sharaf al-Dīn Abū 'Abd Allāh al-Khalīlī, who lived about the year 1400. *Cf.* Suter [1], no. 427. Probably Sharaf al-Dīn should be restored to Shams al-Dīn.
16. *Cf.* ref. 10 above.
17. An analysis is in preparation. On al-Wafā'ī see Suter [1], no. 437, and King [1], 43 and 49.
18. The capital letter notation, now standard, is used to denote medieval trigonometric functions. *Cf.* Kennedy [1], 139b–140a.
19. In the Escorial MS, the maximum value of ϕ is 50°. As stated by Abū Naṣr, beyond latitude 45° there were few people who knew about spherical astronomy or even thought about it. *Cf.* Jensen [1], 5.
20. On which, see Irani [2].
21. *Cf.* refs 26, 29, and 31 below.
22. For some of the modern formulae, consult Smart [1], 25–52. For the derivation of these formulae by medieval methods, see, for example, King [1], 90–316.
23. On which, see Kennedy [1], 132–3 (no. 55), and the edition of C. Nallino (Milan-Rome, 1899–1907). al-Battānī does not, for example, discuss the calculation of the duration of twilight.
24. With the reservation that Ibn Yūnus discusses the calculation of time since sunrise rather than the hour-angle. *Cf.* King [1], 147–52.
25. Generally, I use δ for solar declination (*mayl*) and Δ for the declination of a star (*bu'd*). For most of the problems under discussion, al-Khalīlī notes the validity of his procedures for both the Sun and the fixed stars.
26. al-Khalīlī's instructions (MS B.N. ar. 2558, fol. 62r, ll. 1–7) read:

    Section on the hour-angle. Find the value of the first function for the altitude (as argument) on the page corresponding to the latitude, and find the value of the second function (for the declination as argument) as we described above for the half diurnal arc. Then, if the Sun is in the southern signs, add the two functions; otherwise, take the difference between them. The result will be the auxiliary Sine. Enter it in the table with the auxiliary Sine (as horizontal argument) and enter the declination as (vertical) argument: you will find the hour-angle. However, if the Sun is in the northern signs and the value of the first function is less than the value of the second, subtract the result from 180°: the remainder will be the hour-angle. If the two functions are equal, the hour-angle will be 90°.
27. On Islamic values for these parameters, consult Wiedemann [1], ii, 769 and 777, and King [2].
28. See, in particular, Smart [1], 51–52.
29. al-Khalīlī's instructions for these operations (MS B.N. ar. 2558, fol. 62v, ll. 15–23) read as follows:

    If either one of the meridian or the prime vertical is known, then the azimuth of an observed solar altitude can be found. Enter the altitude and azimuth in the auxiliary Sine tables in the following way. Look for a value in the table equal to the azimuth, which has the altitude as (vertical) argument. The auxiliary Sine is then found (as the horizontal argument) at the head of the appropriate table. Next find the value of the second function for the altitude on the appropriate page for the latitude, and subtract the value from the auxiliary Sine if the Sun is in the southern signs. If it is in the northern signs and the azimuth is northern, add the two values. Otherwise take the difference between them. The result is a value of the first function. Enter the value on the appropriate latitude page and find the corresponding vertical argument. This will be the declination.
30. *Cf.* King [1], 196, for Ibn Yūnus's treatment of this problem.
31. al-Khalīlī describes this method as follows (MS B.N. ar. 2558, fols. 63r, l. 27–63v, l. 4):

    . . . Enter the latitude of the star as argument for the second function for latitude 49°. Then double the result and add it to the auxiliary Sine. . . . Enter the result in the table of the first function for latitude 37° and find the corresponding vertical argument. It will be the declination of the star. . . .

    He does not explain the reason for using the tables for latitudes 49° and 37°.
32. The normed right ascension is defined by:

    $$\alpha'(\lambda) = \alpha(\lambda) + 90°.$$

    This function is tabulated in the *Handy tables*, and similar tables are common in Islamic *zījes*. It is useful for finding the ascendant, H, if the longitude of upper midheaven, M, is known, since:

    $$\alpha_\phi(\lambda_H) = \alpha(\lambda_M) + 90° = \alpha'(\lambda_M),$$

    where α_ϕ denotes oblique ascension for latitude ϕ.

APPENDIX

There is no explicit indication in the texts consulted of the method used by al-Khalīlī to compile the tables. He clearly did not compute all of the entries directly, and the error patterns confirm that he used some kind of interpolation system. It may be that he computed the values of:

$$\frac{\text{Sec }\theta}{R} = \frac{R}{\text{Cos }\theta} \qquad (\theta = 1°, 2°, \ldots, 89°)$$

since these are required to compute:

$$G(1,y) = \text{arc Cos}\left\{\frac{R}{\text{Cos }y}\right\}.$$

It is reasonable to assume that he used the Secant function to compile the tables, rather than continually dividing by the Cosine in the tabulation of f_ϕ and G. No independent table of the Secant is contained in any known Islamic source, and the only Islamic table of the Cosecant function is in MS Berlin 5750, fols. 85r–87v, which may be due to Ḥabash (*cf.* Kennedy [1], 151b).

It is also possible that al-Khalīlī computed a table of arc Cos (z). No table of this kind is attested yet in the sources, and the only Islamic tables of arc Sin (z) known to me are contained in the Yemeni *Mukhtār Zīj* (MS British Museum 768 (Or. 3624), fol. 169v), and in one manuscript of Ibn Yūnus's *Very useful tables* (MS Chester Beatty 3673, fol. 8r). The function is computed to two sexagesimal digits for each 0;30 and 1;0 of argument, respectively, in the two tables. Tables of the Sine and Tangent function, accurately computed to, say, three sexagesimal digits, for each degree of argument were standard in the time of al-Khalīlī. He may have used the trigonometric tables of Ibn al-Shāṭir, in which the Sine is given to four sexagesimal digits for each degree of argument and the Tangent to three digits for each half degree (*cf.* Kennedy [1], 162b–163a). Alternatively, he might have used Sine tables like those of Ibn Yūnus, computed to five sexagesimal digits for each minute of argument (*cf.* King [1], 9 and 85–89), to take the arc Cosine in the compilation of the table of G.

Finally, it is certain that al-Khalīlī had at his disposal a multiplication table containing the sexagesimal products:

$$m \times n \qquad (m, n = 1, 2, \ldots, 60)$$

such as, for example,

$$34 \times 13 = 7{,}22 \text{ and } 41 \times 23 = 15{,}43.$$

Multiplication tables are not generally contained in Islamic *zījes* or tables for timekeeping. Indeed' only one complete Islamic multiplication table has been noted in the literature. P. Luckey has described a table in a work by the Egyptian scholar Sibṭ al-Māridīnī (d. *ca* 1500, *cf.* Suter [1], no. 445, and Luckey [1], 68, and 39, n. 67). The multiplication table referred to by Kūshyār b. Labbān (*fl. ca* 1000, *cf.* Suter [1], no. 192, and Levey [1], 98 and 36–38) has been omitted from the manuscript published by M. Levey. Likewise, the Samarqand mathematician and astronomer al-Kāshī (*fl. ca* 1430, *cf.* Luckey [1], esp. 47) in his treatise on arithmetic describes but does not reproduce the multiplication table which he used for his extensive calculations. R. Irani has discussed an extract from such a table contained in MS Princeton Yahuda 373 (*cf.* Irani [3]). The following examples of complete Islamic sexagesimal multiplication tables have recently come to my attention, and there is every reason to suppose that numerous others survive in the vast manuscript sources. These tables were the standard computational aids of Islamic astronomers and mathematicians.

(*a*) MS B.N. ar. 2531 is a work by the prolific Egyptian scholar Ibn al-Majdī (d. 1447, *cf.* Suter [1], no. 432), dealing with the planetary tables of Ibn Yūnus and Ibn al-Shāṭir. Fols. 115v–127r contain a table of sexagesimal products:

$$m \times n \qquad \begin{cases} m = 1, 2, \ldots, 120 \\ n = 1, 2, \ldots, 60 \end{cases}.$$

(*b*) MS B.N. ar. 2520 is an anonymous *Zīj* comprising tables originally due to Ibn Yūnus and Ibn al-Shāṭir. Fols 167v–173r contain a table of products

$$m \times n \qquad (m, n = 1, 2, \ldots, 60).$$

At the head of each of the sixty columns for m, the value of $\sqrt{m}$ is given to three sexagesimal digits.

(*c*) MS Princeton Yahuda 4072, fols 23r–29r, and

(*d*) MS Aleppo Aḥmadīya 1310. Both these sources contain tables similar to those in (*b*) above. In the second table m^2 is given at the head of each of the sixty columns for m.

(*e*) MSS Munich 865 (313) and 866 (318). I have not inspected these sources, but the description of them in the catalogue (Aumer [1], 380) indicates that they contain a multiplication table.

These tables are referred to in the sources as either *al-jadwal al-sittīnī* or *jadwal al-nisba al-sittīnīya*, "sexagesimal table".

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Smart [1]: W. M. Smart, *Textbook on spherical astronomy* (London, 1972 edn).
Suter [1]: H. Suter, "Die Mathematiker und Astronomen der Araber und ihre Werke", *Abhandlungen zur Geschichte der mathematischen Wissenschaften*, x (1900).
Wiedemann [1]: E. Wiedemann, *Aufsätze zur arabischen Wissenschaftsgeschichte* (2 vols, Hildesheim, 1970).

XII

ASTRONOMICAL TIMEKEEPING IN OTTOMAN TURKEY

The history of astronomy in Ottoman Turkey is a much neglected area of the history of Islamic science.[1] In this paper I shall discuss just one aspect of the activity of the astronomers of Ottoman Turkey, namely, the determination of time by the sun and stars and the regulation of the astronomically defined times of the five daily prayers.[2]

The Ottoman astronomers who specialized in this branch of astronomy were called *muwaqqits*,[3] and they had at their disposal several means of regulating time in general and the prayer-times in particular. Firstly, thanks mainly to their contact with Europe, they had from the sixteenth century onwards mechanical clocks.[4] Secondly, as heirs to the earlier Islamic astronomical tradition, they had sundials for regulating the time of day and the day-time prayers,[5] and quadrants which served as analog computers to determine the time of day or night from the observed altitude of the sun or stars and to derive the times of prayer for a given day.[6] Thirdly, and perhaps of more interest to the history of mathematical astronomy, they used mathematical tables displaying the time of day or night in terms of solar or stellar altitude and displaying the times of the prayers throughout the year. It is these tables, which have not been previously discussed in the modern literature, that concern the present study.

Let us begin with a brief account of the definitions of the times of Muslim prayer which were standard in Ottoman Turkey. The definitions of the times of prayer were fixed in the eight century,[7] and with few attested exceptions, have been in use for over a millenium.[8] The prayers are to be performed within specified intervals of time defined by their endpoints. The Muslim day begins at sunset with the first prayer (Arabic *maghrib,* modern Turkish *akşam*). The second prayer begins at nightfall (A. *ᶜisha',* T. *yatsi*), and the third prayer begins at daybreak (A. *fajr,* T. *tan* or *fecir*) and must be completed by sunrise. The fourth prayer begins when the sun begins to decline from the meridian (A. *zuhr,* T. *öğle*). The fifth prayer is in the afternoon (A; *ᶜaṣr,* T. *ikindi*) and begins when the shadow of an object of length n has increased beyond its midday minimum s_0 by an amount Δs equal to n.[9] Either the beginning of an interval for another prayer, that is, a second °*aşr,* or the end of the interval for the first °*aşr,* takes place when the increase of the shadow is equal to $2n$; most Ottoman sources differentiate between *asr-i-awwal* and *asr-i-sani,* a first and a second °*aşr.* Some Ottoman sources also mention a prayer in the late morning (Arabic *duha* or *dahwa,* Ottoman Turkish *zahwe,* modern Turkish *kuşluk*).

The hour-angle at the times defined by these rules can be determined using formulae known to Muslim astronomers since Abbasid times.[10] The modern formula for the hour-angle t as a function of the instantaneous altitude h and declination δ for latitude φ is

$$t\,(h,\,\delta,\,\varphi)\;=\;\text{arc cos}\left\{\frac{\sin h - \sin\delta\,\sin\varphi}{\cos\delta\,\cos\varphi}\right\}$$

and the Islamic methods for determining the hour-angle, which were mainly derived from stereographic projections of the celestial sphere, were equivalent to this formula. From the hour-angle at the beginning of each prayer one can easily derive the durations of each of the prayer-times, or express the times of the prayers in mean local time or according to the Turkish convention in which sunset is reckoned as 12 o'clock.[11]

Already in the ninth century there began a Muslim tradition of compiling tables displaying the times of prayer for each degree of solar longitude λ or each day of the year. More extensive tables were compiled displaying the hour-angle t or time since rising T of the sun for each degree of solar altitude h and each degree of solar longitude λ, or alternatively for each degree of solar meridian altitude H and each degree of solar altitude h. The Ottomans were the heirs to this earlier tradition.

Before we survey some of the tables compiled by Ottoman astronomers for the purposes of timekeeping, we should point out that the beginnings of astronomy in Ottoman Turkey are very obscure. Indeed, we are as yet completely in the dark concerning astronomical knowledge amongst the Ottomans before the early fifteenth century when Qadi Zade al-Rumi, who was born in Bursa, then the Ottoman capital, joined the staff of the Samarqand Observatory of Ulugh Beg and later became its director.[12] The next director was °Alı Qushjı who, on the death of Ulugh Beg, entered the service of the Sultan Muhammad II in Istanbul.[13] From that time onwards astronomy flourished in Istanbul, and for several centuries there existed a distinctly Ottoman tradition based mainly on the works of al-Jaghmını, Naşır al-Dın al-Tusı, and Ulugh Beg, and commentaries thereon. Alongside this main tradition of theoretical and planetary astronomy there was another concerned with astronomical timekeeping, which was derived from Egypt and Syria. In order to appreciate the Ottoman activity in this branch of astronomy it is necessary to bear in mind that original Muslim activity in astronomical timekeeping reached its peak in fourteenth century Damascus, and we have evidence that the works of al-Khalılı and Ibn al-Shatir, the two main representatives of this Syrian school,[14] were known in Istanbul in the fifteenth century.

The following remarks on some of the tables compiled by Ottoman astronomers reveal for the first time the scope of the interest in astronomical timekeeping in Ottoman Turkey. Most of the tables discussed have been recomputed using an electronic computer to investigate their accuracy (see Plate 6).

Shaykh Vefa

Mustafa ibn Ahmad known as Shaykh Vefa was a celebrated Sufi *shaykh* whose wide academic interests included astronomy. He lived in the time of Sultans Muhammad II and Bayazid II and died in the year 1491.[15] Whilst on a pilgrimage he was captured and imprisoned on Rhodes, and whilst in prison he compiled a set of astronomical tables in the form of an almanac (*ruzname*). This *Ruzname-i-Vefa,* which remained popular for several centuries and exists in dozens of manuscripts,[16] consists of tables for calendar conversion and also for regulating the times of prayer. The latter tables display the length of daylight, length of night, the time from sunrise to midday, the time from midday to the °*asr* and the time from the °*asr* to sunset and the length of twilight (see Plate 1). Entries are given for each day of the year in equinoctial hours and fractions thereof (expressed in equatorial degrees, where 1 hour = 15 degrees), and are computed for latitude 41;30°, which is half a degree too high for Istanbul. Many of the manuscript copies of Vefa's *Ruzname,* including the one published by Velschius in 1676,[17] contain only the calendrical tables and not the prayer-tables, perhaps because the *Ruzname* of Darendeli (see below) was recognized as more accurate.

Muhammad ibn Katib Sinan

Muhammad ibn Yusuf, otherwise known as Muhammad ibn Kâtib Sinan, was a *muwaqqit* at the imperial palace in Istanbul in the time of the Sultan Bayazıd II (*ca.* 1500).[18] Two of his works concern the present study. The first, extant in three copies in the Topkapi and Aya Sofia (Süleymaniye) Libraries,[19] is a Turkish recension of the auxiliary tables of al-Khalili of Damascus for solving all the standard problems of spherical astronomy for all latitudes.[20] Muhammad ibn Katib Sinan translated the introduction of the tables into Turkish and added a special table for latitude 40;30° (for an unspecified locality) to al-Khalili's set for each degree of latitude (see Plate 2). It is clear that al-Khalili's tables were already known in Turkey: a manuscript in the Hamidiye (Süleymaniye) Library contains al-Khalili's tables copied in Edirne in 1464/5.[21]

The other work by Muhammad ibn Katib Sinan is likewise preserved only in two manuscripts in the Topkapi and Aya Sofia (Süleymaniye) Libraries.[22] It consists of tables for reckoning time by the stars, and contains over 500 pages of tables containing about a quarter of a million entries. The main tables display the time since sunset, the time remaining until sunrise, the time remaining until daybreak, and the time remaining until midday, for each degree of the normed right ascension and each degree of solar longitude (see Plate 3). One simply observes a star which is culminating instantaneously, reads its ascension from a separate table also presented by Muhammad ibn Katib Sinan, enters the ascension and the solar longitude in the main table and reads the time of night. This enormous table has no parallel in the known Islamic manuscript sources, the only precedent being a much smaller table compiled in Damascus about 1425 displaying the time remaining until daybreak as a function of solar longitude for seven culminating stars (which vary for each zodiacal sign). This table thus represents the most original Ottoman contribution to astronomical timekeeping by tables.

Taqi l-Din

A manuscript in the Library of Kandilli Observatory is in the hand of the astronomer Taqi l-Din, who founded the Istanbul observatory in 1577.[23] This manuscript contains *inter alia* his small *zıj* (that is, an astronomical handbook with tables) entitled *Sidrat al-muntaha* and his works on stereographic projection and observational instruments, as well as a table for timekeeping, which is not contained in any other known manuscript. I see no reason to doubt that this table was compiled by Taqi l-Din himself. It displays the hour-angle and time since sunrise in equatorial degrees and minutes for each degree of meridian solar altitude and each degree of instantaneous solar altitude (see Plate 4), and the entries are based on parameters $\varphi = 41°$ (Istanbul) and $\epsilon = 23;30°$ (taken from the value of Ulugh Beg). This table, which contains over 6,000 entries, is of the variety known in medieval Arabic as *taylasan*, "shawl," because of their trapezoidal shape (since the two arguments H and h are such that min $H < h < H <$ (max H). The earliest example of such a table dates from tenth century Baghdad, and several later tables of this kind were computed for other latitudes, including one for the latitude of Maragha by the astronomers of Nasır al-Dın al-Tusı.[24] No such *taylasan* tables were compiled for Cairo or Damascus and it seems probable that Taqi l-Din, if he is indeed the compiler of this table, was inspired by the *taylasan* table for Maragha in the *Zij-i-Ilkhani* of al-Tusi.

Simpler versions of such a *taylasan* table for Istanbul, giving entries to, say, degrees for each five degrees of both arguments, were occasionally inscribed on quadrants. One would compute the meridian altitude of the sun, measure the instantaneous altitude of the sun, and use the table to estimate the time since sunrise. An example of such quadrant is preserved in Kandilli Observatory Library.

Darendeli

Nothing of consequence is known about Darendeli, the author of a second *Ruzname* for Istanbul, save that he or his family was associated with the town of Darende in central Anatolia. He appears to have lived in the seventeenth century. His almanac exists in several manuscript copies,[25] and has been published by D'Ohsson in his *Tableau Général de l'Empire Ottoman*, 1787.[26]

Darendeli's tables are arranged for each degree of solar longitude, and display the length of daylight, the length of night, the time of midday, the times of the first and second °*asr*, the times of nightfall and daybreak, the time when the sun is in the azimuth of Mecca, and the time of the *zahwe* (see Plate 5). Times are given in hours and minutes according to the Ottoman convention, and are based on latitude 41°. The tables for twilight are based on the assumption that morning twilight begins and evening twilight ends when the sun is respectively 19° and 17° below the horizon: these are the standard parameters for twilight used by the Ottoman astronomers and were borrowed from al-Khalılı and Ibn al-Shatir.[27] The time when the sun is in the direction of Mecca corresponds to a direction of 42° east of south, indicating that the longitude difference between Istanbul and Mecca was taken as about 17° (the actual value is about 11°).[28] The time of the *zahwe* is taken as midway between daybreak and sunset, a definition still used, for example, in modern Pakistan.[29] In the other few instances where this prayer is mentioned in the medieval astronomical sources a quite different definition is used.[30]

Anonymous

An anonymous manuscript in Istanbul University Library contains an extensive set of prayer-tables for Istanbul after the model of al-Khalılı's prayer-tables for Damascus.[31] Values are given in degrees and minutes for each degree of solar longitude for some twenty-two functions relating to timekeeping, including the solar altitude and hour-angles at the two *ᶜasrs* and when the sun is the direction of Mecca, and the durations of morning and evening twilight. Values are based on latitude 41;15°, and the tables may not have been extensively used because, although they are very accurately computed, they were based on this incorrect latitude.

Ahmad Efendi

A unique manuscript in Kandilli Observatory Library contains a set of tables for timekeeping compiled in 1684 by Ahmad Efendi known as al-Misrı al-Islambulı (the Egyptian living in Istanbul).[32] The set of tables display the hour-angle expressed in equatorial degrees and minutes for each degree of solar altitude and each degree of solar longitude (see Plate 7). These tables, which contain over 10,000 entries, are based on parameters $\varphi = 41°$ (Istanbul) and $\epsilon = 23;30°$ (taken from Ulugh Beg's value). They have identical format to those in the Cairo corpus of tables for timekeeping, compiled several centuries previously. They were superceded a century later by more extensive tables of Salih Efendi (see below).

Salih Efendi

About 1775 the Ottoman astronomer Salih Efendi compiled an enormous corpus of tables for timekeeping by the sun, which is extant in several manuscript copies.[33] These tables, which contain over 80,000 entries, and a simpler version thereof by Sadiq Jihangiri (see below), were used extensively in Istanbul in the nineteenth century. The tables display not only the hour-angle for each degree of solar longitude and for each degree of solar altitude up to the maximum for that longitude, but also the times when the sun has this altitude in the east and in the west in equatorial degrees (*al-da'ir*), equatorial hours (*al-mustawiya*), seasonal hours (*al-zamaniya*), and in equatorial hours according to the Turkish convention (*al-muwafaqa*) (see Plate 8). At the end of the main set of tables the times of the first and second *ᶜasrs,* nightfall and daybreak are given for each degree of solar longitude, expressed in the Turkish convention. Values in all the tables are given to three sexagesimal digits rather than two as was standard, and are very carefully computed; thus, for example, the values of the hour-angle are given to seconds of arcs, usually correctly, and the corresponding times of day, expressed in hours, minutes, and seconds, are also generally correct. All of the values are based on $\varphi = 41°$ (Istanbul) and $\epsilon = 23;28,54°$ (taken from Taqi l-Din). A shorter version of these tables prepared by Sadiq Jihangiri[34] displays only the time since sunrise for eastern and western altitudes expressed in the Turkish convention and in hours and minutes only (see Plate 9).

Anonymous (Edirne)

A unique manuscript in Oxford contains a set of tables displaying the hour-angle and time since sunrise as a function of solar altitude and solar longitude for each degree of both arguments for the latitude of Edirne, taken as 42°.[35] Likewise a unique manuscript in the Süleymaniye Library contains a set of minor tables for timekeeping for the same latitude.[36] No tables for the prayer-times are contained in either of these sources, which are both anonymous.

Miscellaneous

Other prayer-tables of the Ottoman kind exemplified by the *Ruznames* of Shaykh Vefa and Darendeli were compiled for other latitudes in the Ottoman Empire. Examples are known for such localities as Algiers, Cairo, Damietta, Medina, Mecca, Sanaa, Damascus, Aleppo, and Crete. In some cases, such as the tables for Cairo and Damascus, the Ottoman-type prayer-tables were compiled using the earlier prayer-tables which had already been in use for several centuries. In other cases, such as the tables for Algiers and Sanaa, we do not yet know of any earlier prayer-tables for these localities. So far no Ottoman-type prayer-tables have been identified in the manuscript sources for any other locality in Turkey other than Istanbul. This I find rather surprising.

Other Ottoman-type tables for timekeeping by the sun after the model of those of Salih Efendi have been identified only for Cairo, Damascus, Jerusalem, and Mecca. Only in the last case were these tables not based on earlier tables; these tables for Mecca were compiled in the early nineteenth century by Husayn Husni, the second astronomer of the Ottoman Empire.

Postscript

The above examples do not exhaust the known tables compiled by Ottoman astronomers for the purposes of timekeeping and preserved in manuscript form, but they must suffice in this brief survey. In Turkey in the nineteenth century the prayer-times were displayed in annual almanacs or diaries (see Plate 10). Modern Turkish prayer-tables (see Plate 11) also continue an Ottoman tradition by displaying the times of prayer not only in local standard time but also according to the Ottoman convention (*ezani*). Dr. Fatin Gökmen, first Director of Kandilli Observatory, wrote a book on the theory and use of the almucantar quadrant. Dr. Muammer Dizer, the present Director, has conducted detailed investigations of the determination of first visibility of the lunar crescent as well as the computation of the prayer-times by modern methods, and has prepared prayer-tables for longitudes and latitudes all over the world by electronic computer (see Plate 12). Thus the interest in astronomical timekeeping cultivated by the Ottomans has continued to this day in modern Turkey.

Acknowledgements

The research on medieval Islamic science conducted at the American Research Center in Egypt since 1972 has been supported mainly by the Smithsonian Institution (1972-80), the National Science Foundation, Washington, D.C. (1972-80), and the Ford Foundation (1976-78). This support is gratefully acknowledged.

It is a pleasure to express my gratitude to the Directors of Kandilli Observatory, the Topkapı Saray Library, and Princeton University Library, for permission to publish photographs of manuscripts in their collections.

Notes

1. The standard bibliographical sources for Islamic science, namely, H. Suter, "Die Mathematiker und Astronomen der Araber und ihre Werke," ***Abhandlungen zur Geschichte der mathematischen Wissenschaften,*** 10 (1900), and the surveys of Arabic and Persian literature by Brockelmann and Storey, contain very little information on any Ottoman astronomers other than Qadi Zade, ^{c}Ali Qushji, and Taqi l-Din. A. Azzawi's ***History of Astronomy in Iraq,*** Baghdad: Iraqi Academy Press, 1958, contains some useful additional biographical and bibliographical information which is, however, poorly documented. Apart from a series of articles on specific topics that have been published by A. Sayili and S. Tekeli, and also A.S. Ünver, Ottoman astronomy is therefore an untapped field, and there is an obvious need for a bibliographical survey of the vast amount of available manuscript sources and a series of monographs on particular aspects of Ottoman astronomy.

2. This paper is essentially an abridgement of the section on Ottoman Turkey in my forthcoming ***Studies in Astronomical Timekeeping in Medieval Islam,*** which contains an analysis of all known medieval Islamic tables for regulating time by the sun and stars and for regulating the times of prayer. The interested reader may consult my "Astronomical Timekeeping in Medieval Islam," ***Actes du XXIX^e Congrès international des Orientalistes,*** (Paris, 1973), II/2, pp. 86-90; and "Ibn Yunus' ***Very Useful Tables*** for Regulating the Time by the Sun," ***Archive for History of Exact Sciences,*** 10 (1973), pp. 342-394; and "Astronomical Timekeeping in Fourteenth Century Syria," ***Proceedings of the First International Symposium on the History of Arabic Science,*** (Aleppo, 1976).

3. A useful study on the ***muvakkithanes*** (that is, the buildings adjacent to the mosques where the ***muwaqqits*** worked) of Istanbul is A.S. Ünver, "Osmanli Türklerinde İlim Tarihinde Muvakkithaneler," ***Atatürk Konferansları,*** V (1975), pp. 217-257. See also the same author's paper presented at this Symposium.

4. On clocks in Ottoman Turkey see S. Tekeli, *Clocks in the Ottoman Empire in the 16th Century...,* Ankara Üniversitesi Dil ve Tarih-Coğrafya Fakültesi Yayınları, Sayı 171, 1966; A. Y. al-Hassan, *Taqi l-Din and Arab Mechanical Engineering* (in Arabic), Aleppo: Institute for the History of Arabic Science, 1976; and O. Kurz, *European Clocks and Watches in the Near East,* London: The Warburg Institute and Leiden: E. J. Brill, 1975.

5. An inventory of some sundials in Turkey is given in A.S. Ünver, "Sur les candrans solaires horizontaux et verticaux en Turquie," ***Archives Internationales d'Histoire des Sciences,*** 28-29 (1945), pp. 254-266. See now the paper by W. Meyer presented at this Symposium, and also R. Rohr, "Astrolabium und Sonnenuhr in Dienste der Moschee," ***Centaurus,*** 18 (1973), pp. 44-56.

6. On the use of the quadrant by Ottoman astronomers see, for example, J. Würschmidt, "Die Schriften Gedosis über die Hohenparallelen und über die Sinustafel," *Sitzungsberichte der phys. -med. Sozietät zu Erlangen,* 60 (1928), pp. 127-154, and L. Janin and R. Rohr, "Deux astrolabes-quadrants turcs," 19 (1975), pp. 108-124. The astrolabe was much less frequently used by Ottoman astronomers than the almucantar and sinical quadrants.

7. For a brief survey of the definitions found in the Qur'an and the *hadith* see A. J. Wensinck's article *"Mikat" in the Encyclopaedia of Islam* (lst. edition).

8. See J. Frank and E. Wiedemann, "Die Gebetszeiten im Islam," *Sitzungsberichte der phys. -med. Sozietät zu Erlangen,* 58 (1926), pp. 1-32, reprinted in E. Wiedemann, *Aufsätze zur arabischen Wissenschaftsgeschichte,* 2 vols., Hildesheim: Georg Olm, 1970, vol. 2, pp. 757-788.

9. In a paper currently in preparation I demonstrate that the definitions $\Delta s = n$ and $2n$ for the *ʿasr* (and $\Delta s = 1/4\ n$ for the *zuhr* in some Andalusian sources) were intended as practical means of regulating the prayer-times defined originally in terms of the seasonal hours, and also that the Arabic names of the prayers are the same as the names of the corresponding seasonal hours in pre-Islamic Arabic.

10. See, for example, M. L. Davidian, "al-Biruni on the Time of Day from Shadow Lengths," *Journal of the American Oriental Society,* 80 (1960), pp. 330-335.

11. Since the Islamic day begins at sunset, the motivation behind this convention is clear; however, the details of its introduction and adoption all over the Ottoman Empire are obscure. The convention is, of course, not convenient when one is measuring time with mechanical clocks: in order to keep time in the Turkish convention, clocks must be adjusted to 12 o'clock each day at sunset. See further J. Würschmidt, "Die Zeitrechnung im Osmanischen Reiche," *Deutsche Optische Wochenscrift,* 1917, pp. 88-100, and Kurz, *op. cit.* (note 4), pp. 83-84.

12. On Qadi Zade see Suter, *op. cit.* (note 1), no. 430.

13. On °Ali Qushji see Suter, *op. cit.* (note 1), no. 438, and now M. Cunbur, *Ali Kuşçu Bibliyografyasi,* Ankara: Başbakanlık Basımevi, 1974.

14. On al-Khalili and Ibn al-Shatir see my articles in the *Dictionary of Scientific Biography.* See also note 2 above.

15. Shaykh Vefa is not mentioned in the modern sources on Islamic science. This biographical information comes from MS Cairo Dar al-Kutub K 4037 of an anonymous commentary on his *Ruzname.*

16. For example, MSS Istanbul Nurosmaniye 2914,1 and Kandilli Observatory 496, and Paris B.N. turque 186, 187, 188, and 194.

17. G. H. Velschius, *Commentarius in Ruzname Naurus,* Augusta Vindelicorum, 1676.

18. On Muhammad ibn Katib Sinan al-Qunawi (from Konya) see Suter, *op. cit.* (note 1), no. 455. Suter writes of him as *wazir,* but in the astronomical works of his which I have studied he is identified as a *muwaqqit.* He is possibly the son of another Ottoman astronomer Yusuf ibn Khidrbeg (Suter, no. 443) known as Sinan Pasha, who was a *wazir* of Sultan Muhammad II.

19. MSS Istanbul Topkapı T 1618 (Ahmad III 3499) and T 2932 (Hazine 1760) and Aya Sofia 2590.

20. An analysis of these tables is contained in D. A. King, "al-Khalili's Auxiliary Tables for Solving Problems of Spherical Astronomy," *Journal of History of Astronomy,* 4 (1973), pp. 99-110.

21. MS Istanbul Hamidiye 1453.

22. MSS Istanbul Topkapı T 3046 (Ahmad III 3515) and Aya Sofia 2710.

23. MS Kandilli Observatory 208. The tables, which were identified only during the Exhibition of Scientific Manuscripts and Instruments held at Kandilli Observatory during this Symposium, are contained on fols. 96r-100v.

On Taqi l-Din see, for example, Suter, *op. cit.* (note 1), no. 471; and A. S. Ünver, *Istanbul Rasathanesi,* Türk Tarih Kurumu Yayınlarından, VII. Seri, Sayı 54, Ankara, 1969, in which various important papers by S. Tekeli are also listed.

24. *Cf.* E. S. Kennedy, "A Survey of Islamic Astronomical Tables," *Transactions of the American Philosophical Society,* N. S., 46:2 (1956), p. 161.

25. For example, MSS Istanbul Aşir Ef. 470 and Kandilli Observatory 440.

26. M. D'Ohsson, *Tableau Général de l'Empire Othoman, Paris,* 1787, vol. I, facing p. 192.

27. See the references cited in note 2, especially the third article.

28. No survey has ever been made of the *qiblas* of the medieval mosques in Turkey. Such a survey would be of great interest to the history of Ottoman science (let alone to the history of Ottoman architecture!). Some of the mosques would be found to be oriented in the directions which can be computed from the medieval geographical coordinates. Others would be found to be oriented in directions whose explanation would be much more of a challenge. For an introduction to qibla determinations in medieval Islam see the article *Kibla* in the *Encyclopaedia of Islam* (2nd. edition).

29. M. Bagvi, *Determination of the Direction of Qibla and the Islamic Timings,* Lahore, 1972, pp. 54-55.

30. On the *zahwe, Arabic dahwa or duha,* see my article "A Fourteenth Century Tunisian Sundial for Regulating the Times of Muslim Prayer," in Y. Maeyama and W. G. Salzer, eds., *Prismata: Festschrift für Willi Hartner,* Wiesbaden: Franz Steiner, 1977, esp. pp. 189, 194, and 196.

31. MS Istanbul University Library T 1824, 1.

32. MS Istanbul Kandilli Observatory 196.

33. I have no biographical information on Salih Efendi save that he was also known as *Mi°mâri,* "the architect." His tables are extant in, for example, MSS Istanbul Aşir Ef. 224 and Kandilli Observatory 219.

34. These tables are extant in, for example, MSS Istanbul Bağdatlı Vehbi Ef. 990 and Kandilli Observatory 220.

35. MS Oxford Bodleian arab. e. 93.

36. MS Istanbul Hasan Hüsnü Paşa 1286. These tables were identified only during the Exhibition of Astronomical Manuscripts held at the Süleymaniye Library during the Symposium.

The plates which originally appeared on pp. 256 to 269 have been rearranged in their proper order in this reprint.

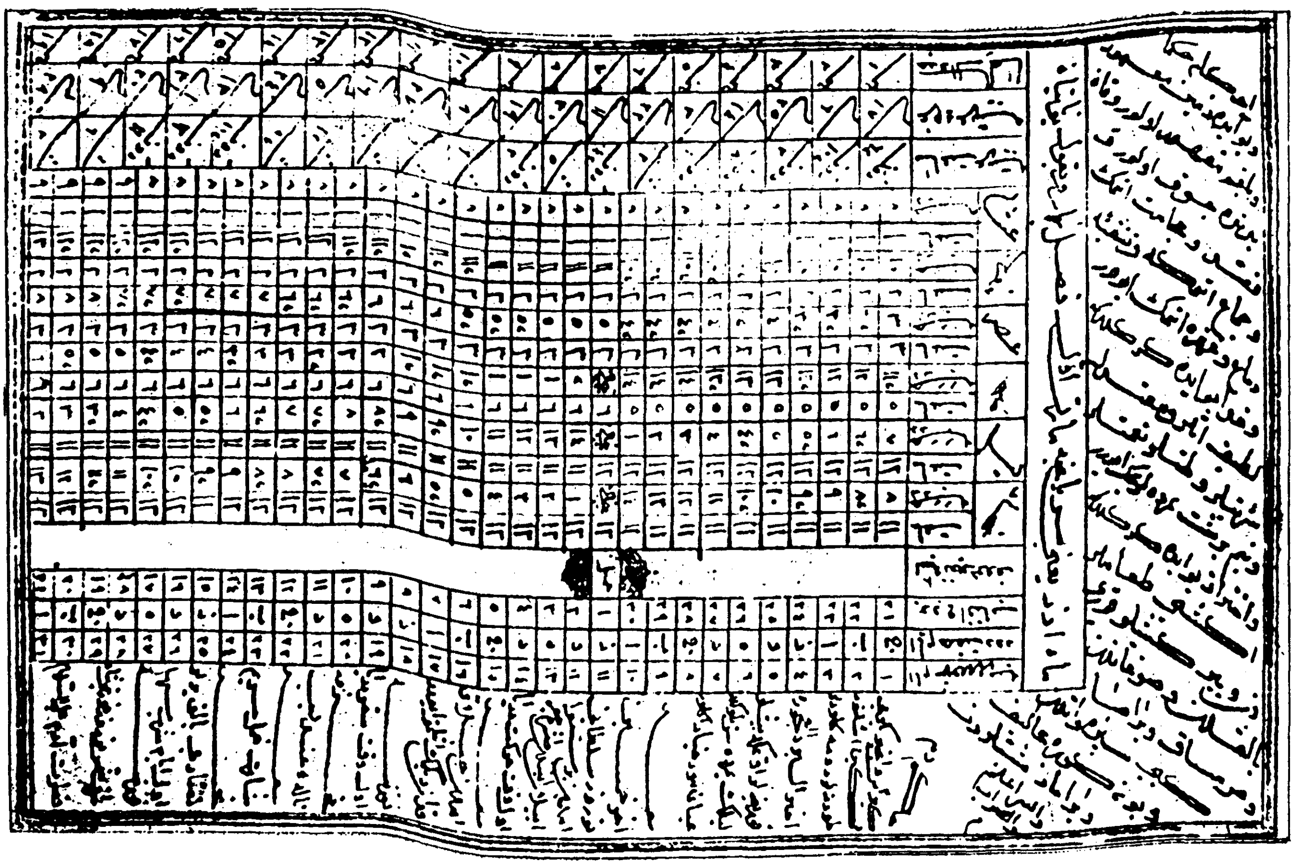


Plate 1: Extract from MS Istanbul Kandilli Observatory 496 of the *Ruzname* of Shaykh Vefa. The extract shows the times of prayer for each day of the month of Adhar (March).

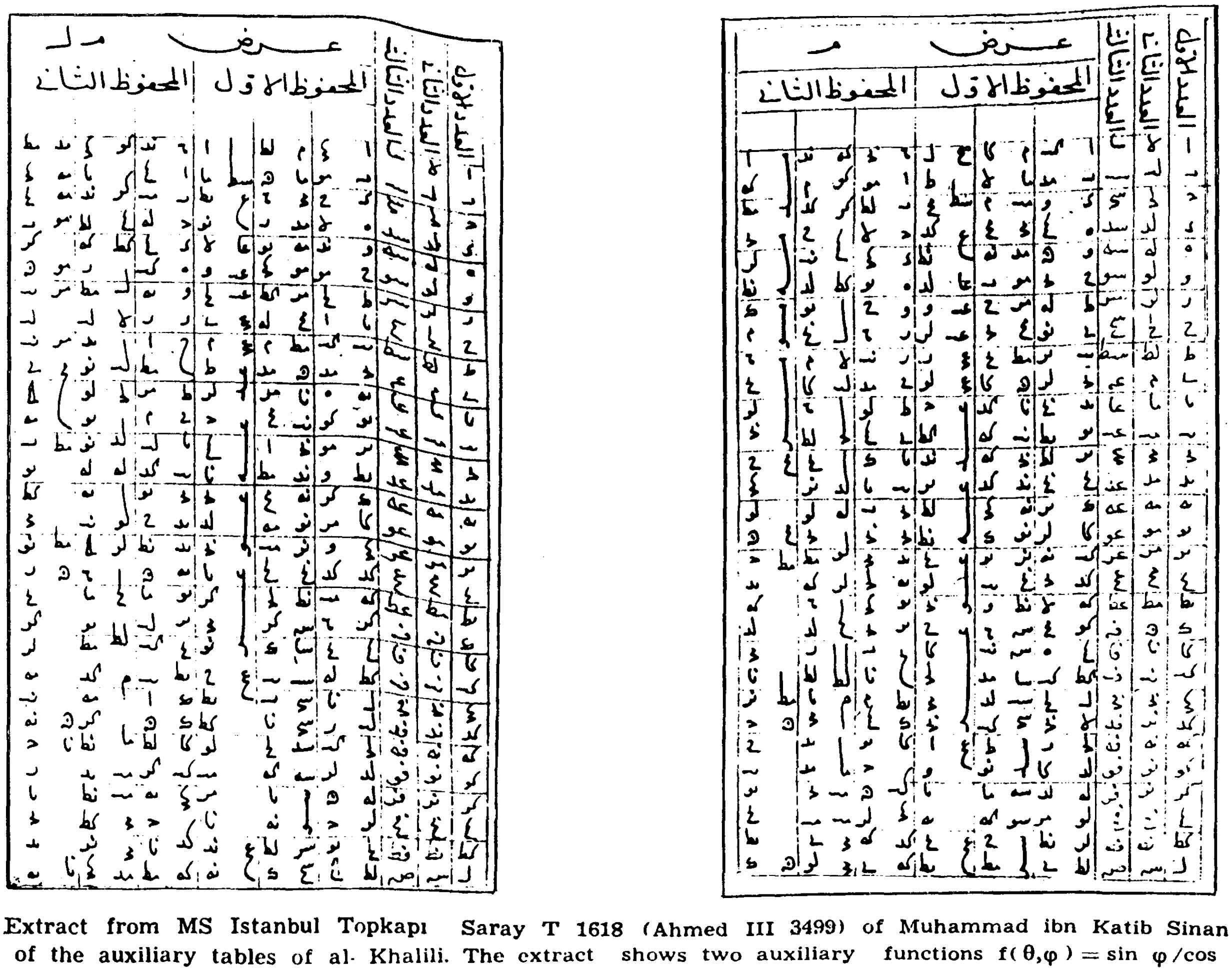

عرض مل
المحفوظ الثاني
المحفوظ الاول
عرض م

Plate 2: Extract from MS Istanbul Topkapı Saray T 1618 (Ahmed III 3499) of Muhammad ibn Katib Sinan's recension of the auxiliary tables of al- Khalili. The extract shows two auxiliary functions f(θ,φ) = sin φ/cos θ and g(θ,φ) = sin θ tan φ computed for each degree of θ for latitudes φ = 40° and 40;30°.

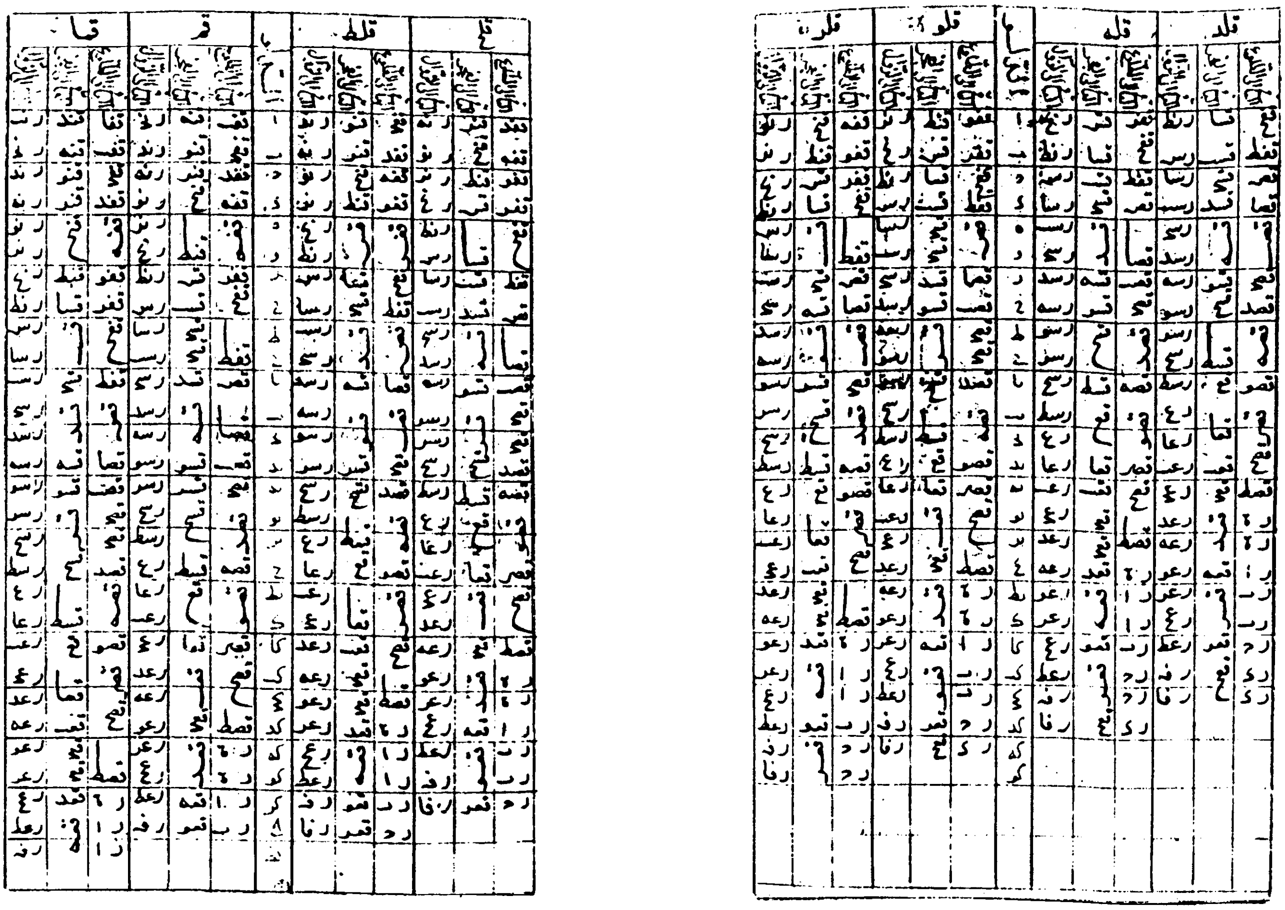


Plate 3: Extract from MS Istanbul Topkapı Saray T 3046 (Ahmad III 3515) of Muhammad ibn Katib Sinan's tables for timekeeping by the stars. The extract shows the time of night for each degree of the ascensions of culminating stars from 134° to 141°, entered horizontally, for each degree of solar longitude in Aquarius, entered vertically.

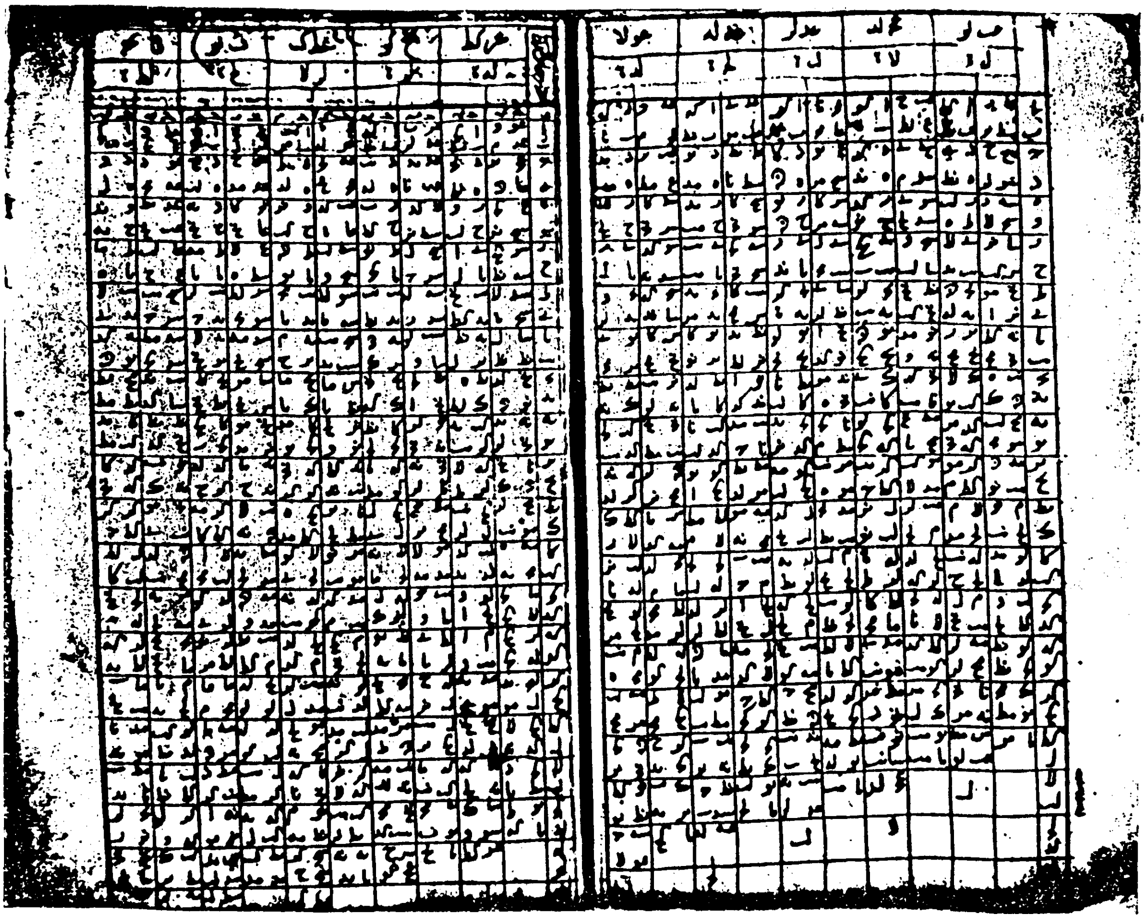


Plate 4: Extract from MS Istanbul Kandilli Observatory 208 of Taqi l-Din's *taylasan* table for Istanbul. The extract shows the hour-angle and time since sunrise for each degree of instantaneous solar altitude, entered vertically, for each degree of solar meridian altitude from 30° to 39°, entered horizontally.

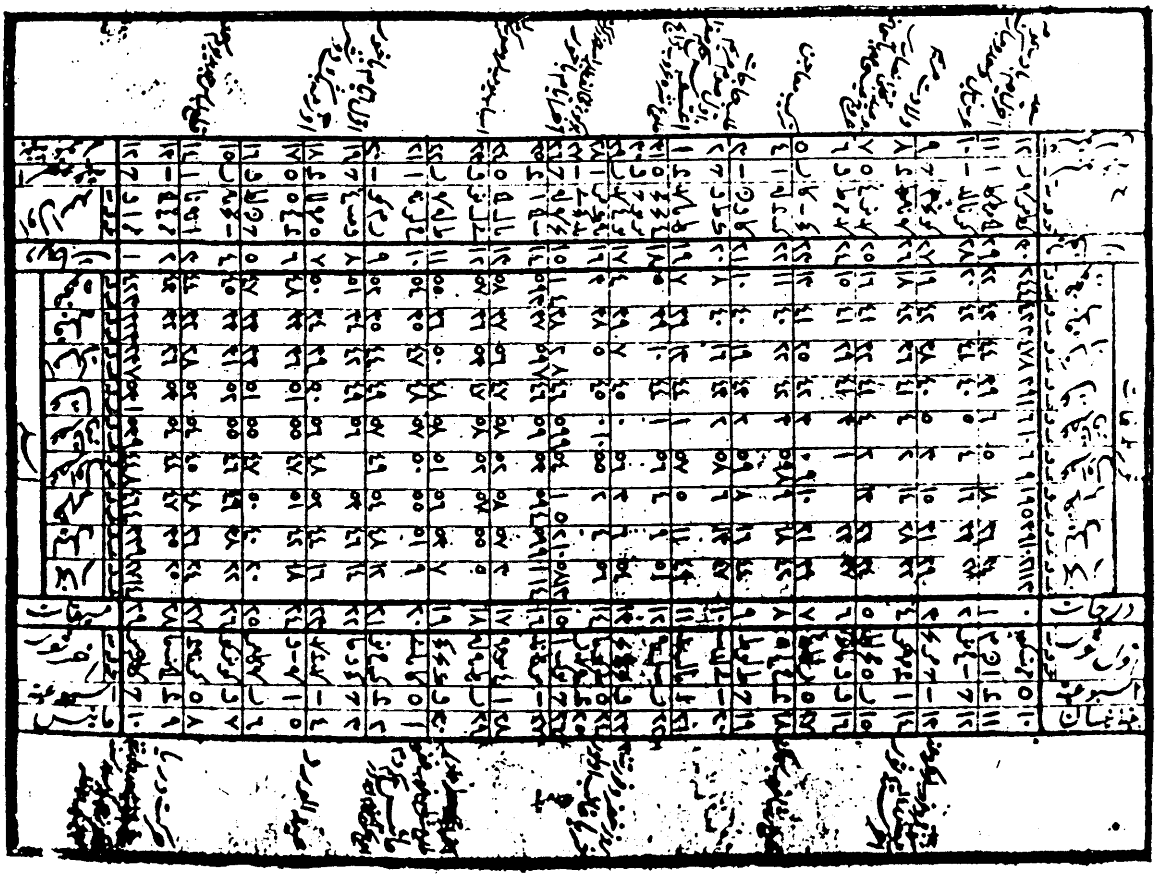
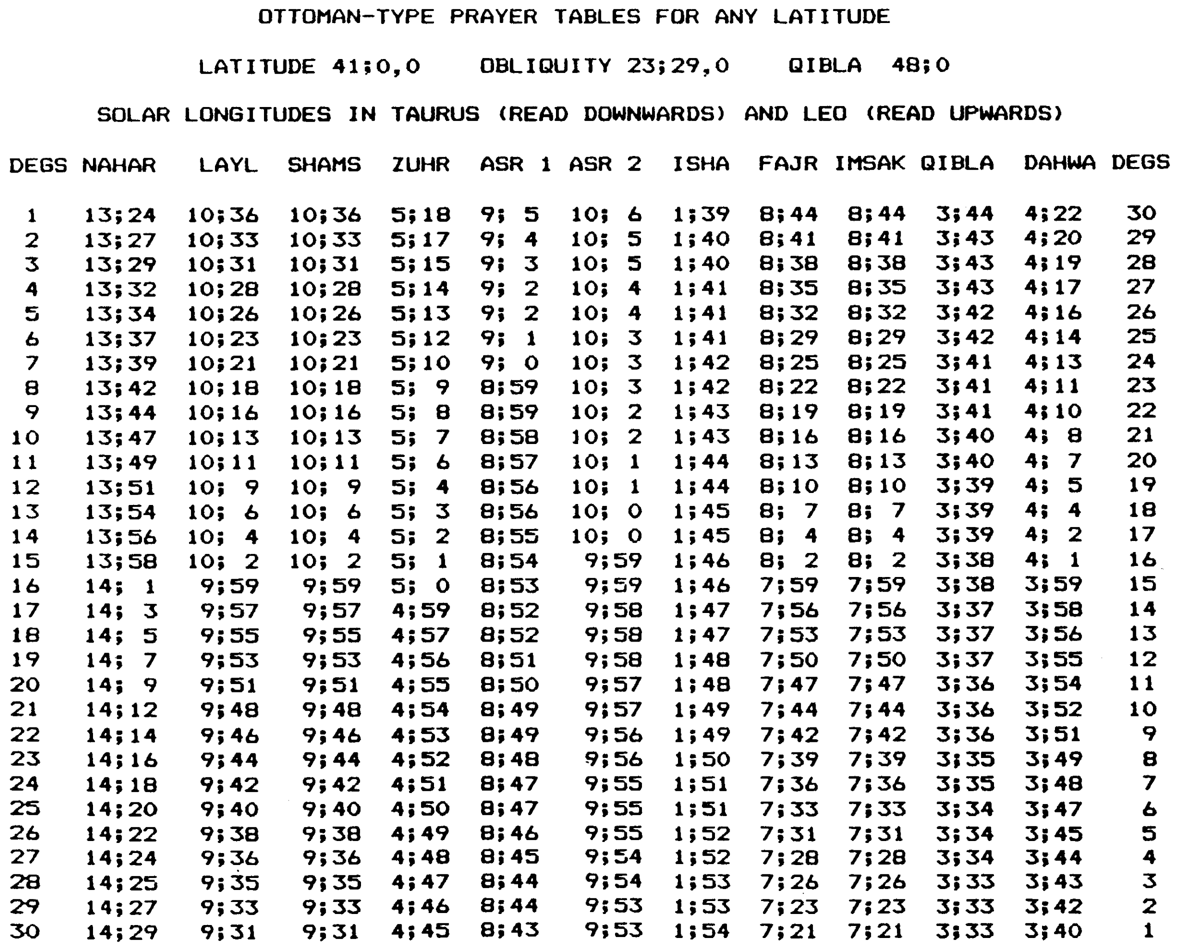


Plate 5: Extract from MS Istanbul Kandilli Observatory 440,2 of the *Ruz-name* of Darendeli. The extract shows the prayer-times for each degree of solar longitude in Taurus.



OTTOMAN-TYPE PRAYER TABLES FOR ANY LATITUDE

LATITUDE 41;0,0 OBLIQUITY 23;29,0 QIBLA 48;0

SOLAR LONGITUDES IN TAURUS (READ DOWNWARDS) AND LEO (READ UPWARDS)

DEGS	NAHAR	LAYL	SHAMS	ZUHR	ASR 1	ASR 2	ISHA	FAJR	IMSAK	QIBLA	DAHWA	DEGS
1	13;24	10;36	10;36	5;18	9; 5	10; 6	1;39	8;44	8;44	3;44	4;22	30
2	13;27	10;33	10;33	5;17	9; 4	10; 5	1;40	8;41	8;41	3;43	4;20	29
3	13;29	10;31	10;31	5;15	9; 3	10; 5	1;40	8;38	8;38	3;43	4;19	28
4	13;32	10;28	10;28	5;14	9; 2	10; 4	1;41	8;35	8;35	3;43	4;17	27
5	13;34	10;26	10;26	5;13	9; 2	10; 4	1;41	8;32	8;32	3;42	4;16	26
6	13;37	10;23	10;23	5;12	9; 1	10; 3	1;41	8;29	8;29	3;42	4;14	25
7	13;39	10;21	10;21	5;10	9; 0	10; 3	1;42	8;25	8;25	3;41	4;13	24
8	13;42	10;18	10;18	5; 9	8;59	10; 3	1;42	8;22	8;22	3;41	4;11	23
9	13;44	10;16	10;16	5; 8	8;59	10; 2	1;43	8;19	8;19	3;41	4;10	22
10	13;47	10;13	10;13	5; 7	8;58	10; 2	1;43	8;16	8;16	3;40	4; 8	21
11	13;49	10;11	10;11	5; 6	8;57	10; 1	1;44	8;13	8;13	3;40	4; 7	20
12	13;51	10; 9	10; 9	5; 4	8;56	10; 1	1;44	8;10	8;10	3;39	4; 5	19
13	13;54	10; 6	10; 6	5; 3	8;56	10; 0	1;45	8; 7	8; 7	3;39	4; 4	18
14	13;56	10; 4	10; 4	5; 2	8;55	10; 0	1;45	8; 4	8; 4	3;39	4; 2	17
15	13;58	10; 2	10; 2	5; 1	8;54	9;59	1;46	8; 2	8; 2	3;38	4; 1	16
16	14; 1	9;59	9;59	5; 0	8;53	9;59	1;46	7;59	7;59	3;38	3;59	15
17	14; 3	9;57	9;57	4;59	8;52	9;58	1;47	7;56	7;56	3;37	3;58	14
18	14; 5	9;55	9;55	4;57	8;52	9;58	1;47	7;53	7;53	3;37	3;56	13
19	14; 7	9;53	9;53	4;56	8;51	9;58	1;48	7;50	7;50	3;37	3;55	12
20	14; 9	9;51	9;51	4;55	8;50	9;57	1;48	7;47	7;47	3;36	3;54	11
21	14;12	9;48	9;48	4;54	8;49	9;57	1;49	7;44	7;44	3;36	3;52	10
22	14;14	9;46	9;46	4;53	8;49	9;56	1;49	7;42	7;42	3;36	3;51	9
23	14;16	9;44	9;44	4;52	8;48	9;56	1;50	7;39	7;39	3;35	3;49	8
24	14;18	9;42	9;42	4;51	8;47	9;55	1;51	7;36	7;36	3;35	3;48	7
25	14;20	9;40	9;40	4;50	8;47	9;55	1;51	7;33	7;33	3;34	3;47	6
26	14;22	9;38	9;38	4;49	8;46	9;55	1;52	7;31	7;31	3;34	3;45	5
27	14;24	9;36	9;36	4;48	8;45	9;54	1;52	7;28	7;28	3;34	3;44	4
28	14;25	9;35	9;35	4;47	8;44	9;54	1;53	7;26	7;26	3;33	3;43	3
29	14;27	9;33	9;33	4;46	8;44	9;53	1;53	7;23	7;23	3;33	3;42	2
30	14;29	9;31	9;31	4;45	8;43	9;53	1;54	7;21	7;21	3;33	3;40	1

Plate 6: Computer printout of the prayer-tables of Darendeli, based on the accurate formulae and Darendeli's parameters.

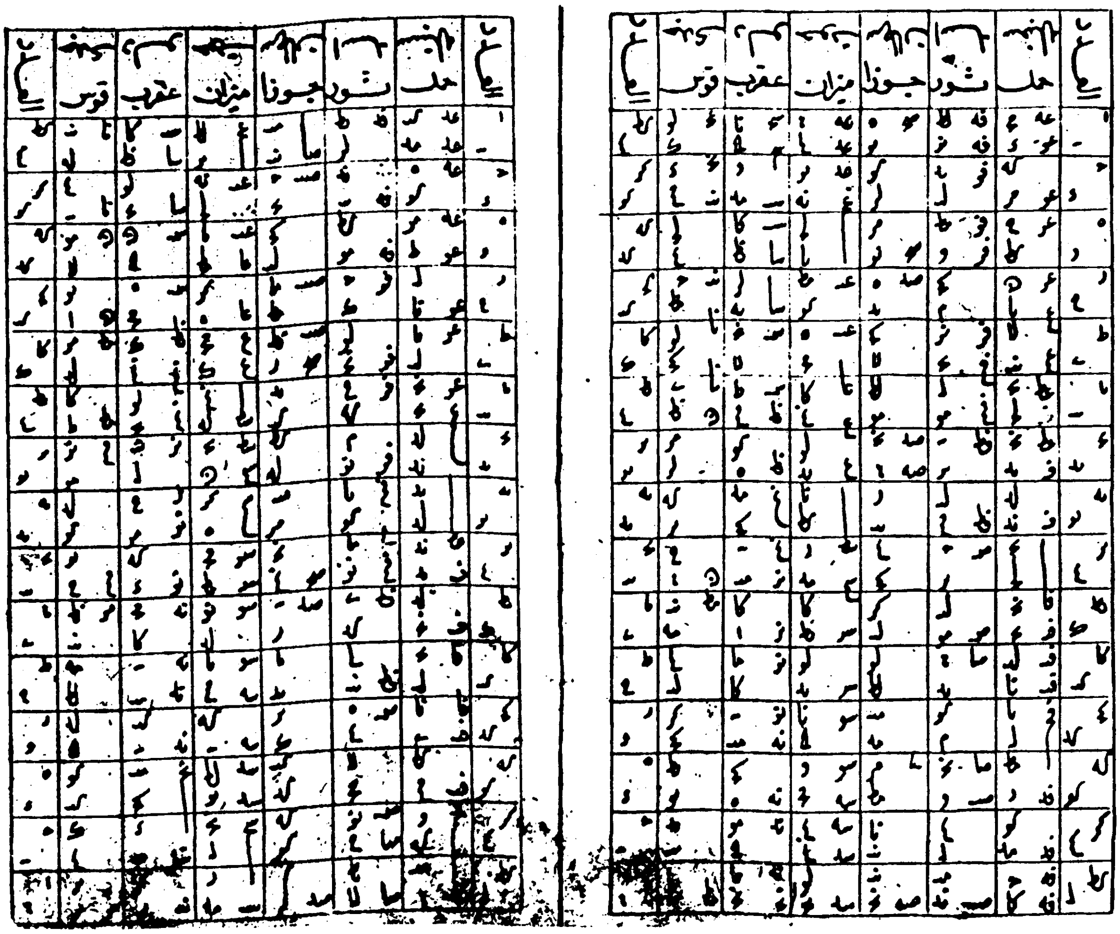


Plate 7: Extract from MS Istanbul Kandilli Observatory 196 of Ahmad Efendi's hour-angle tables for Istanbul. The extract shows the hour-angle for each degree of solar longitude for solar altitude 11° (right hand page) and 12° (left hand page).

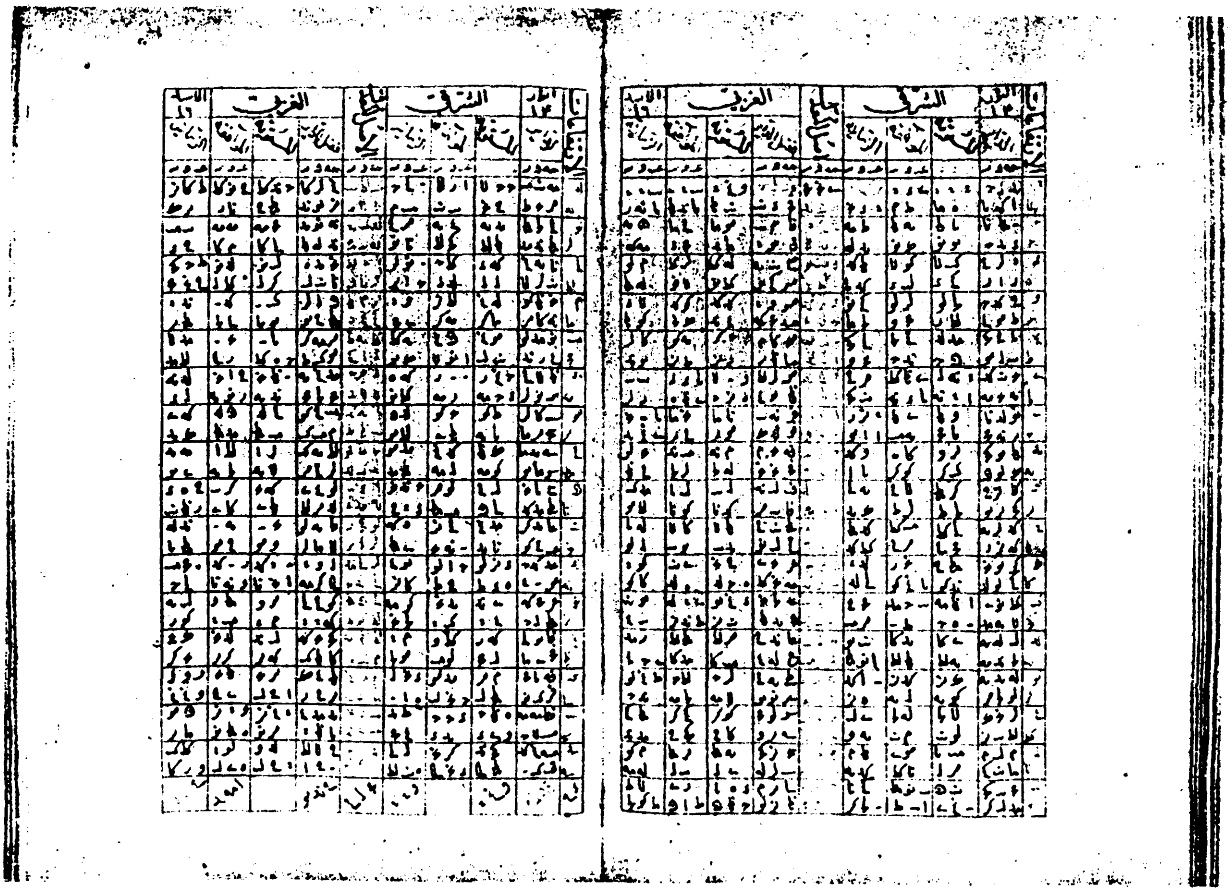


Plate 8: An extract from MS Princeton Yahuda 353 of Salih Efendi's tables for timekeeping. The extract shows the time in various systems for each degree of solar altitude, entered vertically, from 0° to 65° for the solar position Taurus 14°.

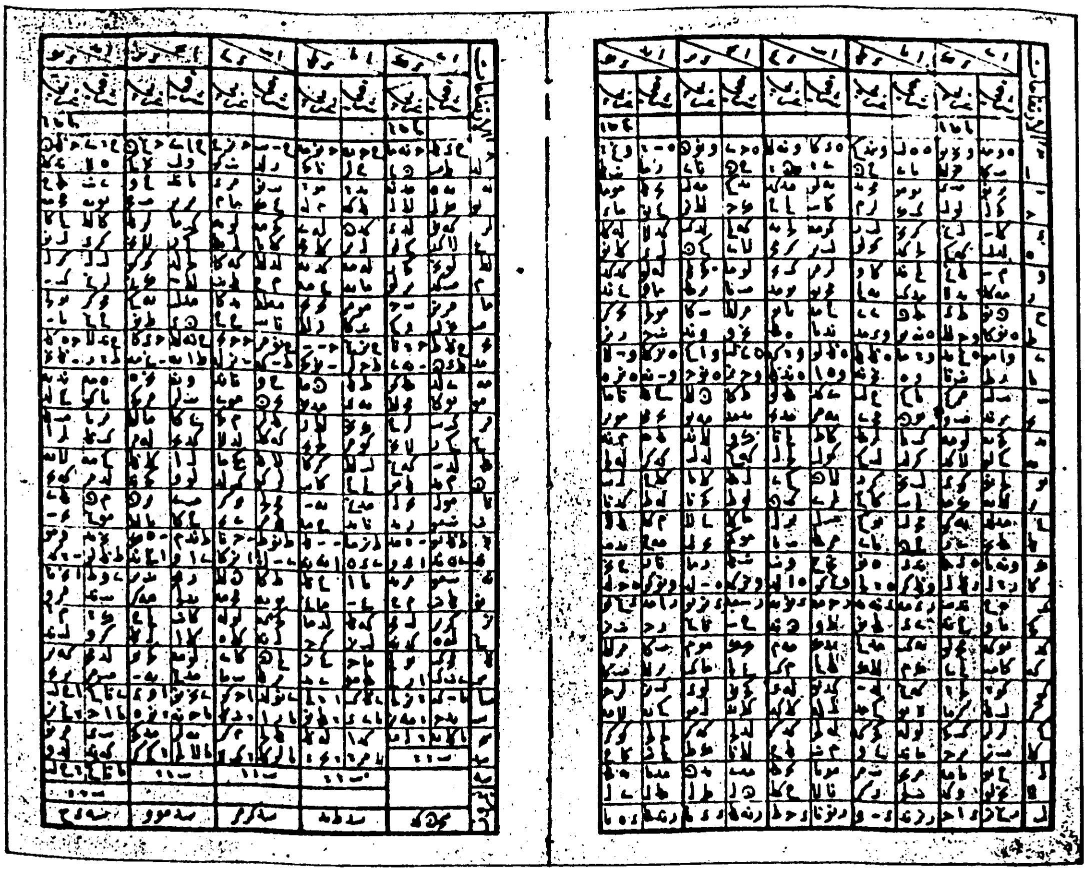
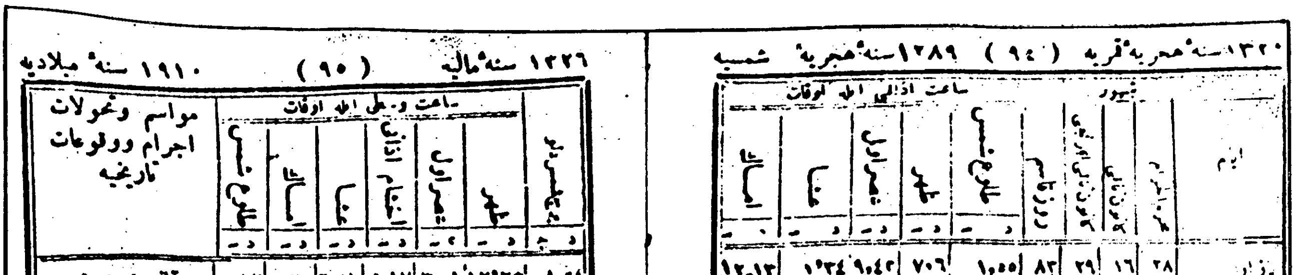


Plate 9: An extract from MS Istanbul Kandilli Observatory 440 of Jihangiri's abridgement of Salih Efendi's tables. The extract shows the two times of day for each degree of solar longitude from 40° to 44°, entered horizontally, when the sun has a given altitude, entered vertically.



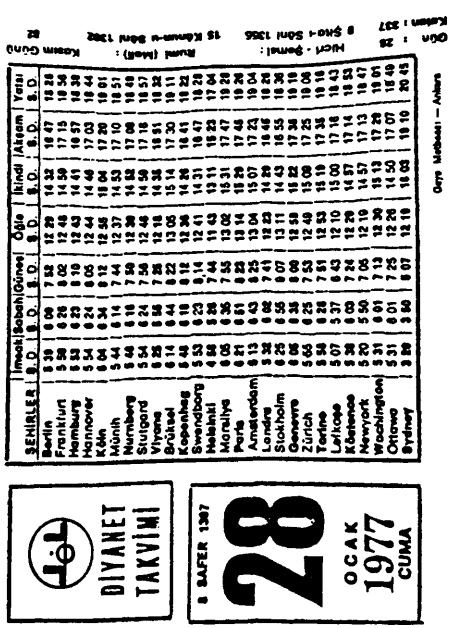
١٣٢٠ سنهٔ هجريهٔ قمريه (٩٤) ١٢٨٩ سنهٔ هجريهٔ شمسيه

ايام	شهور				ساعت اذانى ايله اوقات				
	[illegible]	[illegible]	[illegible]	[illegible]	طلوع شمس	ظهر	عصر اول	عشا	امساك
بازار	٢٨	١٦	٢٩	٨٣	١٫٥٥	٧٫٠٦	٩٫٤٣	١°٣٤	١٢٫١٣
بازارايرتسى	٢٩	١٧	٣٠	٨٤	٥٢	٥	٤٣	٣٤	١١
صالى	٣٠	١٨	٣١	٨٥	٥١	٤	٤٣	٣٤	٩
چهارشنبه	١	١٩	١	٨٦	٤٨	٣	٤٣	٣٤	٧
پنجشنبه	٢	٢٠	٢	٨٧	٤٦	٢	٤٣	٣٤	٥
جمعه	٣	٢١	٣	٨٨	٤٣	١٫٠٠	٤٣	٣٤	٣
جمعه ايرتسى	٤	٢٢	٤	٨٩	٤٠	٠٫٥٩	٤٢	٣٣	١
بازار	[illegible]	٢٣	٥	٩٠	٣٨	٥٨	٤٢	٣٣	١١٫٠٩
بازارايرتسى	٦	٢٤	٦	٩١	٣٦	٥٧	٤٢	٣٣	٥٦
صالى	٧	٢٥	٧	٩٢	٣٣	٥٥	٤٢	٣٣	٥٤
چهارشنبه	٨	٢٦	٨	٩٣	٣١	٥٤	٤١	٣٢	٥٢
پنجشنبه	٩	٢٧	٩	٩٤	٢٩	٥٣	٤١	٣٢	٤٩
جمعه	١٠	٢٨	١٠	٩٥	٢٨	٥٢	٤١	٣٢	٤٧
[illegible] ايرتسى	١١	٢٩	١١	٩٦	٢٥	٥١	٤١	٣٢	٤٥
[illegible]	١٢	٣٠	١٢	٩٧	٢٠	٥٠	٤٠	٣٢	٤٢
[illegible]	١٣	٢١	١٢	٩١	٢٠١ ١٠	٦٫٤٩	٩٫٤٠	١٣٣٢	١١°٤٠

١٣٢٦ سنهٔ ماليه (٩٥) ١٩١٠ سنهٔ ميلاديه

ساعت وسطى ايله اوقات							مواسم وتحولات اجرام ووقوعات تاريخيه
[illegible]	ظهر	عصر اول	اكشام اذانى	عشا	امساك	طلوع شمس	
٨٫٢٤	١٢°٢٣	٣٫٠١	٥٫١٧	٦٫٥١	٥٫٣٠	٧٫١٢	قاندون نورتنمسى
٩٫٢٥	٢٣	١	١٨	٥٢	٢٩	١١	آخر اربعين اجتماع نيرين
١١٫٢٦	٢٣	٢	١٩	٥٣	٢٨	١٠	اول خمسين
١١٫٢٧	٢٤	٤	٢١	٥٥	٢٨	٩	قياط الرحمى مطر الحليم
١٢°٢٨	٢٤	٥	٢٢	٥٦	٢٧	٨	
١٣٫٢٩	٢٤	٦	٢٣	٥٧	٢٥	٧	
١٤°٣٠	٢٤	٦	٢٤	٥٨	٢٥	٦	ازدواج حيوانات
١٥٫٣١	٢٤	٧	٢٥	٥٫١	٢٤	٤	
١٦°٣١	٢٤	٩	٢٧	٧°	٢٣	٤	غايت ارتفاع قمر تربيع اول
١٧٫٣٢	٢٤	١٠	٢٨	١	٢٢	٣	[illegible]
١٨٫٣٣	٢٤	١٠	١٩	١	٢١	١	
١٩٫٣٤	٢٤	١١	٣٠	٢	١٩	٧٫٠٠	
٢٠°٣٤	٢٤	١٢	٣١	٣	١٧	٦٫٥٩	
٢١ ٣٥	٢٤	١٤	٣٣	٥	١٨	٥٨	
٢٢ ٣٦	٢٤	١٠	٣٤	٦	١٦	٥٦	استقبال نيرين
٢٣ ٣٦	٢٤	١٦ ٣	٣٦ ٥	٨ ٦	١١ ٥	٥٥ ٦	

Plate 10: An extract from a Turkish almanac for the year 1910, displaying the prayer-times for each day in both *ezanî* (r. h. s.) and *vasati* (l. h. s.) systems.



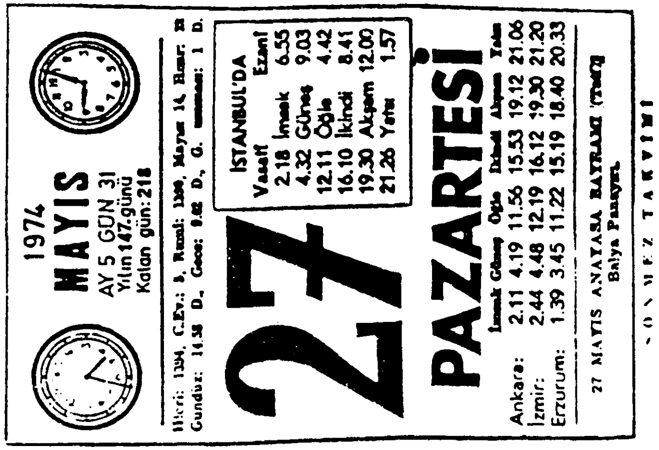
8 SAFER 1397

28

OCAK 1977 CUMA

DİYANET TAKVİMİ

Plate 12: A leaf from a modern Turkish almanac prepared at Kandilli Observatory showing the prayer-times on September 28, 1977, in various cities around the world, calculated with an electronic computer.



Hicri: 1394, C.Evv.: 5, Rumi: 1390, Mayıs 14, Hızır: 22

Gündüz: 14.58 D., Gece: 9.02 D., G. uzaması: 1 D.

1974 MAYIS AY 5 GÜN 31 Yılın 147.günü Kalan gün: 218

27

İSTANBUL'DA

	Vasati	Ezani
İmsak	2.18	6.55
Güneş	4.32	9.03
Öğle	12.11	4.42
İkindi	16.10	8.41
Akşam	19.30	12.00
Yatsı	21.26	1.57

PAZARTESİ

	İmsak	Güneş	Öğle	İkindi	Akşam	Yatsı
Ankara:	2.11	4.19	11.56	15.53	19.12	21.06
İzmir:	2.44	4.48	12.19	16.12	19.30	21.20
Erzurum:	1.39	3.45	11.22	15.19	18.40	20.33

27 MAYIS ANAYASA BAYRAMI (Tatil)
Balya Panayırı

DİYANET TAKVİMİ

Plate 11: A leaf from a modern Turkish almanac, showing the prayer-times for Istanbul, Ankara, Izmir, and Erzurum in both *vasati* and *ezani* systems for May 27, 1974.

AL-KHALĪLĪ'S *QIBLA* TABLE*

The *qibla*, or direction of Mecca, defines the direction of prayer in Islam.[1] The determination of the *qibla* for a given locality is a mathematical problem of some complexity which was of considerable interest to medieval Muslim scholars. They developed numerous alternative exact solutions to the *qibla* problem, using either spherical trigonometry or orthogonal projections,[2] and some also relied on approximate solutions.[3] It

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* My research in Egypt during the year 1972–73 was supported by the Smithsonian Institution and the National Science Foundation, Washington, D.C., and outside Egypt by a grant from the Penrose Fund of the American Philosophical Society. This support is gratefully acknowledged. Microfilms of al-Khalīlī's *qibla* table were kindly provided by the Bibliothèque Nationale, Paris, and the Staatsbibliothek, Berlin. The table was recomputed at the Yale Computer Center, using a grant made available by the Yale University Department of Near Eastern Languages and Literatures. This paper is dedicated to the memory of Dr. Karl Schoy (1877–1925) who, through his numerous valuable publications on different aspects of Islamic astronomy and mathematics (see J. Ruska, "Carl Schoy," *Isis* 9 [1927]: 83–95), was indirectly one of my teachers. Schoy's studies, which were based on a wide variety of manuscript sources, and included the first serious investigations of Islamic spherical astronomy, trigonometry, sundial theory, and the determination of the *qibla*, well merit republication.

[1] See further *Encyclopaedia of Islam*, 1st ed., s.v. "Ḳibla."

[2] On the analemma construction of Ḥabash (ca. A.D. 850) see Y. Id and E. S. Kennedy, "A Letter of al-Bīrūnī: Ḥabash al-Ḥāsib's Analemma for the Qibla," *Historia Mathematica* 1 (1974): 3–11 and on the method of al-Nayrīzī (ca. A.D. 900) involving spherical trigonometry see K. Schoy, "Abhandlung von al-Faḍl b. Ḥātim al-Nayrīzī über die Richtung der Qibla," *Sitzungsberichte der math.-phys. Klasse der Bayerischen Akademie der Wissenschaften zu München*, 1922, pp.55–68. For the procedures described by Ibn Yūnus (ca. A.D. 1000) involving projection methods see K. Schoy, *Gnomonik der Araber* in *Die Geschichte der Zeitmessung und der Uhren*, ed. E. von Bassermann-Jordan, vol. 1F (Berlin and Leipzig, 1923), pp. 35–42; D. A. King, "The Astronomical Works of Ibn Yūnus" (Ph.D. diss., Yale University, 1972), pp. 256–66, and section 5 below; and on the geometrical construction of Ibn al-Haytham (ca. A.D. 1025) see K. Schoy, "Abhandlung des al-Ḥasan ibn al-Ḥasan ibn al-Haiṯam (Alhazen) über die Bestimmung der Richtung des Qibla," *Zeitschrift der Deutschen Morgenländischen Gesellschaft* 75 (1921): 242–53 and Appendix A below. For the various procedures of al-Bīrūnī (ca. A.D. 1025) see idem, *Die trigonometrischen Lehren des persischen Astronomen Abu l-Raiḥān Muḥ. Ibn Aḥmad al-Bīrūnī dargestellt nach al-Qānūn al-Masʿūdī* (Hannover, 1927), pp. 70–71; J. Ali, *The Determination of the Coordinates of Cities: al-Bīrūnī's Taḥdīd al-amākin* (Beirut, 1966), pp. 241–63; and the commentary in E. S. Kennedy, *A Commentary Upon Bīrūnī's Kitāb Taḥdīd al-Amākin* (Beirut, 1973). On the method of Ulugh Beg (ca. A.D. 1440) see L.-A. Sédillot, *Prolégomènes des tables astronomiques d'Oulog-Beg*, 2 vols. (Paris, 1847–53), 2:120–22, and K. Schoy, "Mittagslinie und Qibla," *Zeitschrift der Gesellschaft für Erdkunde zu Berlin*, 1915 pp. 574–76.

[3] On approximate *qibla* methods in the treatises of al-Battānī (ca. A.D. 900) and al-Jaghmīnī (ca. A.D.

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has not been previously noted that certain Muslim scholars compiled tables for finding the *qibla*, and the purpose of this paper is to describe such a table prepared by the fourteenth century Syrian astronomer al-Khalīlī, which is the most sophisticated trigonometric table known to me from the entire medieval period. I also mention some simpler *qibla* tables based on approximate formulae.

The problem of determining the *qibla* for a given locality can be understood from figure 1, which shows a locality X and Mecca M on the terrestrial surface. The north pole P and the equator AB are also shown. The inclination of the great circle arc XM to PXA, the meridian at X, defines the direction of Mecca. If the latitude and longitude of X are denoted by φ and L, and the coordinates of Mecca by φ_M and L_M, then

$$XA = \varphi; \qquad MB = \varphi_M; \qquad \text{and} \qquad AB = L_M - L = \Delta L.$$

The angle MXA, called in Arabic *inḥirāf al-qibla*, is here denoted by q. The formula defining q, expressed in modern notation, is

$$q = \text{arc cot}\left\{\frac{\sin \varphi \cos \Delta L - \cos \varphi \tan \varphi_M}{\sin \Delta L}\right\} \tag{1}$$

which follows by an application of the modern cotangent rule[4] to the spherical triangle PXM. The various exact solutions devised by the Muslim scholars are equivalent to this formula. The standard medieval approximation is equivalent to

$$q = \text{arc tan}\left\{\frac{\sin \Delta L}{\sin \Delta\varphi}\right\} \tag{2}$$

where

$$\Delta\varphi = \varphi - \varphi_M.$$

In section 1 below, I describe al-Khalīlī's *qibla* table and reproduce it in tables 1–8. The error in each entry is also indicated. In section 2, I describe a method for determining the *qibla*, known to al-Khalīlī but originally derived by the thirteenth century Moroccan astronomer al-Marrākushī. In section 3, I describe a procedure for computing *qibla* values by means of another set of mathematical tables compiled by al-Khalīlī and the formulae of al-Marrākushī. I have not been able to establish that the entries in al-Khalīlī's *qibla* table were derived in this way. In section 4, I discuss a particular kind of quadrant invented by the Muslim astronomers, with which al-Khalīlī could find the *qibla* without any computation whatsoever. I have been unable to consult his treatise on the use of this instrument, and so in section 6 I describe a procedure for finding the *qibla* with a quadrant attributed to the fifteenth century Egyptian astronomer Sibṭ al-Māridīnī. This procedure is based on formulae which were originally derived by the tenth century Egyptian astronomer Ibn Yūnus and which are presented in section 5. In Appendix A, I describe the geometrical construction for the *qibla* suggested by the Egyptian scholar Ibn al-Haytham, a late contemporary of Ibn Yūnus. In Appendix B, I present a theoretical construction for the *qibla* using a quadrant and the formulae of al-Marrākushī. Finally, in Appendix C, I discuss certain other Islamic *qibla* tables which are based on approximate formulae.

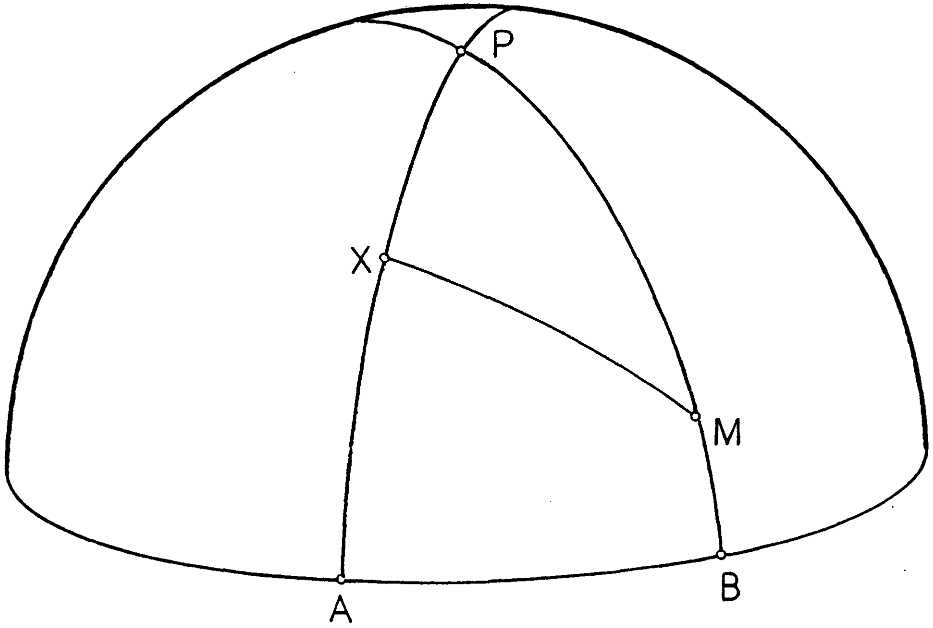

FIG. 1

1. *al-Khalīlī's* Qibla *Table*

Shams al-Dīn Muḥammad ibn Muḥammad al-Khalīlī was a timekeeper (*muwaqqit*) associated with the Umayyad Mosque in Damascus.[5] He lived about the year A.D. 1365 and was a contemporary of the celebrated astronomer Ibn al-Shāṭir.[6] al-Khalīlī's major works were two sets of mathematical tables, containing a total of about 40,000 entries. Each of al-Khalīlī's tables are outstanding examples of the various categories of tables for astronomical timekeeping which were compiled by the medieval Muslim astronomers, and

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1325) see C. A. Nallino, *al-Battani sive Albatenii Opus Astronomicum*, 3 pts. (Milan and Rome, 1899–1907) 1:318–19, 2:xxvii, 3:206–7; and A. Hochheim and G. Rudloff, "Die Astronomie des Maḥmūd ibn Muḥammad ibn 'Omar al-Ǧagmīnī," *Zeitschrift der Deutschen Morgenländischen Gesellschaft* 47 (1893): 271–72. See also Schoy, "Mittagslinie und Qibla," pp. 572–74, and idem, *Gnomonik der Araber*, pp. 35–36; King, "The Astronomical Works of Ibn Yūnus," pp. 265–66; and P. Schmalzl, *Zur Geschichte des Quadranten bei den Arabern* (Munich, 1929), p. 99; and Appendix C below.

[4] On this see, for example, W. M. Smart, *Text-Book on Spherical Astronomy* (Cambridge, 1971), p. 12. The formula for q follows immediately from Ibn al-Haytham's geometrical construction described in Schoy, "Abhandlung des al-Ḥasan ibn al-Ḥasan ibn al-Haiṯam (Alhazen)": see Appendix A below.

[5] On al-Khalīlī see H. Suter, "Die Mathematiker und Astronomen der Araber und ihre Werke," *Abhandlungen zur Geschichte der mathematischen Wissenschaften* 10 (1900), no. 418; C. Brockelmann, *Geschichte der arabischen Litteratur*, 2d ed., 2 vols. (Leiden, 1943–49), 2:156–57; idem, *Geschichte der arabischen Litteratur*, Supplement, 3 vols. (Leiden, 1937–42), 2:157; G. Sarton, *Introduction to the History of Science*, 3 vols. (Baltimore, 1927–48), 3/2: 1526–27 (the discussion of al-Khalīlī's tables is full of errors); and my forthcoming article in the *Dictionary of Scientific Biography*. On his tables see notes 7 and 9 below.

[6] On Ibn al-Shāṭir see E. Wiedemann, *Aufsätze der arabischen Wissenschaftsgeschichte*, 2 vols. (Hildesheim, 1970), 2:729–38; E. S. Kennedy, "A Survey of Islamic Astronomical Tables," *Transactions of the American Philosophical Society*, n.s. 46 (1956), no. 11, and idem, "Late Medieval Planetary Theory," *Isis* 57 (1966): 365–78. The latter article refers to the research of the past twenty years on Ibn al-Shāṭir.

which I have recently analyzed in some detail.[7] Some of his tables are computed for the latitude of Damascus and enable the user to find the time and the direction of the sun from the solar altitude and to find the astronomically defined times of Muslim prayer: these are similar in conception to the tables for timekeeping attributed to the tenth century Egyptian astronomer Ibn Yūnus, computed for the latitude of Cairo-Fustat.[8] Of particular scientific interest are al-Khalīlī's *Universal Tables* of auxiliary functions, which can be used to solve all of the standard problems of spherical astronomy for any latitude.[9] However, his most outstanding achievement in computational mathematics was his table displaying the *qibla* as a function of geographical latitude and longitude.

Two copies of al-Khalīlī's *qibla* table are known to me: MS Paris Bibliothèque Nationale, ar. 2558, fols. 53v–61r, and MS Berlin Ahlwardt 5754 (Wetzstein 1138), fols. 47v–55r.[10] Both of these manuscripts contain other tables by al-Khalīlī for astronomical timekeeping, but they are the only known manuscripts of the Damascus corpus of such tables which contain the *qibla* table. The Paris manuscript is a magnificent example of Arabic penmanship, copied in A.H. 811 (= A.D. 1408) by Aḥmad ibn ʿAlī al-Khabbāz; the Berlin manuscript is also copied in an elegant hand but is undated. The tables are preceded by a short introduction in both sources.

The function tabulated by al-Khalīlī is here denoted by q, although in fact it defines the acute angle which the arc XM makes with the arc PA in figure 1. Values of the function $q(\varphi, L)$ are given to two sexagesimal digits for the domains

$$\varphi = 10°, 11°, \ldots, 56° \quad \text{and} \quad 33;30°$$

and

$$\Delta L = 1°, 2°, \ldots, 60°.$$

The 2,880 entries in the table are expressed in standard Arabic sexagesimal notation.[11] For each value of φ there is a subtable in which L is entered vertically. The double vertical argument runs from 7° to 66° (longitudes west of Mecca, for which the *qibla* is easterly), and from 127° to 68° (longitudes east of Mecca, for which the *qibla* is westerly). These two sets of longitudes correspond to a change in ΔL from 60° to 1°, since the longitude of Mecca is taken as

$$L_{\mathrm{M}} = 67;0°.$$

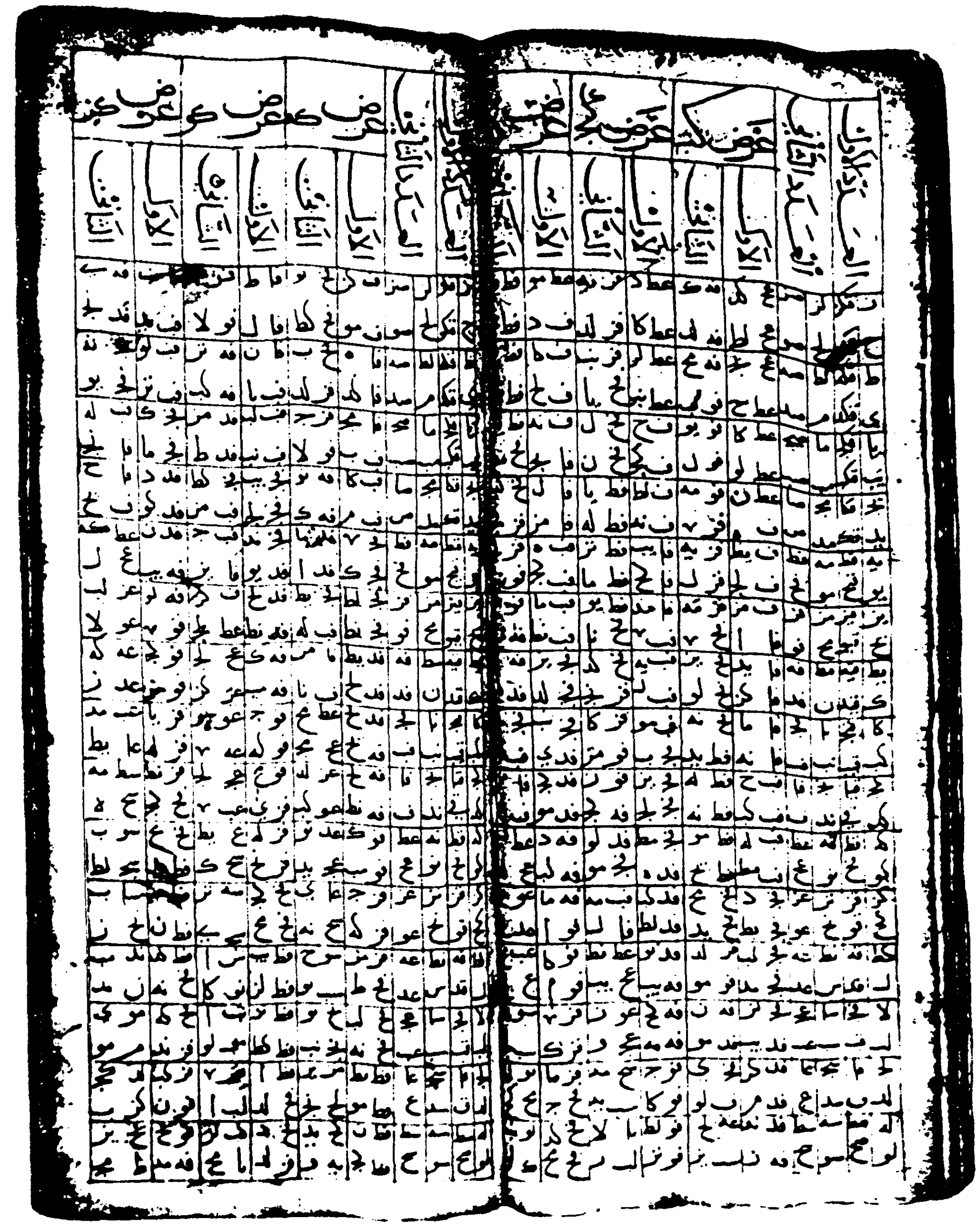

FIG. 2.—Extract from al-Khalīlī's *qibla* table in MS Paris Bibliothèque Nationale ar. 2558. Courtesy of the Keeper of Manuscripts, Bibliothèque Nationale, Paris

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[7] I have prepared two monographs on astronomical timekeeping in medieval Islam (*ʿilm al-mīqāt*) which are currently being submitted for publication. The first describes all known medieval tables for regulating the times of the five daily prayers of Islam, and the second describes a large corpus of medieval tables for reckoning time by the sun and stars. In particular, the first paper contains an analysis of al-Khalīlī's prayer-tables for Damascus which were used there until the turn of the present century, and the second contains an analysis of al-Khalīlī's various tables for timekeeping.

[8] For a detailed analysis of the tables attributed to Ibn Yūnus see D. A. King, "Ibn Yūnus' *Very Useful Tables* for Reckoning Time by the Sun," *Archive for History of Exact Sciences* 10 (1973): 342–94. I have discussed the problems of their attribution in my study of medieval Islamic prayer-tables (see n. 7 above).

[9] An analysis of these tables is contained in D. A. King, "Al-Khalīlī's Auxiliary Tables for Solving Problems of Spherical Astronomy," *Journal for the History of Astronomy* 4 (1973): 99–110. Earlier examples of such auxiliary tables are discussed in R. A. K. Irani, "The *Jadwal al-Taqwīm* of Ḥabash al-Ḥāsib" (Master's diss., American University of Beirut, 1956), and in C. Jensen, "Abū Naṣr's Approach to Spherical Trigonometry as Developed in his Treatise 'The Table of Minutes'," *Centaurus* 16 (1971): 1–19. Several later examples as well as another set attributed to al-Khalīlī are analyzed in my study of medieval Islamic tables for timekeeping (see n. 7 above and n. 25 below).

[10] The manuscripts are described in MacG. de Slane, *Catalogue des manuscrits arabes* (Paris, 1883–95), p. 460, and in W. Ahlwardt, *Die Handschriften-Verzeichnisse der Königlichen Bibliothek zu Berlin*. Vol. 5. *Verzeichniss der arabischen Handschriften* (Berlin, 1893), p. 207. [*Note added in proof:* MS Paris Bibliothèque Nationale ar. 2560,11 is probably another copy of al-Khalīlī's *qibla* table. It is catalogued as "tableaux qui donnent l'argument des latitudes, depuis le 10^{e} degré jusqu'au 56^{e}."]

[11] On this notation see R. A. K. Irani, "Arabic Numeral Forms," *Centaurus* 4 (1955): 1–12.

The underlying latitude of Mecca[12] is

$$\varphi_M = 21;30°.$$

The entries for each latitude are arranged in two columns of thirty entries, and two facing pages of both manuscripts serve six consecutive latitudes. Figure 2 shows an extract from the Paris manuscript, for latitudes 22°, 23°, 24°, on the right hand page, and 25°, 26°, 27°, on the left hand page.

Tables 1–8 show the entries in al-Khalīlī's table. In his introduction he mentions that the *qibla* is southerly if the entry in the table is in red ink and northerly if it is in black ink. In tables 1 and 2 all entries are northerly, and in tables 3, 4, and 5 all entries above the broken line are northerly. All other entries are southerly. The critical apparatus on page 95 shows the significant variants in the two manuscripts.

Figures 3 and 4 show the behavior of a function $q'(\varphi, \Delta L)$ defining the inclination of the *qibla* to the south-point, for

$$\varphi = 10°, 15°, \ldots, 55°$$

and

$$\Delta L = 5°, 10°, 20°, \ldots, 60°,$$

respectively. al-Khalīlī's function q is related to this as follows:

$$q = \begin{cases} 180° - q' & (q' \geqslant 90°) \\ q' & (q' \leqslant 90°). \end{cases}$$

The curve for the limiting case

$$\varphi = \varphi_M = 21;30°$$

is also shown in figure 3, although values for this latitude are not tabulated by al-Khalīlī. For the critical case

$$\Delta L = 0°$$

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[12] The values 21;30° and 67;0° for the latitude and longitude of Mecca occur in a list of geographical coordinates preserved in MS Paris Bibliothèque Nationale ar. 2558, fol. 51v, preceding al-Khalīlī's *qibla* table. The geographical coordinates are attributed to al-Mizzī (see n. 34 below) and the *qibla* values which are given for each city are attributed to al-Khalīlī. The *qibla* values are slightly less accurate than those which can be derived from al-Khalīlī's *qibla* table by interpolation, which suggests that they were computed before the *qibla* table. Further details are contained in my study of medieval prayer-tables (see n. 7 above).

al-Khalīlī's coordinates for Mecca are attested in various later Syrian and Egyptian sources (e.g., MS Paris Bibliothèque Nationale ar. 2520, fols. 162v and 82v, respectively). Most Islamic sources give 21;0° or 21;40° for the latitude of Mecca, but Ibn al-Shāṭir used 21;20° (MS Vatican Borg. ar. 105, fol. 41r). The modern value is 21;26°.

Several Islamic coordinate lists give the *qibla* for each locality. The list in MS Bodleian Seld. Arch. A30, fols. 155r–157v appended to the *zīj* of Ibn al-Shāṭir (which may not be attributable to him) contains *qibla* values for about 175 localities. The list of about 100 *qibla* values published in Schoy, "Abhandlung von al-Faḍl b. Ḥātim al-Nayrīzī," pp. 67–68 is taken from MS Gotha Forschungsbibliothek A1403, where it is attributed to the Damascus astronomer Ibn Zurayq, a successor of Ibn al-Shāṭir and al-Khalīlī (see Suter, "Die Mathematiker und Astronomen der Araber," no. 426).

The geographical coordinates given in numerous Islamic astronomical handbooks have been data processed and subjected to a preliminary analysis: see F. I. Haddad and E. S. Kennedy, "Geographical Tables of Medieval Islam," to appear in *al-Abḥāth* for further details. On medieval measurements of geographical coordinates see K. Schoy, "Längenbestimmung und Zentralmeridian bei den älteren Völkern," *Mitteilungen der kaiserserlich-königlichen geographischen Gesellschaft* 12 (1915): 27–62, and Ali, *The Determination of the Coordinates of Cities.*

TABLE 1

AL-KHALĪLĪ'S *QIBLA* TABLE

		10°	11°	12°	13°	14°	15°
7°	127°	70;50°	71;26 +1	72; 2 +2	72;37 +1	73;11 -2	73;49 -1
8	126	70;49 +1	71;26 +2	72; 3 +2	72;39 +1	73;15 -1	73;54 -1
9	125	70;47 +1	71;25 +2	72; 4 +2	72;41 +1	73;18 -2	73;59
10	124	70;44 +1	71;23 +1	72; 3 +1	72;42	73;21 -1	74; 4
11	123	70;41 +1	71;21 +1	72; 2 +1	72;43	73;23 -2	74; 8
12	122	70;37	71;19 +1	72; 1 +1	72;43	73;24 -3	74;11
13	121	70;32	71;17 +2	71;59	72;43 -1	73;25 -4	74;13 -1
14	120	70;27 -1	71;13 +1	71;57	72;43	73;26 -4	74;16 -1
15	119	70;23 +1	71;10 +2	71;55	72;42	73;28 -3	74;20 +1
16	118	70;18 +2	71; 5 +1	71;53 +1	72;41	73;29 -2	74;23 +2
17	117	70;11 +1	71; 0 +1	71;50 +1	72;40 +1	73;29 -2	74;24 +1
18	116	70; 3	70;54 +1	71;46 +1	72;38 +1	73;28 -2	74;24
19	115	69;55	70;47	71;41 +1	72;35 +1	73;27 -2	74;24 -1
20	114	69;46	70;40	71;35	72;30 -1	73;25 -2	74;23 -2
21	113	69;37	70;33	71;30 +1	72;26 -1	73;23 -2	74;22 -3
22	112	69;27	70;25 +1	71;24 +1	72;23 +1	73;20 -2	74;21 -3
23	111	69;16	70;15	71;17 +2	72;16 -1	73;17 -2	74;20 -2
24	110	69; 4	70; 4 -1	71; 7	72; 9 -1	73;12 -3	74;19 -1
25	109	68;51	69;53 -1	70;58	72; 3 -1	73; 8 -2	74;17 -1
26	108	68;37	69;42	70;48	71;55 -1	73; 3 -2	74;13 -1
27	107	68;23 +1	69;29	70;37 -1	71;47	72;57 -1	74;11 +1
28	106	68; 7 +1	69;15	70;26	71;39 +1	72;51	74; 5 -1
29	105	67;50 +1	69; 0	70;13	71;27	72;44 +1	73;59 -1
30	104	67;31 +1	68;43 -1	69;59	71;18 +2	72;35 +1	73;53 -1
31	103	67;10	68;26	69;43 -1	71; 3	72;24	73;44 -2
32	102	66;48 -1	68; 8 +1	69;27	70;49	72;12 -1	73;35 -3
33	101	66;26	67;48 +1	69;11 +2	70;35 +1	72; 0	73;26 -2
34	100	66; 2 +1	67;24	68;51 +1	70;17	71;48 +1	73;17 -1
35	99	65;35 +1	67; 1 +1	68;31 +2	69;59	71;34 +2	73; 7 +1
36	98	65; 6	66;35	68; 8 +2	69;40 +1	71;18 +3	72;54 +1
37	97	64;37 +2	66; 6 -1	67;42 +1	69;18	70;57	72;39 +1
38	96	64; 4 +2	65;38 +1	67;14	68;56 +2	70;36 -1	72;22
39	95	63;27	65; 5 +1	66;45	68;30 +2	70;14 -1	72; 3 -1
40	94	62;48 -1	64;31 +2	66;13	68; 1 +1	69;51	71;43 -1
41	93	62; 5 -3	63;54 +3	65;39	67;29 -1	69;25 +1	71;22
42	92	61;20 -3	63;11 +1	65; 1	66;55 -1	68;56 +1	70;58 +1
43	91	60;33 -2	62;24 -2	64;20 -1	66;18 -2	68;23	70;30
44	90	59;42 -2	61;35 -3	63;36	65;38 -2	67;47 -1	69;59 -1
45	89	58;48	60;44 -1	62;49 +1	64;55 -1	67; 7 -2	69;26
46	88	57;50 +3	59;48	61;57 +2	64; 8	66;23 -3	68;49
47	87	56;45 +4	58;46	60;58 +1	63;13 -2	65;35 -3	68; 8
48	86	55;30	57;36 -3	59;51 -3	62;13 -3	64;43 -3	67;20 -2
49	85	54;10 -3	56;21 -4	58;39 -6	61; 6 -6	63;48 +1	66;28 -2
50	84	52;46 -2	55; 1 -3	57;24 -4	59;57 -4	62;43 +1	65;31 -2
51	83	51;16 -1	53;34 -2	56; 3 -1	58;41 -1	61;28 -2	64;26 -2
52	82	49;36 -1	51;56 -3	54;30 -1	57;13 -2	60; 7 -2	63;12 -3
53	81	47;49 +2	50;11 -1	52;49	55;38 +1	58;37 -1	61;51 -1
54	80	45;50 +2	48;15	50;54 -1	53;49 +1	56;55 -1	60;17 -1
55	79	43;37 -1	46; 4 -2	48;47 -2	51;44 -2	55; 0	58;27 -4
56	78	41;18 +2	43;45	46;29	49;30	52;50 +1	56;25 -3
57	77	38;42 +1	41; 6 -3	43;52 -1	46;55 -1	50;17 -3	54; 8 +1
58	76	35;53 +1	38;15 -2	40;56 -4	44; 0 -4	47;26 -5	51;21 -3
59	75	32;49 +1	35;10 +2	37;50 +2	40;53 +3	44;20 +2	48;17 +2
60	74	29;29	31;44 +2	34;17 +2	37;13 +1	40;38 +1	44;35 -1
61	73	25;54	27;59 +1	30;22 +1	33; 9 +1	36;26	40;19 -1
62	72	22; 4	23;54 -1	26; 4	28;38 +1	31;40 -1	35;25 +1
63	71	17;58 -2	19;32 -2	21;24 -1	23;37 -1	26;18 -3	29;46 +4
64	70	13;41 -2	14;57 +1	16;24 -1	18; 9 -3	20;21 -4	23;14 +1
65	69	9;15	10; 5 -1	11; 7	12;23	13;57	16; 1 +2
66	68	4;42 +3	5; 8 +3	5;40 +3	6;21 +5	7;10 +5	8;18 +8

TABLE 2

AL-KHALĪLĪ'S *QIBLA* TABLE

		16°	17°	18°	19°	20°	21°
7°	127°	74;27 -1	75; 6	75;46 +2	76;22 -1	77; 4 +1	77;43
8	126	74;33 -1	75;14 +1	75;54 +1	76;34	77;16 +1	77;57 +1
9	125	74;40	75;22 +1	76; 3 +1	76;45 +1	77;28 +1	78;10 +1
10	124	74;45	75;30 +2	76;11	76;55 +1	77;38	78;23 +1
11	123	74;50 -1	75;36 +1	76;19	77; 5 +1	77;49	78;35
12	122	74;55 -1	75;42 +1	76;26 -1	77;13 -1	78; 0 -1	78;48
13	121	75; 0 -1	75;49 +2	76;34 -1	77;22 -1	78;11 -1	79; 0 -1
14	120	75; 5	75;54 +1	76;41 -2	77;32	78;22	79;12 -1
15	119	75; 9	76; 0 +1	76;50	77;40 -1	78;34 +1	79;25 -1
16	118	75;13	76; 6 +2	76;59 +2	77;50	78;45 +1	79;38
17	117	75;17 +1	76;10 +1	77; 5 +1	77;59 +1	78;55 +1	79;50
18	116	75;20 +1	76;14	77;10	78; 8 +1	79; 6 +2	80; 2
19	115	75;22 +1	76;19 +1	77;16	78;16 +1	79;14	80;14
20	114	75;23	76;21 -1	77;21 -1	78;23	79;23 -1	80;25 -1
21	113	75;25	76;24 -2	77;28	78;31 +1	79;33	80;37
22	112	75;26	76;28 -1	77;34 +1	78;37 -1	79;44 +1	80;48 -1
23	111	75;26 -1	76;30 -2	77;37 -1	78;44 -1	79;52	80;59 -1
24	110	75;26 -1	76;32 -2	77;40 -2	78;51	80; 0 -1	81;10 -2
25	109	75;26	76;34 -2	77;45 -1	78;57 -1	80; 8 -2	81;22 -1
26	108	75;25	76;36 -1	77;49 -1	79; 4	80;17 -2	81;33 -1
27	107	75;24	76;38	77;52 -1	79;10	80;26 -1	81;44 -1
28	106	75;22 +1	76;40 +2	77;55 -1	79;16 +1	80;37 +2	81;55 -1
29	105	75;19 +1	76;38	77;57 -2	79;21 +1	80;44 +1	82; 6
30	104	75;14 -1	76;36 -1	78; 0	79;25	80;49 -2	82;17
31	103	75;10	76;35	78; 2	79;29	80;57 -1	82;27
32	102	75; 6 +1	76;33	78; 3 +1	79;33	81; 4 -1	82;38
33	101	74;59 +1	76;29 -1	78; 3 +1	79;37	81;11 -1	82;49 +1
34	100	74;51	76;25 -1	78; 3 +1	79;40 +1	81;19 +1	83; 0 +2
35	99	74;43 +1	76;21	78; 2 +2	79;43 +1	81;26 +2	83;10 +2
36	98	74;34 +1	76;16 +1	78; 0 +2	79;44 +1	81;32 +2	83;20 +2
37	97	74;22	76; 9 +1	77;57 +2	79;44	81;37 +2	83;29 +2
38	96	74;11 +2	76; 2 +3	77;52 +1	79;45	81;41 +1	83;38 +2
39	95	73;56	75;53 +3	77;46	79;45 +1	81;46 +2	83;46
40	94	73;38 -2	75;41 +2	77;39 -1	79;44 +1	81;50 +2	83;54 -1
41	93	73;19 -3	75;28 +2	77;31 -1	79;40 -1	81;53 +2	84; 2 -1
42	92	72;59 -4	75;12	77;21 -2	79;37	81;54	84;11 -1
43	91	72;38 -3	74;55	77;10 -3	79;31 -2	81;55 -1	84;19 -1
44	90	72;14 -2	74;36	76;58 -2	79;25 -2	81;56	84;27 -1
45	89	71;48 -1	74;15	76;45 -1	79;17 -3	81;57 +1	84;35
46	88	71;19 +1	73;49 -2	76;29	79; 8 -3	81;55	84;43
47	87	70;44 +1	73;19 -5	76; 9 -1	78;58 -2	81;53	84;50 +1
48	86	70; 3 -1	72;47 -6	75;47	78;45 -1	81;48 -2	84;55 -1
49	85	69;18 -3	72;13 -5	75;19 -2	78;30 -1	81;42 -2	84;59 -2
50	84	68;29 -2	71;34 -4	74;47 -4	78;11 -1	81;35 -2	85; 3 -3
51	83	67;33 -2	70;49 -3	74;12 -5	77;47 -2	81;25 -3	85; 7 -4
52	82	66;31	69;58 -1	73;33 -3	77;19 -3	81;13 -3	85;12 -2
53	81	65;20 +1	68;58	72;50 +1	76;49 -2	80;59 -1	85;15 -1
54	80	63;57 +1	67;46 -2	71;57 +3	76;13 +1	80;41	85;15 -2
55	79	62;16 -4	66;24 -2	70;49	75;28 +2	80;18 +1	85;12 -4
56	78	60;29 +1	64;49	69;35 +4	74;31	79;48 +2	85; 9 -4
57	77	58;17 -2	62;55	68; 3 +6	73;20 -3	79; 8	85; 6 -1
58	76	55;40 -7	60;36 -4	66;10 +6	71;55 -4	78;16 -3	84;59 +1
59	75	52;47	57;52 -4	63;50 +5	70;12 -1	77;10 -6	84;46 +3
60	74	49;15 +1	54;37 -1	60;58 +6	67;52 -6	75;48 -4	84;20 -1
61	73	45; 0	50;31 -4	57;10 -3	64;53 -9	73;48 -11	83;40 -8
62	72	39;56 -1	45;30 -4	52;29 -3	60;54 -12	71;10 -10	82;44 -14
63	71	33;55 -1	39;24 +2	46;23 -3	55;38 -1	67;29 +2	81;30 -8
64	70	26;47 -4	31;38 -4	38;15 -9	47;44 -9	61;14 -7	79;19
65	69	18;40 -1	22;29 +3	28; 1 +5	36;31 -2	50;55	74;41 +4
66	68	9;44 +7	11;48 +7	15; 6 +13	20;36 +12	32; 7 +21	62;17 +41

TABLE 3

AL-KHALĪLĪ'S *QIBLA* TABLE

		22°	23°	24°	25°	26°	27°
7°	127°	78;24 +1	79; 4	79;46 +1	80;27	81; 9	81;52 +1
8	126	78;39 +1	79;21 +1	80; 4 +1	80;46	81;30 +1	82;14 +1
9	125	78;53	79;37 +1	80;21	81; 5	81;50	82;36 +1
10	124	79; 8 +1	79;52	80;38	81;24	82;11 +1	82;57
11	123	79;21 -1	80; 8	80;55 -1	81;43	82;32 +1	83;20 +1
12	122	79;36	80;23 -1	81;13	82; 2	82;52	83;41
13	121	79;50	80;39 -1	81;30 -1	82;21	83;12	84; 4
14	120	80; 5 +1	80;55 -1	81;47 -1	82;40	83;33	84;26
15	119	80;19 +1	81;12	82; 5 -1	83; 0	83;54	84;50 +1
16	118	80;33 +1	81;28	82;23	83;20 +1	84;16 +1	85;12
17	117	80;47 +1	81;44 +1	82;41	83;39	84;38 +1	85;36 +1
18	116	81; 1 +1	82; 0 +1	82;59 +1	83;59 +1	84;59 +1	86; 0 +2
19	115	81;14	82;15	83;17 +1	84;19 +1	85;20	86;23 +1
20	114	81;27 -1	82;30 -1	83;34	84;38	85;42	86;47 +1
21	113	81;41 -1	82;46 -1	83;52	84;58	86; 3 -1	87;11 +1
22	112	81;55	83; 2	84;10	85;18	86;25 -1	87;35 +1
23	111	82; 8 -1	83;17 -1	84;28	85;38	86;48 -1	87;59
24	110	82;22 -1	83;33 -1	84;46	85;59	87;10 -2	88;23 -2
25	109	82;35 -1	83;49 -1	85; 4 -1	86;20	87;35	88;48 -2
26	108	82;49 -1	84; 5 -1	85;22 -1	86;42 +1	87;58	89;14 -2
27	107	83; 4 +1	84;22 -1	85;41 -1	87; 3 +1	88;23	89;42 -1
28	106	83;19 +2	84;39	86; 1	87;25 +1	88;48 +1	89;50
29	105	83;32 +1	84;56 +1	86;21	87;47 +1	89;12	89;24 +2
30	104	83;44	85;12	86;40	88; 9	89;37	88;55 +1
31	103	83;57 -1	85;28 -1	87; 0	88;32	89;57 +1	88;25
32	102	84;12 +1	85;45 -1	87;20	88;55	89;29 -1	87;54 -1
33	101	84;27 +2	86; 3	87;41	89;19	89; 1 -1	87;22 -2
34	100	84;40 +1	86;21 +1	88; 3 +1	89;46 +2	88;34	86;50 -2
35	99	84;54 +2	86;39 +1	88;24 +1	89;50 -1	88; 4 -1	86;18 -2
36	98	85; 7 +1	86;57 +2	88;48 +3	89;23 -2	87;34 -1	85;44 -2
37	97	85;20	87;15 +1	89; 9 +1	88;56 -2	87; 3 -1	85;10 -1
38	96	85;34	87;34 +2	89;33 +2	88;29 -1	86;31 -1	84;33 -1
39	95	85;48	87;52 +1	89;56 +1	88; 2	85;57 -2	83;55 -1
40	94	86; 2	88;11	89;42 +1	87;34 +2	85;22 -2	83;16 -1
41	93	86;16 -1	88;30 -1	89;15	87; 3 +2	84;47	82;35
42	92	86;30 -1	88;50 -1	88;49	86;31 +2	84; 9	81;53 +2
43	91	86;45 -1	89;11 -1	88;22 +1	85;56 +1	83;29	81; 8 +3
44	90	87; 0 -1	89;35 +1	87;53 +1	85;20 +1	82;47	80;18 +1
45	89	87;15 -1	89;57	87;23 +1	84;41	82; 3 +1	79;25
46	88	87;30 -1	89;41 +2	86;52 +2	84; 1	81;17 +2	78;30
47	87	87;45 -2	89;16 +2	86;18 +2	83;19	80;27 +3	77;32
48	86	88; 0 -4	88;51 +3	85;42 +3	82;35 +2	79;33 +4	76;31 +2
49	85	88;17 -3	88;24 +4	85; 5 +4	81;47 +3	78;33 +3	75;25 +4
50	84	88;36 -2	87;53 +3	84;22 +3	80;51 +1	77;27 +1	74; 7
51	83	88;55 -1	87;21 +3	83;33	79;48 -4	76;16	72;44 -3
52	82	89;14 -1	86;47 +4	82;42 -2	78;43 -5	75; 0	71;19
53	81	89;35	86; 7 +2	81;47 -2	77;34 -4	73;33 -2	69;45 +3
54	80	89;55 -2	85;23 -1	80;48	76;22 +3	72; 0	68; 5 +11
55	79	89;46 +6	84;36 -1	79;42 +3	74;56 +6	70;19 +5	66; 2 +9
56	78	89;18 +3	83;46 +2	78;24/+4	73;14 +6	68;20 +7	63;39 +2
57	77	88;48 +1	82;45 +2	76;48	71;10 -1	65;57 +3	61; 2
58	76	88;14 -1	81;30 -1	74;58 -3	68;55 +2	63;10 -4	58; 7 +2
59	75	87;34 -4	79;59 -6	72;48 -4	66; 8 -2	60; 1 -4	54;42 +1
60	74	86;46 -8	78;12 -6	70;11 -2	62;56 +4	56;21 -1	50;44
61	73	85;50 -9	76; 7 +5	66;55 +3	58;56 +8	52; 1 +6	46;10 +3
62	72	84;46 -1	73; 6 +6	62;33 +2	53;52 +10	46;36 +5	40;46 +3
63	71	83;10 +5	68;45 +2	56;39 -1	47;17 +5	40; 0 +3	34;23 -1
64	70	80;36 +13	62;15 +3	48;28 -1	38;53 +4	32; 1	27; 2 -2
65	69	75;38 +20	51;31 +9	36;52 +3	28;14 +8	22;37 +4	18;47 +1
66	68	62;17 +24	32; 7 +15	20;36 +9	15; 6 +11	11;48 +6	9;43 +6

TABLE 4

AL-KHALĪLĪ'S *QIBLA* TABLE

		28°	29°	30°	31°	32°	33°
7°	127°	82;34	83;16 -1	84; 0	84;43	85;26	86;11 +1
8	126	82;58 +1	83;41	84;26	85;11 +1	85;56 +1	86;41 +1
9	125	83;22 +2	84; 7 +1	84;53 +1	85;39 +1	86;26 +2	87;11
10	124	83;44	84;33 +2	85;19	86; 7 +1	86;55 +1	87;44 +2
11	123	84; 8	84;57 +1	85;46 +1	86;35 +1	87;25 +1	88;14 +1
12	122	84;32 +1	85;22	86;13 +1	87; 3	87;54	88;45
13	121	84;55	85;48 +1	86;41 +2	87;32	88;24	89;17
14	120	85;20	86;13	87; 7	88; 0 -1	88;55	89;49
15	119	85;45 +1	86;41 +2	87;35	88;29 -1	89;25 -1	89;38
16	118	86; 9	87; 7 +1	88; 3	89; 0	89;57	89; 4 -1
17	117	86;36 +2	87;34 +2	88;34 +3	89;31 +1	89;31	88;30 -1
18	116	87; 0 +1	88; 0	89; 1 +1	89;58 -1	88;57 -1	87;58 +1
19	115	87;25 +1	88;27	89;30	89;28	88;25	87;21 -1
20	114	87;50	88;54 -1	89;59	88;56	87;51 -1	86;47
21	113	88;16	89;22 -1	89;31 +1	88;24	87;18 +1	86;10 -1
22	112	88;43	89;53 +1	89; 0	87;51	86;44 +1	85;33 -1
23	111	89; 9 -1	89;40 +1	88;28	87;19 +1	86; 8 +1	84;57
24	110	89;37 -1	89;10 +1	87;57 +1	86;45 +2	85;31	84;19
25	109	89;55 +1	88;40 +1	87;25 +1	86;10 +1	84;54	83;41 +2
26	108	89;26	88; 9 +1	86;52 +2	85;34 +1	84;19 +3	83; 0 +1
27	107	88;57	87;37 +1	86;16	84;57	83;38 +1	82;18
28	106	88;27	87; 4	85;40 -1	84;20 +1	82;56 -1	81;36
29	105	87;57 +1	86;31	85; 5 -1	83;43 +2	82;19 +3	80;53
30	104	87;26 +1	85;56 -1	84;29	83; 1	81;34	80; 9 +1
31	103	86;54 +1	85;21 -1	83;52 +1	82;21	80;52 +1	79;24 +1
32	102	86;20	84;46	83;12	81;38 -1	80; 7 +1	78;39 +4
33	101	85;45 -1	84; 9 +1	82;30 -1	80;54 -1	79;18 -2	77;48 +2
34	100	85; 9 -2	83;31 +1	81;47 -3	80;10 -1	78;33	76;56
35	99	84;33 -1	82;50	81; 4 -3	79;23 -1	77;41 -2	76; 3 -1
36	98	83;56 -1	82; 8 -1	80;19 -3	78;34 -2	76;49 -3	75; 7 -2
37	97	83;17 -1	81;26	79;33 -2	77;46	75;58 -1	74;11 -2
38	96	82;37	80;39 -2	78;45 -2	76;52 -2	75; 0 -3	73;12 -2
39	95	81;54 -1	79;52 -3	77;55 -1	75;58 -2	74; 2 -3	72;10 -3
40	94	81;10	79; 5 -1	77; 3	75; 3	73; 4 -1	71; 6 -4
41	93	80;24	78;16 +1	76; 8	74; 2 -1	72; 1	70; 0 -3
42	92	79;35	77;22 +1	75;10 +1	73; 1	70;57 +2	68;51 -2
43	91	78;44	76;25 +1	74; 8	71;56 +1	69;47 +2	67;39
44	90	77;50 +1	75;25 +1	73; 3	70;45	68;31	66;22
45	89	76;53 +2	74;21	71;52 -2	69;32 +1	67;14	65; 0
46	88	75;52 +2	73;14 +1	70;38 -3	68;15 +2	65;54 +3	63;35 +1
47	87	74;46 +2	72; 5 +4	69;22 -1	66;51 +1	64;24	62; 5 +2
48	86	73;34 +1	70;47 +4	68; 0 +1	65;24 +2	62;52 +1	60;26 -1
49	85	72;19 +2	69;23 +3	66;32 +2	63;49 +2	61;13 +1	58;42 -2
50	84	70;58 +3	67;52 +2	64;54 +1	62; 7 +2	59;27 +1	56;52 -3
51	83	69;27 +2	66;15 +2	63; 9	60;15 -1	57;31 -1	54;58 -1
52	82	67;49 +2	64;32 +5	61;16 -1	58;18	55;32 +1	52;55 +1
53	81	66; 2 +2	62;35 +4	59;14	56;11	53;21 +1	50;45 +4
54	80	64; 4 +3	60;23	57; 1 +1	53;55 +3	51; 1 +2	48;17 -2
55	79	61;53 +4	58; 2 -1	54;36 +2	51;26 +4	48;31 +4	45;49 +3
56	78	59;25 +4	55;28 +1	51;55 +2	48;42 +3	45;46 +3	43; 5 +2
57	77	56;37 +2	52;34	48;57 +1	45;41	42;46 +1	40; 8
58	76	53;24 -4	49;14 -7	45;39 -2	42;25 -2	39;35 +1	37; 0
59	75	49;50 -5	45;41 -4	42; 3 -3	38;53 -2	36; 4 -3	33;40
60	74	45;54 +1	41;43	38; 7 -2	35; 5	32;26 +1	30; 7 +1
61	73	41;21 +5	37;14 +1	33;49 +1	30;58 +3	28;29 +2	26;21 +2
62	72	36; 5 +4	32;13 +1	29; 5 +2	26;28 +3	24;13	22;21 +1
63	71	30; 6 +2	26;37 -1	23;51 -1	21;37 +1	19;43	18; 7 -1
64	70	23;23	20;30 -3	18;17 -1	16;31 +1	15; 0 -1	13;45 -1
65	69	16; 6 +4	13;57 -3	12;26 +1	11;10 +1	10; 8 +1	9;17 +1
66	68	8;15 +5	6;58 -8	6;20 +4	5;41 +4	5; 9 +3	4;41 +2

TABLE 5

AL-KHALĪLĪ'S *QIBLA* TABLE

		33;30°	34°	35°	36°	37°	38°
7°	127°	86;32	86;54	87;37 -1	88;22	89; 6	89;50
8	126	87; 3	87;27 +2	88;11	88;57 +1	89;42 +1	89;53
9	125	87;33 -1	87;58 +1	88;44	89;31	89;42	88;55 -1
10	124	88; 5 -1	88;30	89;19 +1	89;54	89; 6	88;18
11	123	88;37 -1	89; 3 +1	89;52	89;19	88;29	87;40
12	122	89; 9 -1	89;36	89;34 +1	88;43	87;52	87; 2 +1
13	121	89;41 -2	89;52 +1	88;59	88; 7 +1	87;15 +1	86;23 +1
14	120	89;45 +1	89;18 +1	88;24 +1	87;30 +1	86;38 +3	85;43 +1
15	119	89;12 +1	88;44 +1	87;48 +1	86;52	85;58 +2	85; 2 +1
16	118	88;37	88; 8	87;11	86;15 +1	85;18 +1	84;21 +1
17	117	88; 2	87;34 +1	86;35 +1	85;36 +1	84;37	83;39
18	116	87;27	86;57	85;58 +2	84;56	83;56	82;56
19	115	86;53 +2	86;22 +2	85;19 +1	84;16	83;14	82;13
20	114	86;17 +2	85;44 +1	84;39	83;36 +1	82;32	81;30 +1
21	113	85;40 +2	85; 6 +1	83;59	82;54	81;49	80;45 +1
22	112	85; 2 +2	84;27 +1	83;19	82;12	81; 5	79;59
23	111	84;23 +1	83;48 +1	82;38 +1	81;29 +1	80;20	79;12
24	110	83;43	83; 9 +2	81;56 +1	80;45 +1	79;36 +2	78;26 +1
25	109	82;59 -3	82;29 +4	81;12	80; 0 +1	78;49 +2	77;37 +1
26	108	82;17 -4	81;46 +3	80;27 -1	79;13	78; 1 +2	76;48 +2
27	107	81;37 -2	81; 0	79;42 -1	78;26	77; 9 -1	75;56 +1
28	106	80;55 -1	80;14 -2	78;56	77;37	76;18 -2	75; 2 -1
29	105	80; 9 -2	79;28 -2	78; 9 +1	76;47	75;27 -1	74; 7 -2
30	104	79;30 +4	78;42 -1	77;18 -1	75;56	74;34 -1	73;11 -3
31	103	78;38 -1	77;55	76;26 -3	75; 2 -2	73;40	72;16 -2
32	102	77;48 -2	77; 6 +1	75;37	74; 8 -1	72;44 +1	71;20 +1
33	101	76;55 -5	76;12 -2	74;41 -2	73;11 -2	71;44 -1	70;19
34	100	76; 2 -6	75;18 -3	73;46 -1	72;13 -2	70;43 -2	69;16 -1
35	99	75;10 -5	74;22 -4	72;49 -1	71;14 -2	69;41 -2	68;12 -1
36	98	74;16 -3	73;24 -5	71;49 -1	70;12 -2	68;38 -1	67; 6 -1
37	97	73;22 +1	72;25 -5	70;48	69; 8 -1	67;32 -1	65;58 -1
38	96	72;22 +1	71;25 -3	69;43 -1	68; 2 -1	66;22 -2	64;47 -1
39	95	71;20 +2	70;23 -1	68;36 -1	66;52 -2	65;12 -1	63;33 -2
40	94	70;14 +1	69;18 +1	67;26 -2	65;40 -2	63;59	62;18 -1
41	93	69; 5	68; 8 +1	66;15	64;26 -1	62;41 -1	61; 0
42	92	67;52 -1	66;55 +1	64;59	63; 9 +1	61;20 -1	59;38
43	91	66;36 -2	65;39 +1	63;39 -1	61;45 -2	59;56 -2	58;13
44	90	65;16 -3	64;17	62;16 -1	60;19 -2	58;29 -1	56;45 +1
45	89	63;53 -3	62;52	60;50	58;50 -2	56;58 -1	55;12
46	88	62;27 -1	61;25 +2	59;18	57;13	55;24	53;36 +1
47	87	60;55	59;50 +1	57;42 +1	55;41 +1	53;46 +2	51;56 +2
48	86	59;20 +3	58;10	56; 2 +2	53;59 +3	52; 2 +2	50;10 +1
49	85	57;36 +3	56;25 +1	54;14 +2	52;10 +2	50;12 +2	48;20 +1
50	84	55;43	54;33	52;19	50;14 +1	48;17 +2	46;25 +1
51	83	53;43 -2	52;34	50;20 +1	48;13	46;14 -1	44;24
52	82	51;34 -6	50;30 +2	48;15 +3	46; 8 +2	44; 9 +1	42;18 -1
53	81	49;20 -6	48;16 +2	46; 0 +2	43;54 +2	41;56	40; 8
54	80	47; 0 -4	45;52	43;38 +2	41;33 +2	39;38 +1	37;53 +2
55	79	44;30 -1	43;23 +3	41; 8 +2	39; 6 +3	37;12 +1	35;31 +3
56	78	41;49	40;42 +4	38;26 -1	36;29 +2	34;40 +2	33; 2 +3
57	77	38;55	37;46	35;38	33;43	31;59	30;26 +2
58	76	35;49 -1	34;42 -1	32;40 -1	30;50 -1	29;12	27;43
59	75	32;30 -3	31;28 -1	29;34	27;51	26;18 -1	24;55 -1
60	74	29; 5 +2	28; 7 +2	26;19 +1	24;44 +1	23;20 +1	22; 3 -1
61	73	25;22	24;31 +2	22;54 +2	21;30 +2	20;14 +1	19; 8 +2
62	72	21;31 +2	20;44 +1	19;20 +1	18; 7 +2	17; 2 +2	16; 4 +1
63	71	17;31 +5	16;49 +2	15;38 +1	14;36	13;44 +1	12;55 -1
64	70	13;22 +9	12;43	11;48 -1	11; 1 -1	10;21	9;45
65	69	9; 0 +7	8;33	7;56	7;24	6;57 +1	6;32
66	68	4;29 +1	4;19 +1	4; 1 +2	3;45 +2	3;33 +4	3;17 +1

TABLE 6

al-Khalīlī's *Qibla* Table

		39°	40°	41°	42°	43°	44°
7°	127°	89;26	88;43 +1	87;58	87;15 +1	86;31 +1	85;48 +1
8	126	88;43	88; 3	87;17	86;33 +1	85;48 +1	85; 3 +1
9	125	88; 9	87;23	86;37 +1	85;50	85; 4	84;18
10	124	87;31 +1	86;44 +2	85;54	85; 6 -1	84;20 +1	83;32
11	123	86;51 +1	86; 2 +1	85;12	84;24 +1	83;35	82;47
12	122	86;11 +1	85;20	84;30 +1	83;39	82;50	82; 1 +1
13	121	85;30	84;38	83;46	82;55	82; 5 +1	81;14 +1
14	120	84;49 +1	83;55	83; 3 +1	82;10	81;18 +1	80;26
15	119	84; 7 +1	83;12	82;19 +1	81;24	80;30	79;37
16	118	83;24	82;29 +1	81;34 +2	80;39 +2	79;43	78;48
17	117	82;41	81;44 +1	80;43 +2	79;51 +1	78;54	77;59
18	116	81;56 -1	80;57 -1	80; 1 +1	79; 2	78; 5	77; 8
19	115	81;11 -1	80;10 -2	79;12	78;13	77;14 -1	76;16 -1
20	114	80;26 -1	79;25	78;24	77;24 +1	76;23 -1	75;24
21	113	79;41	78;39 +2	77;35	76;33	75;31 -1	74;30 -1
22	112	78;54 +1	77;50 +1	76;44	75;42 +1	74;38 -1	73;35 -2
23	111	78; 6 +1	77; 0 +1	75;54 +1	74;48 -1	73;44 -1	72;40 -2
24	110	77;16	76; 8	75; 2 +1	73;54 -1	72;50	71;45 -1
25	109	76;25 -1	75;16	74; 8	73; 0	71;54	70;49
26	108	75;34	74;22 -1	73;13	72; 4	70;56 -1	69;51 +1
27	107	74;41	73;26 -3	72;17	71; 7	69;57 -1	68;51 +1
28	106	73;46 -1	72;31 -2	71;20	70; 9	68;57 -1	67;49 -1
29	105	72;51 -1	71;36	70;21 -1	69; 9	67;56 -1	66;47
30	104	71;55	70;38	69;21 -1	68; 7	66;55	65;44 +1
31	103	70;56 -1	69;38	68;20	67; 6 +2	65;52 +2	64;40 +2
32	102	69;56 -1	68;36	67;17	66; 2 +2	64;46 +1	63;33 +2
33	101	68;53 -2	67;31 -2	66;13 +1	64;54	63;36 -1	62;23
34	100	67;49 -2	66;26 -1	65; 6 +1	63;45 -1	62;26 -2	61;10 -3
35	99	66;44 -2	65;19 -1	63;56 -1	62;35 -1	61;15 -2	59;58 -3
36	98	65;37 -1	64; 9 -2	62;44 -2	61;22 -2	60; 3 -1	58;46 -1
37	97	64;28	62;58 -1	61;33	60; 9 -1	58;48 -1	57;30 -1
38	96	63;13 -2	61;43 -2	60;17 -1	58;53 -1	57;31 -1	56;13 -1
39	95	61;57 -4	60;28 -1	59; 1	57;35	56;12 -1	54;54
40	94	60;41 -2	59;10	57;41	56;15 +1	54;51	53;33 +1
41	93	59;22 -1	57;49 +1	56;19 +1	54;51	53;26 -1	52; 8 +1
42	92	57;59	56;25 +1	54;52	53;24 -1	52; 0 -1	50;40
43	91	56;33	54;57 +1	53;24	51;55 -1	50;31 -1	49;11
44	90	55; 4 +1	53;27 +2	51;53	50;25 +1	49; 0	47;40 +1
45	89	53;30 +1	51;52 +1	50;18	48;52 +3	47;25	46; 5
46	88	51;53 +1	50;15 +2	48;41 +1	47;13 +2	45;48 +1	44;28
47	87	50;12 +2	48;34 +2	47; 1 +2	45;32 +2	44; 8 +1	42;49 +1
48	86	48;27 +2	46;48 +2	45;15 +2	43;46	42;24 +1	41; 6 +1
49	85	46;37 +2	44;58 +1	43;25	41;57 -1	40;37 +1	39;19
50	84	44;42 +1	43; 4 +1	41;32	40; 6	38;47 +1	37;30
51	83	42;42 +1	41; 4 -1	39;35	38;10 -1	36;52	35;38 -1
52	82	40;38 +1	39; 3	37;35 +1	36;11 -1	34;55	33;44
53	81	38;31 +3	36;57 +2	35;31 +2	34;11 +1	32;56 +1	31;47 +1
54	80	36;16 +2	34;46 +2	33;22 +2	32; 6 +3	30;53 +1	29;47 +2
55	79	33;57 +3	32;29 +2	31; 9 +2	29;56 +3	28;47 +2	27;43 +1
56	78	31;31 +2	30; 9 +3	28;51 +1	27;41 +2	26;37 +2	25;36 +1
57	77	28;59 +1	27;40	26;28	25;21 -1	24;21	23;25 -1
58	76	26;23	25; 9	24; 2	23; 0 -1	22; 4 -1	21;12 -2
59	75	23;42	22;36 +2	21;32 -1	20;37	19;46	18;57 -2
60	74	20;58 +2	19;57 +2	19; 0	18;11 +1	17;25 +1	16;41 -1
61	73	18; 8 +2	17;13 +1	16;26 +2	15;42 +2	15; 1 +1	14;23
62	72	15;14 +2	14;26	13;45	13; 7	12;34 +1	12; 3 +1
63	71	12;14	11;37	11; 1 -2	10;31 -2	10; 5	9;40
64	70	9;12 -1	8;44 -1	8;17 -2	7;55 -1	7;34 -1	7;16
65	69	6;11 +1	5;51	5;35 +1	5;18	5; 4	4;52 +1
66	68	3; 7 +1	2;57 +1	2;49 +2	2;41 +2	2;33 +1	2;26

TABLE 7

al-Khalīlī's *Qibla* Table

		45°	46°	47°	48°	49°	50°
7°	127°	85; 3	84;21 +1	83;37 +1	82;53 -1	82;11	81;29 +1
8	126	84;18	83;33	82;49	82; 5	81;22 +1	80;38
9	125	83;32	82;46	82; 1	81;16	80;32	79;48 +1
10	124	82;46 +1	81;59	81;12 -1	80;26 -1	79;41	78;56
11	123	81;59	81;11	80;24	79;36 -1	78;51 +1	78; 5 +1
12	122	81;11	80;22	79;34	78;46	77;59	77;12
13	121	80;23	79;33	78;44	77;55	77; 6	76;18 -1
14	120	79;35 +1	78;43	77;53	77; 3	76;12 -2	75;24 -1
15	119	78;46 +1	77;53	77; 1	76;11 +1	75;18 -2	74;30
16	118	77;56 +1	77; 3 +2	76; 9	75;18 +1	74;24 -2	73;35
17	117	77; 4	76;11 +2	75;15 -1	74;24 +1	73;30 -1	72;40
18	116	76;12	75;17	74;20 -2	73;28	72;35	71;43
19	115	75;20	74;22 -1	73;25 -2	72;32 -1	71;39	70;46
20	114	74;25 -1	73;27 -2	72;30 -2	71;36	70;42 +1	69;48 +1
21	113	73;30 -2	72;32 -1	71;35 -1	70;39	69;43	68;49 +1
22	112	72;35 -2	71;37	70;39 +1	69;41	68;44	67;48
23	111	71;40	70;40	69;41 +1	68;41	67;43 -1	66;47 -1
24	110	70;43	69;42 +1	68;41	67;41	66;43	65;46
25	109	69;45	68;43 +1	67;40	66;40	65;41	64;44 +1
26	108	68;45	67;43 +2	66;39	65;39 +2	64;38	63;39
27	107	67;44	66;41 +2	65;36	64;35 +1	63;33	62;34
28	106	66;42	65;37 +1	64;32	63;30 +1	62;28	61;28
29	105	65;39	64;32	63;27	62;25 +2	61;21	60;20 -1
30	104	64;33 -1	63;26	62;20	61;18 +2	60;13 -1	59;11 -2
31	103	63;27 -1	62;19	61;12	60; 8	59; 4	58; 2 -1
32	102	62;20	61;10 -1	60; 2 -1	58;57 -1	57;54	56;52
33	101	61;11	60; 0 -1	58;50 -2	57;45 -1	56;42	55;40
34	100	59;59 -1	58;49	57;37 -3	56;32 -2	55;28 -1	54;26 -1
35	99	58;46 -1	57;35 -1	56;24 -2	55;18 -1	54;13 -2	53;11 -1
36	98	57;32 -1	56;18 -2	55; 9 -2	54; 2 -1	52;57 -1	51;55 -1
37	97	56;15 -1	55; 0 -4	53;52 -1	52;44 -2	51;40 -1	50;39 +1
38	96	54;57 -1	53;40 -5	52;33 -1	51;25 -2	50;21 -1	49;21 +2
39	95	53;37 -1	52;19 -5	51;12 -2	50; 5 -1	49; 1	48; 1 +3
40	94	52;16 +1	50;57 -5	49;50 -1	48;43	47;40 +2	46;40 +4
41	93	50;51 +1	49;35 -2	48;26	47;19	46;15 +1	45;16 +4
42	92	49;24	48;10	47; 0	45;52	44;48	43;48 +1
43	91	47;55 +1	46;41	45;31	44;24	43;20	42;19
44	90	46;24 +1	45;11 +1	44; 2 +2	42;54	41;51	40;50 -1
45	89	44;49	43;36	42;28 +1	41;22	40;21 +1	39;20
46	88	43;13 +1	42; 2 +2	40;54 +2	39;47 -1	38;48 +2	37;49 +1
47	87	41;34 +1	40;24 +2	39;18 +3	38;11 -1	37;12 +1	36;16 +2
48	86	39;52 +1	38;43 +2	37;37 +1	36;34 +1	35;35	34;40 +1
49	85	38; 7	36;58	35;53 -1	34;54 +1	33;56	33 2
50	84	36;19	35;15 +2	34; 9 -1	33;11	32;15	31;22 -1
51	83	34;29 -1	33;26 +1	32;24	31;26 -1	30;33	29;42
52	82	32;37	31;35 +1	30;36 +1	29;40	28;50 +1	28; 1 +1
53	81	30;43 +1	29;43 +2	28;46 +1	27;53 +1	27; 5 +2	26;18 +2
54	80	28;46 +2	27;48 +2	26;54 +2	26; 3 +1	25;17 +2	24;33 +2
55	79	26;45 +2	25;51 +3	24;59 +2	24;11 +1	23;27 +1	22;47 +3
56	78	24;41 +1	23;50 +2	23; 2 +1	22;17 +1	21;35	20;58 +2
57	77	22;35 +1	21;47 +1	21; 2	20;21	19;42	19; 7
58	76	20;25 -1	19;42	19; 0 -1	18;22 -2	17;48 -1	17;16
59	75	18;14 -2	17;36	16;59	16;22 -3	15;53	15;25 +1
60	74	16; 1 -3	15;30 +2	14;56 +1	14;24 -1	13;56 -1	13;32 +1
61	73	13;45 -5	13;21 +2	12;51 +1	12;24	11;59	11;38 +1
62	72	11;29 -5	11; 9 +1	10;45 +1	10;22	10; 1	9;43 +1
63	71	9;13 -4	8;56	8;37 +1	8;18	8; 2	7;46
64	70	6;57 -2	6;42 -1	6;28	6;14	6; 2	5;49 -1
65	69	4;40	4;28 -1	4;19	4; 9 -1	4; 2	3;52 -2
66	68	2;21 +1	2;15	2;10	2; 6 +1	2; 2 +1	1;55 -2

TABLE 8

AL-KHALĪLĪ'S *QIBLA* TABLE

		51°	52°	53°	54°	55°	56°
7°	127°	80;47 +1	80; 5	79;22 -1	78;42	78; 3 +1	77;22 +1
8	126	79;55	79;13	78;29 -1	77;49	77; 8 +1	76;26
9	125	79; 3 -1	78;21 +1	77;37	76;55	76;12 -1	75;30 -1
10	124	78;11	77;27	76;43	76; 1 +1	75;17	74;34 -1
11	123	77;19	76;34	75;49	75; 5	74;21	73;38 -1
12	122	76;26 +1	75;41 +2	74;54	74; 9	73;24 -1	72;42
13	121	75;32 +1	74;45	73;58	73;13	72;28	71;44
14	120	74;38 +1	73;49	73; 3 +1	72;17 +1	71;31	70;46
15	119	73;43 +2	72;53	72; 7 +1	71;20 +1	70;34 +1	69;47
16	118	72;47 +1	71;57 +1	71;10 +2	70;22 +1	69;35 +1	68;48
17	117	71;49	71; 0 +1	70;11 +1	69;22	68;34	67;48
18	116	70;51	70; 0 -1	69;11	68;22	67;33 -1	66;47
19	115	69;53	69; 0 -2	68;11 -1	67;21 -1	66;33 -1	65;47 +1
20	114	68;54	68; 0 -2	67;11	66;21	65;33 +1	64;45 +1
21	113	67;54 -1	67; 0 -2	66;11 +1	65;20	64;31 +1	63;42
22	112	66;53 -1	65;59 -2	65; 9 +1	64;17	63;27	62;38
23	111	65;52	64;58	64; 7 +1	63;13 -1	62;23	61;34
24	110	64;50	63;55	63; 3 +1	62; 9 -1	61;18 -1	60;29
25	109	63;46	62;51	61;58 +1	61; 4 -1	60;13 -1	59;23 -1
26	108	62;41 -1	61;46	60;51 -1	59;58 -1	59; 7	58;17
27	107	61;35 -2	60;40	59;44 -2	58;52	58; 0	57;10
28	106	60;29 -1	59;33	58;37 -1	57;44 -1	56;53 +1	56; 3 +1
29	105	59;21 -1	58;25	57;30	56;36	55;44	54;54 +1
30	104	58;13 -1	57;16	56;21 +1	55;27 +1	54;34	53;44 +1
31	103	57; 4	56; 6	55;11 +1	54;17 +1	53;24 +1	52;33 +1
32	102	55;53	54;54 -1	54; 0 +2	53; 5 +1	52;12 +1	51;21
33	101	54;41 +1	53;41 -1	52;45 -1	51;50 +1	50;58 -1	50; 7 -1
34	100	53;27 +1	52;27 -1	51;30 -2	50;35 -3	49;43 -2	48;53 -1
35	99	52;11 -1	51;12 -1	50;15 -2	49;20 -3	48;28 -2	47;38 -2
36	98	50;54 -1	49;56 -1	49; 0 -1	48; 5 -2	47;13 -2	46;23 -1
37	97	49;37 -1	48;39	47;43	46;50	45;57 -1	45; 7 -1
38	96	48;18 -1	47;20 -1	46;24 -1	45;33 +2	44;40	43;50
39	95	46;58	46; 0	45; 4 -1	44;14 +2	43;21	42;31 -1
40	94	45;36	44;39	43;43 -1	42;52 +1	42; 1	41;12 -1
41	93	44;13	43;16	42;21 -1	41;29 -1	40;40	39;52
42	92	42;48	41;52	40;58	40; 6 -1	39;18	38;31
43	91	41;21	40;26	39;34 +1	38;42 -1	37;55 +1	37; 9 +1
44	90	39;53	38;59	38; 8 +1	37;17	36;31 +1	35;45
45	89	38;24	37;31 +1	36;41 +2	35;50 -1	35; 5 +1	34;21 +1
46	88	36;54 +1	36; 1 +1	35;12 +2	34;22 -1	33;39 +1	32;55
47	87	35;21 +1	34;30 +1	33;42 +2	32;54	32;12 +2	31;29 +1
48	86	33;47 +1	32;57 +1	32;10 +1	31;25 +1	30;43 +2	30; 2 +1
49	85	32;11 +1	31;22	30;36	29;54 +1	29;12 +1	28;34 +2
50	84	30;33	29;46	29; 2	28;21 +1	27;40 -1	27; 4 +1
51	83	28;55 +1	28;10 +1	27;27	26;47	26; 8 -1	25;33
52	82	27;16 +2	26;32 +1	25;51	25;12	24;35 -1	24; 2
53	81	25;37 +4	24;52	24;14 +1	23;37	23; 2	22;30
54	80	23;56 +6	23;12 +1	22;36 +2	22; 1 +1	21;28	20;58 +1
55	79	22;10 +5	21;30 +1	20;56 +1	20;23	19;53 +1	19;24
56	78	20;22 +2	19;47 +1	19;15 +1	18;44	18;18 +2	17;50 +1
57	77	18;34 +1	18; 3 +1	17;33 +1	17; 5	16;42 +3	16;15 +1
58	76	16;46 +1	16;18 +1	15;52 +2	15;25	15; 4 +3	14;39
59	75	14;57 +1	14;32 +2	14; 6	13;44	13;24 +1	13; 3
60	74	13; 7 +1	12;44	12;22	12; 3 +1	11;44	11;26
61	73	11;17 +1	10;56	10;38 +1	10;21 +1	10; 4	9;49
62	72	9;26 +2	9; 8	8;53 +1	8;38	8;24	8;12 +1
63	71	7;34 +2	7;19	7; 7 +1	6;55	6;43 -1	6;34
64	70	5;41 +1	5;30 +1	5;20	5;11	5; 2 -1	4;55
65	69	3;47	3;40	3;34	3;27 -1	3;21 -1	3;16 -1
66	68	1;53	1;51 +1	1;48 +1	1;43 -1	1;42 +1	1;38 -1

CRITICAL APPARATUS

The notation (φ, L) a;b c;d used below means that the Paris manuscript has the entry a;b for $q(\varphi, L)$ and the Berlin manuscript has the entry c;d. A dash indicates that there is no entry in the manuscript, and a question mark indicates that the entry is illegible. All of the second digits in the entries for $\varphi = 12°, 18°, 24°, 30°, 35°, 41°, 47°$, and $53°$ and $L = 37°, 38°, \ldots, 66°$ in the microfilm of the Berlin manuscript which I have used are illegible. Also, the entries in the Berlin manuscript are less liberally provided with diacritical points than the corresponding entries in the Paris manuscript. I note below all of the entries which are clearly miscopied in either one or the other source.

(13,63)	23;37	23;27	(14,35)	71;3?	71;34	(14,49)	63;48	64;43
(14,63)	26;58	26;18	(15,26)	74;18	74;13	(16,36)	74;34	74;39
(16,40)	73;28	73;38	(16,60)	59;15	49;15	(19,39)	79;45	79;42
(19,40)	79;44	79;—	(20,23)	79;42	79;52	(20,65)	50;35	50;55
(21,12)	78;45	78;48	(21,33)	82;19	82;49	(21,35)	83;10	—
(21,36)	83;20	—	(21,39)	83;46	83;47	(21,54)	81;15	81;14
(23,10)	79;52	79;32	(24,23)	84;23	84;28	(25,20)	84;38	84;33
(27,64)	27; 2	25; 2	(27,65)	18;17	18;17	(28,57)	56;27	56;37
(31,33)	80;44	80;54	(31,51)	60;55	60;15	(31,56)	58;42	48;42
(33,54)	58;17	48;17	(34,32)	76; 6	77; 6	(34,37)	72;25	72;22
(35,40)	66;26	?	(36,66)	3;40	3;45	(37,17)	84;37	84;34
(37,43)	59;56	59; 6	(37,49)	50;12	50; 2	(37,53)	41;46	41;56
(38,45)	45;12	55;12	(39,37)	64;28	64;23	(39,50)	44;52	44;42
(39,60)	20;18	20;18	(41,23)	75;54	75;55	(41,43)	58;24	53;??
(41,66)	2;59	2;4?	(43,55)	28;57	28;47	(44,39)	54;??	54;54
(46,25)	68;43	68;13	(48,49)	34;44	34;54	(49,36)	2;57	52;57
(49,40)	47;40	47;47	(50,62)	9;48	9;43	(50,65)	3;42	3;5?
(50,66)	1;55	1;—	(51,26)	62;35	62;41	(51,27)	61;29	61;35
(51,28)	60;—	60;29	(51,44)	39;53	39;43	(51,58)	16;46	16;47
(52,50)	29;56	29;46	(52,65)	4;40	3;40	(53,14)	73;20	73; 3
(53,58)	15;32	—	(54,35)	59;20	49;20	(54,48)	31;25	31;35
(54,55)	20;23	—	(56,29)	54;54	—	(56,30)	53;44	—
(56,57)	16;15	16;?5	(56,58)	4;39	—	(56,59)	13; 3	—
(56,60)	11;26	—	(56,61)	9;59	9;49			

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we have

$$q = 0° \text{ and } q' = \begin{cases} 180° & \varphi < \varphi_M \\ 0° & \varphi > \varphi_M. \end{cases}$$

The entire table was recomputed in a few seconds at the Yale Computer Center. Tables of the function $h(\varphi, L)$, the altitude of the zenith of Mecca above the local horizon, and the auxiliary function $x(\varphi, L)$ (see section 2) were also computed, to facilitate hand computations of q using al-Khalīlī's auxiliary tables (see section 3). al-Khalīlī's *qibla* values are remarkably accurate: the errors in extensive sections of the table are less than $\pm 0;2°$, although larger errors occur in certain other sections. The errors in the second digit of al-Khalīlī's values are shown after each entry, computed according to the convention

$$\text{error} = \text{text} - \text{recomputation}.$$

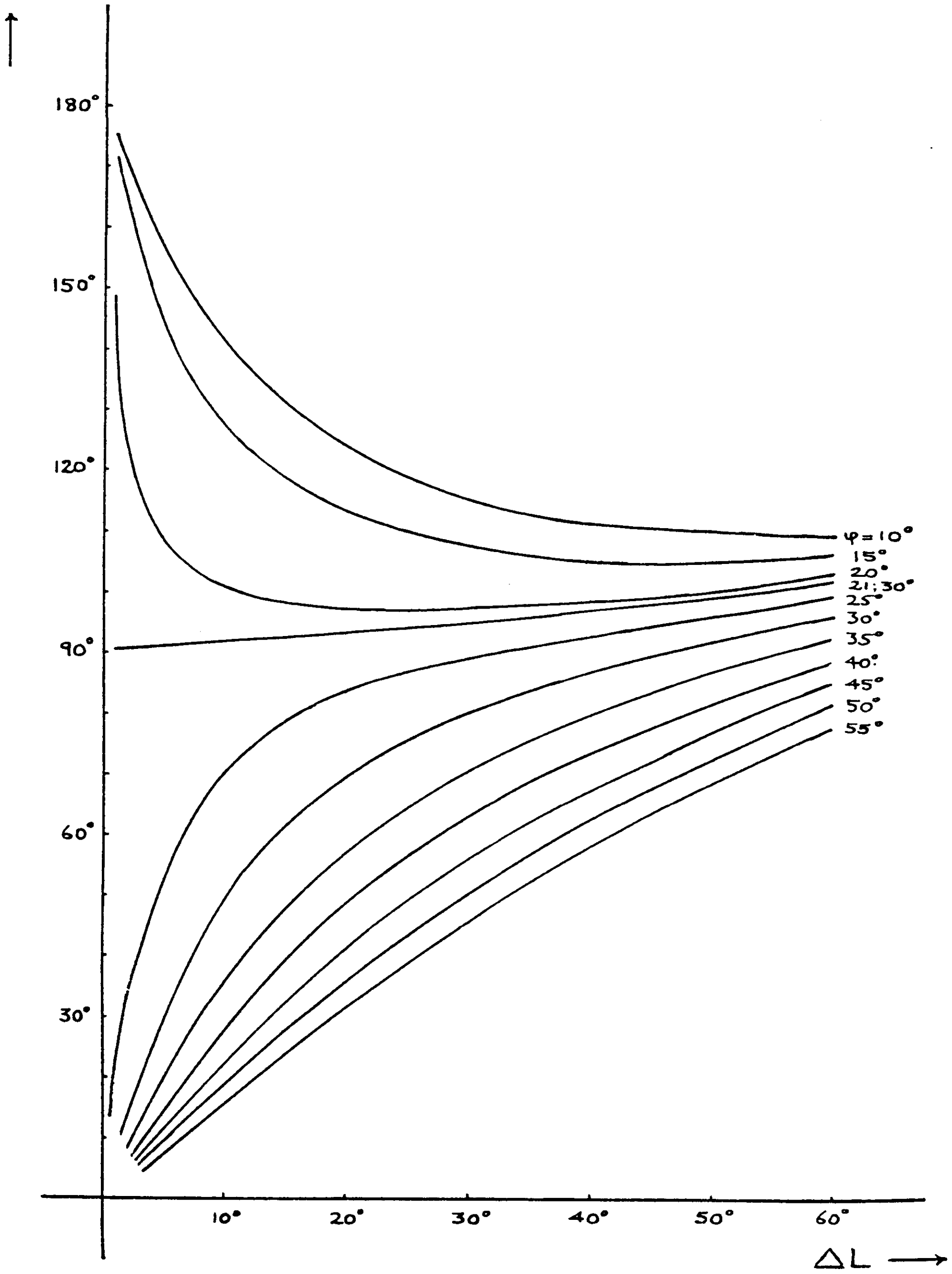

FIG. 3.—Graph showing the behavior of the function q' (ΔL) for different values of φ.

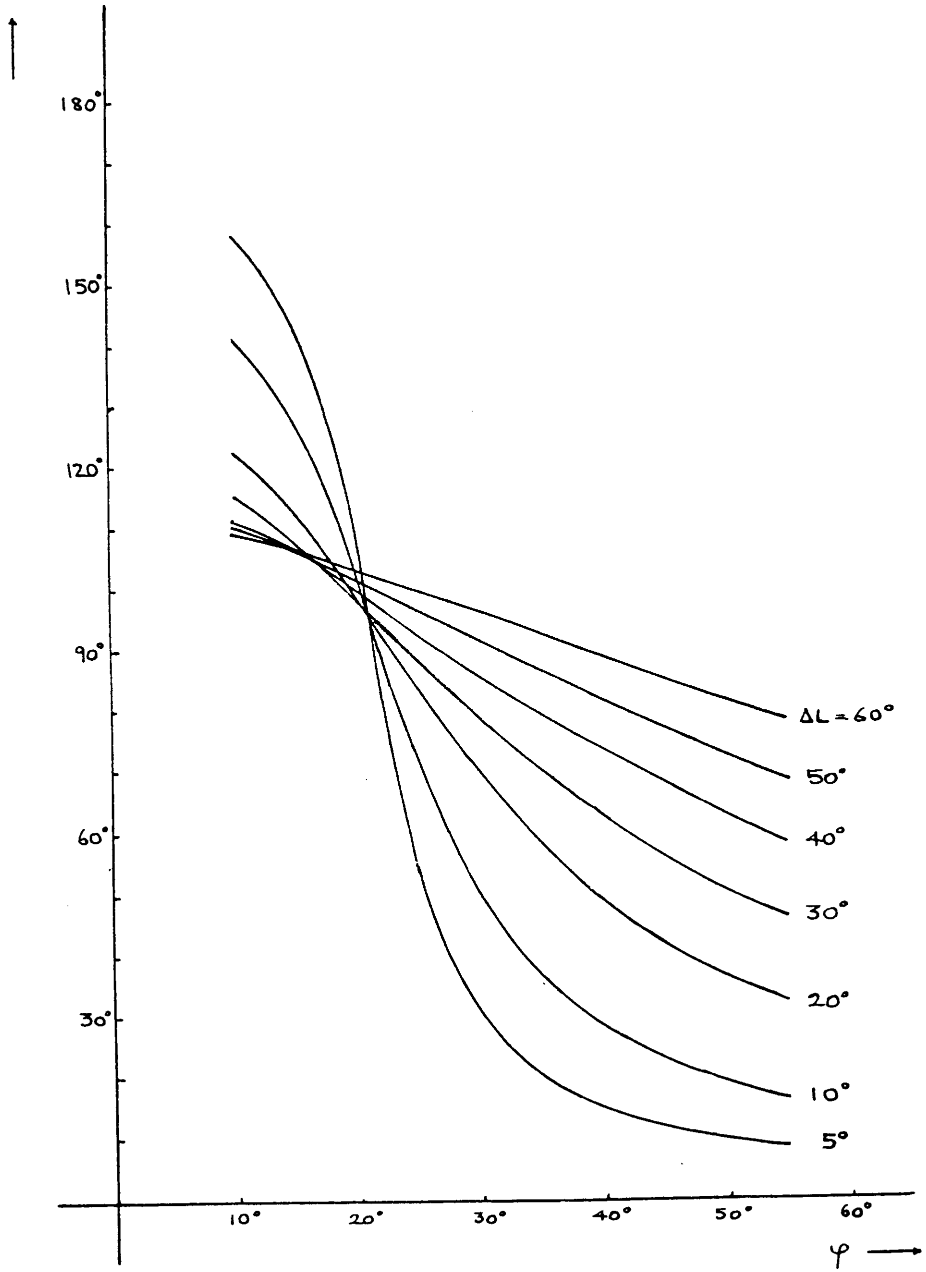

FIG. 4.—Graph showing the behavior of the function q' (φ) for different values of ΔL.

It is reasonable to assume that al-Khalīlī would have computed independently a matrix of values for, say, each degree of φ and each 5° of ΔL, and that he would then have used an interpolation scheme to compute intermediate values. The error patterns in the table confirm this, and the interpolation scheme used, which was more sophisticated than a second order scheme,[13] generally works rather well. Note, however, the relatively large errors in the table for small values of ΔL and values of φ close to φ_M: for these arguments q is very sensitive to small changes in ΔL. Note also the following minor point: the entries for values of φ from 21° through 17° are identical to those for values of φ from 22° through 26° for

$$\Delta L = 1^\circ \ (L = 66^\circ, 68^\circ)$$

and the *qibla* differs only in direction for these values close to φ_M. It is clear from the errors that the entries for φ from 21° through 17° were copied from those for values of φ from 22° through 26°, which were computed.

It is of interest that al-Khalīlī has tabulated $q(\varphi, L)$ for the latitude of Damascus, 33;30°. The latitude of Baghdad is also close to this, but the only reason for computing such a table would appear to be that al-Khalīlī had already computed his auxiliary functions for this latitude and that he used his auxiliary tables to compile his *qibla* table. On the other hand, he also tabulated $q(\varphi, L)$ for latitude 56°, and as far as we know, he computed the auxiliary functions only up to latitude 55°. See further section 3.

To illustrate how the arguments are to be entered in the table, al-Khalīlī considers the case

$$\varphi = 35^\circ \quad \text{and} \quad L = 25^\circ.$$

The value given in the table is, as he says, 81;12° east of south. This value is, in fact, accurate. He also mentions that he used a special kind of quadrant (*rubᶜ mujayyab*) to perform this calculation, and found the values

$$h = 51;0^\circ \quad \text{and} \quad q = 81;10^\circ.$$

Actually, this value of h is in error by only $+0;3^\circ$ and the value of q by only $-0;2^\circ$. al-Khalīlī does not describe the procedure which he used to arrive at these values, or the size of the instrument which he used. The determination of the *qibla* with such a quadrant is discussed in sections 4–6. The procedure described in Appendix B is my own, and should be compared with that of Sibṭ al-Māridīnī, described in section 6.

al-Khalīlī states next that when he used the well-known method of approximation involving the latitude and longitude differences, he found the "terribly inaccurate" value 68;50°. However, my own calculation with the standard approximation, formula (2), yields 70;46°. [*Note added in proof:* It is now clear that al-Khalīlī was referring to one of the other approximate *qibla* formulae which underlie the tables mentioned in Appendix C.]

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[13] On non-linear interpolation schemes in the Islamic sources, see, for example, Schoy, *Die trigonometrischen Lehren des persischen Astronomen Abu l-Raiḥān Muḥ. Ibn Aḥmad al-Bīrūnī*, pp. 40–42; E. S. Kennedy, "A Medieval Interpolation Scheme Using Second Order Differences," *A Locust's Leg: Studies in Honour of S. H. Taqizadeh* (London, 1962), pp. 117–20, and idem, "The Chinese-Uighur Calendar as Described in the Islamic Sources," *Isis* 55 (1964): 435–43; J. Hamadanizadeh, "A Medieval Interpolation Scheme for Oblique Ascensions," *Centaurus* 9 (1963): 257–65; and King, "Ibn Yūnus' *Very Useful Tables*," pp. 354–56.

al-Khalīlī does not describe the method by which he computed his table. However, in his introduction he remarks that he knows of no better method for finding the *qibla* than the one presented in chapter 67 of the astronomical treatise of Abū ᶜAlī al-Marrākushī. To this we now turn.

2. *al-Marrākushī's Method for Determining the* Qibla

Abū ᶜAlī al-Marrākushī, in some sources called Abū l-Ḥasan, is an individual of great importance for the history of Islamic astronomy.[14] Virtually nothing is known about his life, save that his family was associated with Marrakesh, and that he traveled widely and spent some time in Egypt. His major work, an extensive treatise on spherical astronomy and instruments, was compiled about the year A.D. 1280, probably in Cairo, and was published in French translation by J.-J. Sédillot in 1834–35.[15] al-Marrākushī quotes various of his predecessors from the Muslim West, and his work became widely known in the Muslim East and influenced such scholars as al-Khalīlī and Ibn al-Shāṭir.[16] His treatise has generally been neglected by historians of science, although A. von Braunmühl and K. Schoy have discussed the sections on trigonometry and sundials, and C. A. Nallino referred to al-Marrākushī's spherical astronomy in his analysis of the astronomical work of al-Battānī.[17]

al-Marrākushī's solution to the *qibla* problem is analogous to one of the several Islamic methods for calculating the solar altitude from the hour-angle, a problem which is mathematically equivalent to the determination of the altitude of the zenith of Mecca above the local horizon. This method al-Marrākushī claims as his own discovery. Before presenting his method, we consider the *qibla* problem transferred to the celestial sphere.

Figure 5 represents the celestial sphere about the observer at O. The local horizon is shown as NES, the celestial equator as EQ, the celestial pole as P, the local zenith as Z, and the local meridian as NPZQS. The point Z_M represents the zenith of Mecca, and the azimuth of Z_M, measured by the arc SK, defines the direction of Mecca. The declination circle or day-circle through Z_M, namely AB, is also shown, and PZ_M is produced to meet the celestial equator at T. Thus

$$ZQ = \varphi; \qquad Z_MT = \varphi_M; \qquad \text{and} \qquad QT = \Delta L.$$

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[14] On al-Marrākushī see Suter, "Die Mathematiker und Astronomen der Araber," no. 363 and p. 219, n. 74, as well as Brockelmann, *Geschichte der arabischen Litteratur*, 1:625 and ibid., Supp. 1:866. Suter is correct in suspecting that Sédillot misdated al-Marrākushī to ca. A.D. 1230: his solar longitude tables and star catalogue are compiled for A.D. 1275/76 and 1282, respectively. However, it has not been previously stressed that al-Marrākushī's treatise was compiled for Cairo.

[15] J.-J. Sédillot, *Traité des instruments astronomiques des arabes*, 2 vols. (Paris, 1834–35). In the sequel I have translated from the French, rather than from the original Arabic text which is extant in several manuscripts. The title of the work is *Jāmiᶜ al-mabādiʾ wa-l-ghāyāt*, [Compendium of Principles and Results]. al-Khalīlī refers to it as *Risālat al-Jayb*, [Treatise on the Sine (Quadrant)]. On this instrument see section 4 below.

[16] For example, both Ibn al-Shāṭir and al-Khalīlī present lists of standard formulae of spherical astronomy in the same fashion as al-Marrākushī. The formulae are each represented by the four terms of an appropriate proportion: the terms are expressed verbally in the highly technical vocabulary of late Islamic astronomy, but without recourse to symbols. See J.-J. Sédillot, *Traité des instruments astronomiques*, 1:352–59 for al-Marrākushī's list of 62 proportions; MS Bodleian Seld. Arch. A30, fols. 77r–82v for Ibn al-Shāṭir's list of 184; and MS Chester Beatty 4091, fol. 174v for al-Khalīlī's list of 24.

[17] See A. von Braunmühl, *Vorlesungen über Geschichte der Trigonometrie*, 2 vols. (Leipzig, 1900–3), 1:83–86; Schoy, *Gnomonik der Araber*; and Nallino, *al-Battani*, 2: index, s.v. Aboul Hhassan. I have included an analysis of the spherical astronomical tables of al-Marrākushī in my study of medieval Islamic prayer-tables (see n. 7 above.).

Several Islamic methods, including al-Marrākushī's, involve first finding the altitude of Z_M

$$Z_M K = h$$

and then finding

$$SK = q.$$

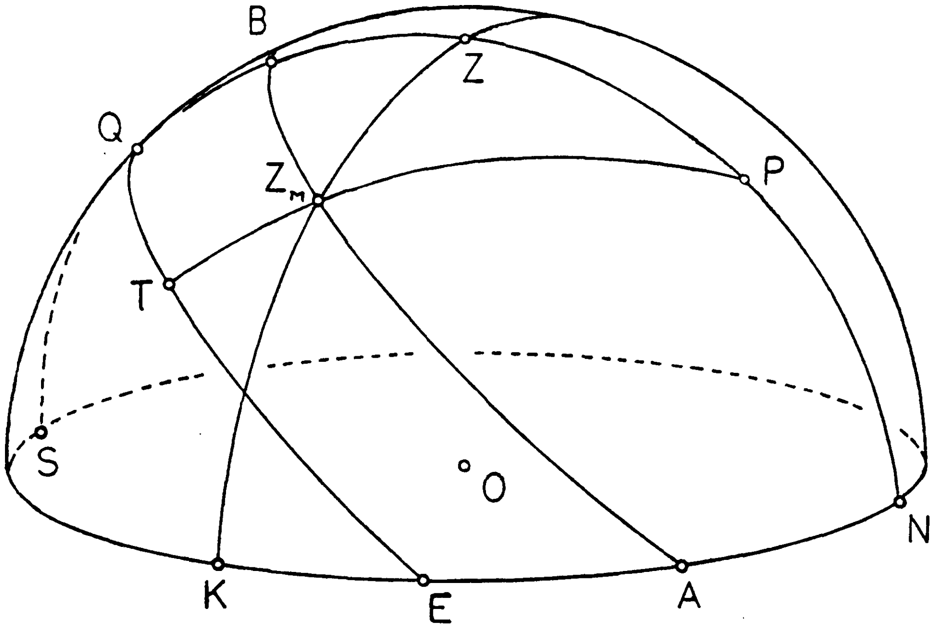

FIG. 5

Note that if Z_M represents an instantaneous solar position with altitude h, declination δ, azimuth a, and hour-angle t, then we have

$$t = \Delta L; \qquad \delta = \varphi_M; \qquad \text{and} \qquad a = q.$$

In chapter 67 of his treatise, al-Marrākushī prescribes the following method for the general problem of the determination of the direction of any locality from a given locality, which may be compared with the methods of Ibn Yūnus, discussed in section 5, and of Ibn al-Haytham, discussed in Appendix A. I use the standard capital notation for medieval trigonometric functions to base R (= 60).[18]

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[18] Thus,

$$\text{Sin}\,\vartheta = R \sin\vartheta, \qquad \text{Cos}\,\vartheta = R\cos\vartheta,$$
$$\text{Tan}\,\vartheta = R\tan\vartheta$$

and

$$\text{Vers}\,\vartheta = R - \text{Cos}\,\vartheta.$$

See further, Wiedemann, *Aufsätze der arabischen Wissenschaftsgeschichte*, 1:564–77; Kennedy, "A Survey of Islamic Astronomical Tables," pp. 139–40; and King, "The Astronomical Works of Ibn Yūnus," pp. 60–65.

al-Marrākushī is a little careless in occasionally neglecting to state "multiply by 60" or "divide by 60" in his trigonometric operations. Further east, astronomers such as Abū l-Wafāʾ (ca. A.D. 975), Abū Naṣr (ca. A.D. 1000), and al-Bīrūnī (ca. A.D. 1025) had long since recognized that the use of $R = 1$ would simplify their trigonometric procedures. See, for example, K. Schoy, "Beiträge zur arabischen Trigonometrie," *Isis* 5 (1923): 393 and 396, and Jensen, "Abu Naṣr's Approach to Spherical Trigonometry."

> To find the azimuth of any locality, multiply the Cosine of the latitude of the locality whose azimuth is required by the Cosine of the local latitude [and divide by the square of 60]: the result will be the "base" (*aṣl*). Take the Versed Sine of the longitude difference between the two localities and multiply it by the "base." Next subtract the product from the Sine of the meridian altitude of the small-circle through the zenith of the other locality and parallel to the celestial equator. The remainder will be the Sine of the altitude of the zenith of the other locality above the local horizon. Finally, find the azimuth corresponding to this altitude by one of the methods explained previously.

al-Marrākushī's method may be rendered in mathematical notation as follows. Firstly, note that the meridian altitude of the declination circle through Z_M, measured by the arc SB in figure 5 is

$$\bar{\varphi} + \varphi_M$$
$$(\text{where } \bar{\varphi} = 90° - \varphi).$$

Secondly, the determination of time from solar altitude involves a division by the quantity

$$\frac{\text{Cos}\,\delta\,\text{Cos}\,\varphi}{R}.$$

This auxiliary function was generally known in Islamic astronomy as the "absolute base" (*al-aṣl al-muṭlaq*), and tables of it with solar longitude as argument are common in Islamic astronomical handbooks.[19] al-Marrākushī prescribes forming the "base"

$$\frac{\text{Cos}\,\varphi_M\,\text{Cos}\,\varphi}{R^2}.$$

Thus, his method may be represented by the formula

$$\text{Sin}\,h = \text{Sin}\,(\bar{\varphi} + \varphi_M) - \text{Vers}\,\Delta L\,\frac{\text{Cos}\,\varphi_M\,\text{Cos}\,\varphi}{R^2}. \tag{3}$$

The corresponding formula for the solar altitude, known to Islamic astronomers originally from Indian sources, is[20]

$$\text{Sin}\,h = \text{Sin}\,(\bar{\varphi} + \delta) - \text{Vers}\,t\,\frac{\text{Cos}\,\delta\,\text{Cos}\,\varphi}{R^2}. \tag{4}$$

Formula (4) is easily derived from figure 6, which shows an analemma construction[21]

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[19] Several such tables are listed in my study of medieval tables for timekeeping (see n. 7).

[20] See, for example, von Braunmühl, *Vorlesungen über Geschichte der Trigonometrie*, 1:41 on the procedure of the *Sūryasiddhānta*, and Nallino, *al-Battani*, 1:192 (Ḥabash and al-Battānī); King, "The Astronomical Works of Ibn Yūnus," p. 153 (Ibn Yūnus); N. Nadir, "Abū al-Wafaʾ on the Solar Altitude," *The Mathematics Teacher* 53 (1960): 460–63 (Abū l-Wafāʾ); M.-L. Davidian, "Al-Bīrūnī on the Time of Day from Shadow Lengths," *Journal of the American Oriental Society* 80 (1960): 330–35 (al-Bīrūnī); and L.-A. Sédillot, *Prolégomènes des tables astronomiques*, 2:125–26 (Ulugh Beg). Note that the expression

$$\frac{R\ \text{Sin (meridian altitude)}}{\text{Vers (semi-diurnal arc)}}$$

used in most of these sources is equal to the "absolute base." On this, see further King, "The Astronomical Works of Ibn Yūnus," pp. 132–33.

[21] On the analemma, see, for example, P. Luckey, "Das Analemma von Ptolemäus," *Astronomische Nachrichten* 230 (1927): 18–46 and O. Neugebauer, *The Exact Sciences in Antiquity*, 2d ed. (New York, 1969), pp. 214–18 and 226. On its use in Islamic mathematics, see K. Schoy, "Abhandlung über die Ziehung der Mittagslinie, dem Buch über das Analemma entnommen, samt dem Beweis dazu von Abū Saʿīd aḍ-Ḍarīr," *Annalen der Hydrographie und maritimen Meteorologie* 50 (1922): 265–71; Y. Id, "An Analemma Construction for Right and Oblique Ascensions," *The Mathematics Teacher* 62 (1969): 669–72; E. S. Kennedy, "Bīrūnī's Graphical Determination of the Local Meridian," *Scripta*

derived from figure 5. Firstly we project the celestial sphere orthogonally in the plane of the meridian. Primes are used in figure 6 to denote the projections of corresponding points in figure 5. The declination circle of Z_M is then rotated through 90° about its diameter BC into the meridian plane to form the semicircle BMC, where the arc MB measures ΔL. Thus M represents the position of Z_M on the declination circle with respect to the local meridian BC. The radius of this semicircle, whose center is the point G, is Cos φ_M, since the arc QB measures φ_M. Thus we have

$$BZ'_M = \text{Cos } \varphi_M \frac{\text{Vers } \Delta L}{R}$$

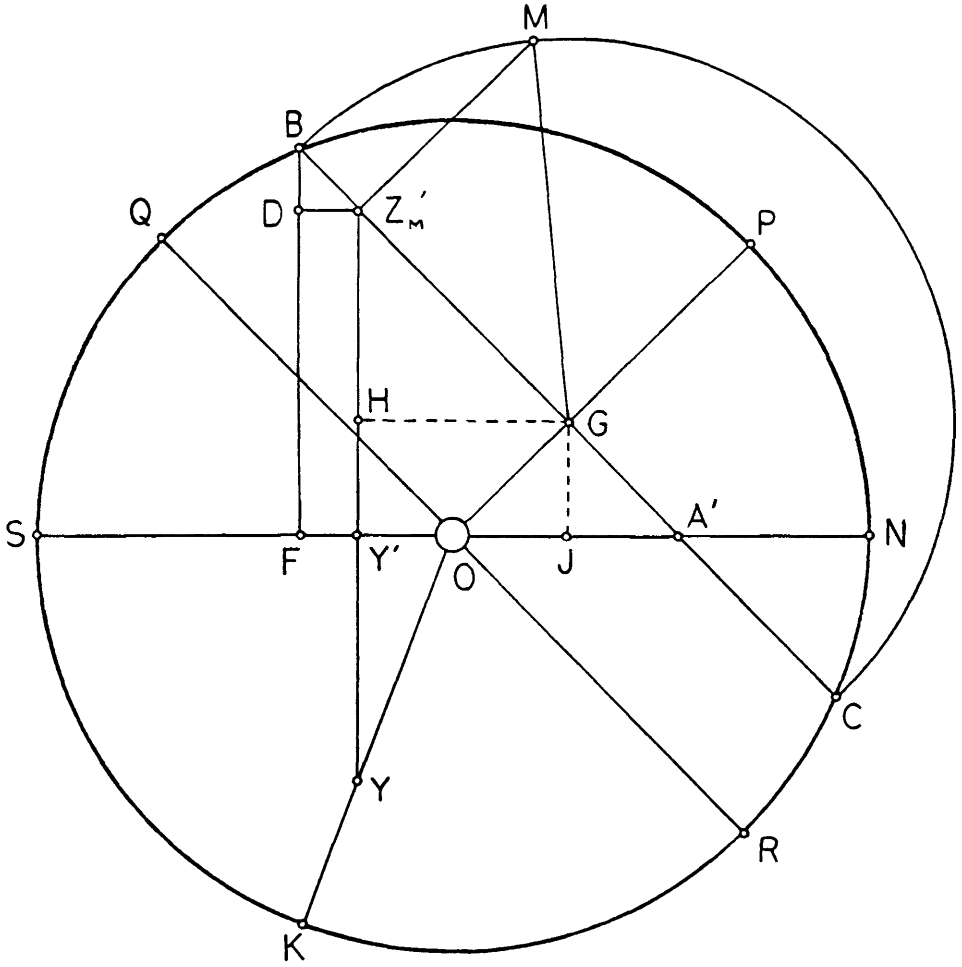

FIG. 6

Mathematica 24 (1959): 251–55; idem, "al-Bīrūnī on Determining the Meridian," *The Mathematics Teacher* 56 (1963): 635–37; P. Luckey, "Beiträge zur Erforschung der islamischen Mathematik," *Orientalia*, n.s. 17 (1948): 498; King, "The Astronomical Works of Ibn Yūnus," pp. 26–27; and Id and Kennedy, "A Letter of al-Bīrūnī," (see n. 2 above).

and hence

$$BD = \text{Vers } \Delta L \frac{\text{Cos } \varphi_M \text{ Cos } \varphi}{R^2}. \tag{5}$$

Furthermore, the perpendiculars BF and Z'_MY' onto the horizon measure Sin ($\bar{\varphi} + \varphi_M$) and Sin h respectively, so that

$$BD = BF - Z'_MY' = \text{Sin } (\bar{\varphi} + \varphi_M) - \text{Sin } h. \tag{6}$$

Formula (3) follows from (5) and (6).

al-Marrākushī's method for finding the solar azimuth from the solar altitude is the standard Islamic method, which had been known for several centuries.[22] It is contained in chapter 62 of his treatise:

> To determine the solar azimuth, multiply the Sine of the altitude by the Sine of the local latitude and divide the product by the Cosine of the local latitude: the quotient will be the "equation of the azimuth" (*taʿdīl al-samt*) if the sun has no declination, otherwise it will be the "horizontal variation" (*ḥiṣṣat al-samt*). If the declination of the sun and the local latitude are in different directions, add the Sine of the rising amplitude (*saʿat al-mashriq*) to the horizontal variation, and if the declination and the latitude are in the same direction, take their difference. The result in both cases will be the equation of the azimuth. Finally divide [the product of] the equation of the azimuth [and 60] by the Cosine of the altitude, and the quotient will be the Sine of the azimuth.

Finally, his method for determining the solar rising or ortive amplitude is outlined in chapter 58 of the treatise:

> Multiply the Sine of the declination by 60, and divide the product by the Cosine of the latitude: the quotient will be the Sine of the rising amplitude.

In this method certain standard Islamic auxiliary functions are used to facilitate the computation. Firstly, the "horizontal variation," also called *ikhtilāf al-ufq* in the Arabic sources,[23] and here denoted by k, is defined by

$$k(h, \varphi) = \frac{\text{Sin } h \text{ Sin } \varphi}{\text{Cos } \varphi}. \tag{7}$$

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[22] On Islamic azimuth calculations see also Luckey, "Beiträge zur Erforschung," p. 502 (al-Māhānī, ca. A.D. 860); Nallino, *al-Battani*, 1:183–84 (Ḥabash and al-Battānī); King, "The Astronomical Works of Ibn Yūnus," pp. 186–87 (Ibn Yūnus); L.-A. Sédillot, *Prolégomènes des tables astronomiques*, 2:106–7 (Ulugh Beg). Several Islamic tables of solar azimuth as a function of solar altitude and solar longitude, with entries for each degree of both arguments, are discussed in my paper on Islamic tables for timekeeping (see n. 7), and in King, "Ibn Yūnus' *Very Useful Tables*," pp. 362–64.

[23] See P. Luckey, "Ṯābit b. Qurra's Buch über die ebenen Sonnenuhren," *Quellen und Studien zur Geschichte der Mathematik, Astronomie, und Physik*, 4 (1937–38): 141–42, on the terminology, and Kennedy "al-Bīrūnī on Determining the Meridian," p. 636, for the usage of al-Bīrūnī, and King, "The Astronomical Works of Ibn Yūnus," pp. 176–80 for the usage of Ibn Yūnus. Ibn Yūnus tabulates $k(h)$ for the latitudes of Cairo and Baghdad for each degree of h: cf. King, "The Astronomical Works of Ibn Yūnus," pp. 170–71, 173, 179–80, and Schoy, *Gnomonik der Araber*, p. 81. al-Khalīlī's table of g_φ (cf. section 3) defines $k(h, \varphi)$ for each degree of both arguments.

Secondly, the rising amplitude, here denoted by ψ, is defined by means of the standard formula[24]

$$\operatorname{Sin} \psi(\delta, \varphi) = \frac{R \operatorname{Sin} \delta}{\operatorname{Cos} \varphi}. \tag{8}$$

With these we then form the "equation of the azimuth," here denoted by e, and defined by

$$e(h, \delta, \varphi) = k(h, \varphi) - \operatorname{Sin} \psi(\delta, \varphi), \tag{9}$$

and finally the azimuth, a, *measured from the meridian* (al-Marrākushī measures it from the east-west line), is given by

$$a(h, \delta, \varphi) = \operatorname{arc Cos} \left\{ \frac{R\, e(h, \delta, \varphi)}{\operatorname{Cos} h} \right\}, \tag{10}$$

which may also be written

$$a(h, \delta, \varphi) = \operatorname{arc Cos} \left\{ \frac{R \left[\frac{\operatorname{Sin} h \operatorname{Tan} \varphi}{R} - \frac{R \operatorname{Sin} \delta}{\operatorname{Cos} \varphi} \right]}{\operatorname{Cos} h} \right\}. \tag{11}$$

Finally, since

$$q(\varphi, L) = a(h, \delta, \varphi)$$

for

$$h = h(\varphi, L) \quad \text{and} \quad \delta = \varphi_M,$$

we have

$$q(\varphi, L) = \operatorname{arc Cos} \left\{ \frac{R \left[\frac{\operatorname{Sin} h(\varphi, L) \operatorname{Tan} \varphi}{R} - \frac{R \operatorname{Sin} \varphi_M}{\operatorname{Cos} \varphi} \right]}{\operatorname{Cos} h(\varphi, L)} \right\}. \tag{12}$$

Formula (12) can be explained with reference to figure 6. We rotate the celestial sphere 90° about the diameter NS so that the projection of Z_M moves from Z'_M to Y. Note that both angles PON and $A'Z'_MY'$ measure φ. The functions used by al-Marrākushī are respectively

$$k = A'Y' = \frac{\operatorname{Sin} h \operatorname{Tan} \varphi}{R}, \tag{13}$$

$$\operatorname{Sin} \psi = A'O = \frac{R \operatorname{Sin} \varphi_M}{\operatorname{Cos} \varphi}, \tag{14}$$

and

$$e = OY' = k - \operatorname{Sin} \psi. \tag{15}$$

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[24] Cf. the similar treatment by Ḥabash, al-Battānī, and Ibn Yūnus, described in Nallino, *al-Battani*, 1:178 and King, "The Astronomical Works of Ibn Yūnus," p. 167. Ibn Yūnus tabulates both ψ and Sin ψ as functions of solar longitude for the latitude of Cairo. Cf. King, "The Astronomical Works of Ibn Yūnus," pp. 169–70 and Schoy, *Gnomonik der Araber*, p. 81. al-Khalīlī's table of f_φ (cf. section 3) defines Sin ψ (δ, φ) for each degree of both arguments. On the rising amplitude, see also E. S. Kennedy and H. Sharkas, "Two Medieval Methods for Determining the Obliquity of the Ecliptic," *The Mathematics Teacher* 55 (1962): 286–90.

But Y is the projection of Z_M in the horizon plane, so that OY measures Cos h and the arc KS measures q. Thus we have

$$OY' = \frac{\operatorname{Cos} h \operatorname{Cos} q}{R}. \tag{16}$$

Formula (12) follows from (15) and (16).

3. *On the Use of al-Khalīlī's Auxiliary Tables for Computing the* Qibla

It is clear from formula (3) that for given φ, the function $h(\varphi, L)$ is defined by the simple relation

$$\operatorname{Sin} h = c_1 + c_2 \operatorname{Cos} \Delta L, \tag{17}$$

where c_1 and c_2 are constants. Given $h(\varphi, L)$, it remains to find the corresponding azimuth $q(\varphi, L)$.

Now al-Khalīlī's auxiliary tables can be used to find $a(h, \delta, \varphi)$ for one degree intervals of each argument, with no more calculation than an addition or a subtraction. The functions tabulated by al-Khalīlī are the following:[25]

$$\left.\begin{aligned} f_\varphi(\vartheta) &= \frac{R \operatorname{Sin} \vartheta}{\operatorname{Cos} \varphi} \\ g_\varphi(\vartheta) &= \frac{\operatorname{Sin} \vartheta \operatorname{Tan} \varphi}{R} \\ \text{and} \qquad G(x, y) &= \operatorname{arc Cos} \left\{ \frac{R\, x}{\operatorname{Cos} y} \right\} \end{aligned}\right\} \tag{18}$$

computed to two sexagesimal digits for the domains

$$\vartheta = 1°, 2°, \ldots, 90°$$

$$\varphi = 1°, 2°, \ldots, 55° \text{ and } \begin{cases} 21;30° \text{ (Mecca)} \\ 33;30° \text{ (Damascus)} \end{cases}$$

$$x = 1, 2, \ldots, 59$$

and

$$y = 0°, 1°, \ldots, n(x),$$

where $n(x)$ is the largest integer such that

$$R\, x \leqslant \operatorname{Cos} n(x).$$

---

[25] See King, "al-Khalīlī's Auxiliary Tables for Solving Problems of Spherical Astronomy" for further details and my paper on medieval prayer-tables (see n. 7) for additional manuscript sources. In MS Escorial ar. 931 the tables of f_φ and g_φ are given only for latitudes 1° through 50°. In all other known sources values are given for latitudes up to 55°, although MS Berlin 5755 (Wetzstein 1138) fol. 35r contains an additional page ruled and titled for latitude 56° but there are no entries. The values extracted from the auxiliary tables in this section are taken from MS Paris Bibliothèque Nationale ar. 2558.

Thus the tables contain nearly 13,500 entries. The solar azimuth for any altitude, declination, and terrestrial latitude is found by the simple procedure

$$a(h, \delta, \varphi) = G\{[g_\varphi(h) - f_\varphi(\delta)], h\}, \tag{19}$$

which is clearly equivalent to formula (10) above. Thus the *qibla* is defined by

$$q(\varphi, L) = G\{x(\varphi, L), h\} \tag{20}$$

where

$$x(\varphi, L) = g_\varphi(h) - f_\varphi(\varphi_M) \tag{21}$$

and $h(\varphi, L)$ has been previously determined using (17) or an equivalent formula. Note that q is northerly or southerly according as $x \lessgtr 0$. It is the absolute value of x which is entered in the auxiliary tables.

The main problem encountered in using the auxiliary tables for *qibla* determinations results from the fact that the quantities

$$\varphi_M, \qquad h(\varphi, L), \qquad \text{and} \qquad x(\varphi, L)$$

are generally non-integral, and one must perform several interpolations to obtain the value of q. Also the function $G(x, y)$ is very sensitive to small changes in x for certain arguments, and an error of, say, 0;1 introduced in calculating the argument $x(\varphi, L)$ might influence the final value of q considerably. Furthermore, the auxiliary tables themselves contain occasional small errors, and a combination of these in each of the three sets of tables might introduce a relatively large error in the final value of q.

The following examples show how al-Khalīlī's auxiliary tables can be used to compute *qibla* values. In computing the values of his functions, linear interpolation is considerably facilitated by having at hand a sexagesimal multiplication table, such as would probably have been available to al-Khalīlī.[26] I use the rounding procedure[27]

$$p;q \rightarrow \begin{cases} p & (q \leqslant 29) \\ p+1 & (q \geqslant 30) \end{cases}$$

and indicate the errors in the second digit of each number in square brackets. Note that we have no way of ascertaining the values of $h(\varphi, L)$ and $x(\varphi, L)$ which might have been computed by al-Khalīlī, if indeed he used this method at all. There is also the possibility that he had at his disposal a set of auxiliary tables giving entries to three sexagesimal digits, and that he might have performed all his calculations with three digit numbers before rounding. Thus, it is hardly surprising that computation with al-Khalīlī's auxiliary tables does not generally yield his *qibla* values precisely.

Firstly, consider the entry for arguments

$$\varphi = 28^\circ \qquad \text{and} \qquad \Delta L = 10^\circ \; (L = 57^\circ, 77^\circ).$$

al-Khalīlī has the rather inaccurate value

$$q = 56;27^\circ \text{ S} [-8].$$

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[26] On such tables in the Islamic sources see D. A. King, "On Medieval Islamic Multiplication Tables," *Historia Mathematica* 1 (1974): 317–23.

[27] This is the procedure of Ibn Yūnus: see King, "The Astronomical Works of Ibn Yūnus," p. 59.

Now the accurate value of h for these arguments is 78;50°, and so

$$q = G\{[g_{28}(78;50) - f_{28}(21;30)], 78;50\}.$$

To compute this we use the following entries in the three auxiliary tables:

f_{28}	
21	24;22 [+1]
22	25;28 [+1]

g_{28}	
78	31;10 [−2]
79	31;17 [−2]

G	6	7
78	61;13 [−2]	55;50 [−2]
79	58;24 [0]	52;19 [+1]

and obtain by linear interpolation

$$f_{28}(21;30) = 24;55\,[+1] \qquad \text{and} \qquad g_{28}(78;50) = 31;16\,[-2]$$

so that

$$x = +6;21\,[-3].$$

However,

$$G(6;21, 78;50) = 56;47^\circ \text{ S} [+12].$$

Thus, in this case, al-Khalīlī's value, which is in error by −0;8°, is more accurate than the value which I derive directly from his auxiliary tables using linear interpolation. However, if x had been more carefully computed to 6;23, the value for q given by the auxiliary tables is the accurate value 56;35°. The yet more precise argument 6;24 yields 56;29° for q, which is close to al-Khalīlī's value.

Secondly, consider the entry for arguments

$$\varphi = 35^\circ \qquad \text{and} \qquad \Delta L = 15^\circ \qquad (L = 52^\circ, 82^\circ).$$

al-Khalīlī has the value

$$q = 48;15^\circ \text{ S} [+3]$$

and the accurate value of h is 71;9°. Thus

$$q = G\{[g_{35}(71;9) - f_{35}(21;30)], 71;9\}.$$

Now using the following entries in al-Khalīlī's tables

f_{35}	
21	26;15 [0]
22	27;25 [−1]

g_{35}	
71	39;44 [+1]
72	39;58 [+1]

G	12	13
71	52; 6 [0]	48;16 [−1]
72	49;39 [−1]	45;28 [−1]

I obtain

$$f_{35}(21;30) = 26;50\,[-1] \qquad \text{and} \qquad g_{35}(71;9) = 39;46\,[0]$$

so that

$$x = +12;56\,[+1]$$

and

$$q = G(12;56,\ 71;9) = 48;6^\circ\ \mathrm{S}\,[-6].$$

Using the more accurate arguments 12;55 and 12;54 for x in the table of G, I obtain respectively

$$q = 48;10^\circ\ \mathrm{S}\,[-2] \qquad \text{and} \qquad q = 48;14^\circ\ \mathrm{S}\,[+2].$$

Finally, note that it is occasionally possible to reproduce al-Khalīlī's actual *qibla* values. Consider, for example, the entry for arguments

$$\varphi = 12^\circ \qquad \text{and} \qquad \Delta L = 18^\circ \qquad (L = 49^\circ,\ 85^\circ).$$

al-Khalīlī has 58;39° for q, whereas the accurately computed value is 58;45°. For these arguments the value of h is 70;21°. Thus,

$$q = G\{[g_{12}(70;21) - f_{12}(21;30)],\ 70;21\},$$

and the relevant entries in the auxiliary tables are

$$f_{12}\ \begin{array}{c|ll} 21 & 21;59 & [0] \\ \hline 22 & 22;58 & [-1] \end{array}, \qquad g_{12}\ \begin{array}{c|ll} 70 & 11;59 & [0] \\ \hline 71 & 12;\ 3 & [-1] \end{array},$$

$$\begin{array}{cc|ll|ll} & & \multicolumn{2}{c|}{10} & \multicolumn{2}{c}{11} \\ \hline G & 70 & 60;49 & [-1] & 57;35 & [0] \\ \hline & 71 & 59;12 & [0] & 55;42 & [-2] \end{array}.$$

Using linear interpolation we have

$$f_{12}(21;30) = 22;29\,[0] \qquad \text{and} \qquad g_{12}(70;21) = 12;0\,[-1]$$

and hence

$$x = -10;29\,[-1]$$

so that

$$q = G(-10;29,\ 70;21) = 58;39^\circ\ \mathrm{N}\,[-6],$$

which is precisely al-Khalīlī's value.

al-Khalīlī's *qibla* values are generally more accurate than those which can be derived from his auxiliary tables in this way. Thus the possibility that he computed his *qibla* values independently of the auxiliary tables cannot be ruled out. Perhaps he computed certain basic values using the analogue computer known as the Sine quadrant, although the one *qibla* value which he says that he computed with a quadrant is less accurate than the corresponding entry in his *qibla* table. In the following three sections I discuss a method for using the Sine quadrant to determine the *qibla*. A treatise on the Sine quadrant by al-Khalīlī survives in the Egyptian National Library in Cairo,[28] but at the time this paper was in preparation the manuscript was inaccessible to me.

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[28] See Suter, "Die Mathematiker und Astronomen der Araber," no. 418. A treatise on the (Sine?) quadrant by one Muḥammad ibn Muḥammad, perhaps al-Khalīlī, was translated into Hebrew by an individual named Moshe Galliano ben Yehuda: see M. Steinschneider, *Die hebraeischen Übersetzungen des Mittelalters und die Juden als Dolmetscher* (1893; reprint ed., Graz, 1956), pp. 575–77.

4. *On the Solution of the* Qibla *Problem using a Sine Quadrant*

The distinctive feature of the instrument called in Arabic *al-rubʿ al-mujayyab*, and here referred to as Sine quadrant, is a grid which can be used as an analogue computer to solve problems of trigonometry without any computation. This instrument has been discussed in some detail by P. Schmalzl (1929) in his valuable study on the development of different types of quadrant in the Islamic world,[29] and the related Arabic technical terminology has been investigated by E. Wiedemann (1909).[30] In passing, we may note that similar instruments, known by the Latin name *sexagenarium*, were also used in late medieval Europe, though to a far lesser extent than the Sine quadrant was used in the Islamic world.[31] The reader is referred to the works of W. Morley (1860), B. Dorn (1865), P. Schmalzl, or L. A. Mayer (1956) for illustrations of the Sine quadrant,[32] and to an article by E. Poulle (1966) for an illustration of a similar European instrument.[33]

In the introduction to the *qibla* table, al-Khalīlī does not describe the Sine quadrant which he used to compute a particular *qibla* value (see section 1). However, the grid on a quadrant made by Muḥammad ibn Aḥmad al-Mizzī,[34] another colleague of Ibn al-Shāṭir, which is illustrated by Dorn and Schmalzl, consists of two sets of equally spaced lines drawn parallel to the axes of the grid. These lines divide each axis into 60 equal parts, and the circumference of the quadrant is divided into 90 equal degree intervals. This particular quadrant has a radius of 7½ inches and the grid a radius of 6 inches. It would have originally been fitted with a thread (*khayṭ*) passing through the hole at the center of the grid, and bearing a movable marker (*murī* or *mūrī*). Some Islamic Sine quadrants have a pair of semicircles drawn with the axes of the grid as diameter: these are particularly useful for determining the Sine and Cosine of a given arc. In figure 7, if the arc AC measures ϑ, then the marker can be moved to D to measure Cos ϑ and to E to measure Sin ϑ. If there are no such semicircles drawn on the grid, Cos ϑ is measured on the horizontal axis by OC′ and Sin ϑ on the vertical axis by OC″. I use this prime and double prime notation in the sequel to denote projections on the horizontal and vertical axes of points on the altitude scale, that is, on the quadrant AB.

In Appendix B I describe a theoretical procedure for determining the *qibla* using a Sine quadrant and the formulae of al-Marrākushī. This procedure is not convenient for certain values of φ and ΔL since it involves operations with quantities whose absolute value may be greater than 60. The method of the fifteenth century astronomer Sibṭ al-Māridīnī for finding the *qibla* by means of a Sine quadrant is mathematically accurate, and will serve to illustrate the facility with which a fairly complicated trigonometric calculation can be performed with such an instrument. His method is also not convenient for certain values of φ and ΔL. Indeed, for a small section of the domain for which al-Khalīlī tabulates $q(\varphi, L)$, both procedures involve the construction of points outside the quadrant. Sibṭ al-Māridīnī's procedure is described in section 6. It is based on formulae originally proposed by his predecessor Ibn Yūnus, and these we consider first.

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[29] See Schmalzl, *Zur Geschichte des Quadranten*, especially pp. 83–99. See also J. Würschmidt, "Die Schriften Gedosis über die Höhenparallelen und über die Sinustafel," *Sitzungsberichte der physikalisch-medizinischen Sozietät zu Erlangen* 60 (1928): 127–54, for a translation and analysis of a late Ottoman treatise on the use of the Sine quadrant.

[30] Wiedemann, *Aufsätze der arabischen Wissenschaftsgeschichte*, 1:560–64.

[31] See L. Thorndike, "Sexagenarium," *Isis* 42 (1951): 130–33, and E. Poulle, "Théorie des planètes et trigonometrie au XV^e siècle, d'après un equatoire inédit, le Sexagenarium," *Journal des Savants*, juillet-septembre 1966, pp. 129–61.

[32] W. H. Morley, "Description of an Arabic Quadrant," *Journal of the Royal Asiatic Society* 17 (1860), fig. 2 following p. 330; B. Dorn, "Drei in der kaiserlichen öffentlichen Bibliothek befindliche astronomische Instrumente mit arabischen Inschriften," *Mémoires de l'Académie Impériale des Sciences de St. Petersbourg* 9 (1865), pls. 1 and 2 following p. 150; Schmalzl, *Zur Geschichte des Quadranten*, pp. 86–87; and L. A. Mayer, *Islamic Astrolabists and their Works* (Geneva, 1956), pl. 26. Islamic astrolabes often display trigonometric grids on the verso: see ibid., pls. 23 and 24.

[33] Poulle, "Théorie des planètes," p. 135, fig. 2.

[34] On al-Mizzī see Suter, "Die Mathematiker und Astronomen der Araber," no. 406; Brockelmann, *Geschichte der arabischen Litteratur*, 1:155–56 and ibid., Supp. 2:156; and Mayer, *Islamic Astrolabists*, pp. 61–62.

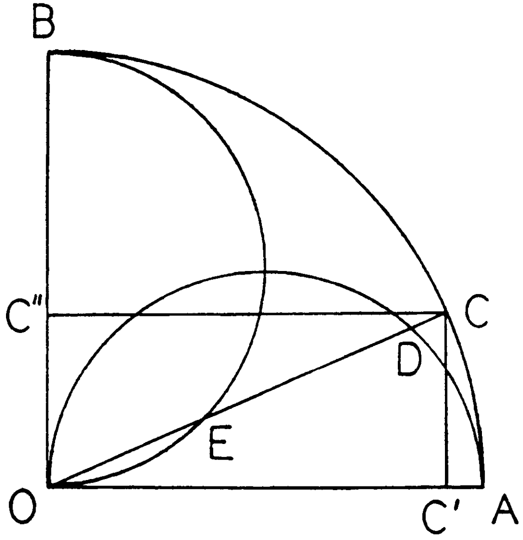

FIG. 7

5. *On the Method of Ibn Yūnus for Determining the* Qibla

Ibn Yūnus worked in Cairo-Fustat at the end of the tenth century. His major work, an astronomical handbook with tables (*zīj*), was called the *Ḥākimī Zīj* because it was dedicated to the Fatimid Caliph al-Ḥākim. This work is one of the finest examples of some one hundred and fifty *zīj*es which are now known to have been compiled by the astronomers of medieval Islam.[35] Among his other compilations are extensive trigonometric tables with values for each minute of arc and part of the corpus of tables for astronomical timekeeping which was used in medieval Cairo.[36] Another extensive table attributed to Ibn Yūnus displays the lunar equation as a function of two arguments which can be taken directly from the lunar mean motion tables, thus solving a computational problem which occupied later Muslim and European astronomers for several centuries.[37]

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[35] On Islamic *zījes* see Kennedy, "A Survey of Islamic Astronomical Tables," and on the *Ḥākimī Zīj* see King, "The Astronomical Works of Ibn Yūnus," currently being submitted for publication.

[36] On the trigonometric tables see King, "The Astronomical Works of Ibn Yūnus," pp. 85–89 and 109, and on the tables for timekeeping see n. 8 above.

[37] An analysis of this table, which contains over 34,000 entries, is presented in D. A. King, "A Double-Argument Table for the Lunar Equation Attributed to Ibn Yūnus," *Centaurus* 18 (1974): 129–46. The table appears to have been derived from a rather simple auxiliary table containing over 10,000 entries.

The major part of the extant chapters of *Ḥākimī Zīj* is concerned with trigonometry and spherical astronomy. In chapter 28 Ibn Yūnus describes in words, and without justification, the following formulae for finding the *qibla*.[38] Firstly,

$$h = \text{arc Sin}\left\{\frac{\dfrac{\text{Cos}\,\varphi\,\text{Cos}\,\Delta L}{R}\,\text{Cos}\,\varphi_{\text{M}} + \text{Sin}\,\varphi_{\text{M}}\,\text{Sin}\,\varphi}{R}\right\}, \tag{22}$$

and then

$$q = \text{arc Sin}\left\{\frac{\text{Sin}\,\Delta L\,\text{Cos}\,\varphi_{\text{M}}}{\text{Cos}\,h}\right\}. \tag{23}$$

These formulae can be explained with reference to figure 6, in which GHY′J is a rectangle. Since

$$Z'_{\text{M}}\text{G} = \frac{\text{Cos}\,\Delta L\,\text{Cos}\,\varphi_{\text{M}}}{R},$$

we have

$$Z'_{\text{M}}\text{H} = \frac{\text{Cos}\,\Delta L\,\text{Cos}\,\varphi_{\text{M}}\,\text{Cos}\,\varphi}{R^2}.$$

Also

$$\text{GJ} = \frac{\text{Sin}\,\varphi_{\text{M}}\,\text{Sin}\,\varphi}{R}.$$

But

$$Z'_{\text{M}}\text{Y}' = \text{Sin}\,h = Z'_{\text{M}}\text{H} + \text{GJ},$$

whence formula (22), which is mathematically equivalent to formula (3). Furthermore,

$$\text{M}Z'_{\text{M}} = \frac{\text{Sin}\,\Delta L\,\text{Cos}\,\varphi_{\text{M}}}{R}$$

and

$$\text{YY}' = \frac{\text{Cos}\,h\,\text{Sin}\,q}{R}.$$

But both $\text{M}Z'_{\text{M}}$ and YY′ measure the distance from Z_{M} to the meridian, and are thus equal. Formula (23) follows immediately.

6. *Sibṭ al-Māridīnī's Method for Finding the* Qibla *by Means of a Sine Quadrant*

Sibṭ al-Māridīnī was the leading Egyptian astronomer of the late fifteenth century. He was an extremely prolific writer, and his works reflect his competence if lack of originality.[39] We are concerned here with his treatise on the Sine quadrant, which was the most popular treatise of its kind during the next four centuries, that is, throughout the period of decline of traditional Islamic astronomy. Literally hundreds of manuscripts of this treatise survive: I have used MS Princeton Yahuda 861.[40] Sibṭ al-Māridīnī's *qibla* method is based on the formulae of Ibn Yūnus described in section 5. His teacher, Ibn al-Majdī, used precisely the same formulae as Ibn Yūnus.[41]

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[38] See Schoy, *Gnomonik der Araber*, pp. 36–41 and King, "The Astronomical Works of Ibn Yūnus," pp. 256–68 for further details. Ibn Yūnus also tabulated the solar altitude in the azimuth of the *qibla*: see Schoy, *Gnomonik der Araber*, no. 42; King, "The Astronomical Works of Ibn Yūnus," pp. 266–68, and idem, "Ibn Yūnus' *Very Useful Tables*," pp. 368–69.

[39] On Sibṭ al-Māridīnī see Suter, "Die Mathematiker und Astronomen der Araber," no. 445 where twenty-five of his works are listed, and Brockelmann, *Geschichte der arabischen Litteratur*, 2:468, and ibid., Supp., 2:484.

In order to understand Sibṭ al-Māridīnī's instructions we must first note that he uses two auxiliary quantities which he calls *al-aṣl al-muṭlaq* and *buʿd al-quṭr*. I denote these by "absolute base" and "center-altitude" respectively. The first quantity is simply

$$B = \frac{\operatorname{Cos} \varphi_M \operatorname{Cos} \varphi}{R}$$

and, but for a factor of R, is identical with the "base" used by al-Marrākushī (see section 2). The second quantity is

$$C = \frac{\operatorname{Sin} \varphi_M \operatorname{Sin} \varphi}{R}$$

and measures the altitude of the center of the day-circle of the zenith of Mecca above the local horizon, that is, GJ in figure 6 or KJ in figure 11. The expression *buʿd al-quṭr* literally means "distance of the diameter." Tables of both

$$B(\lambda) = \frac{\operatorname{Cos} \delta(\lambda) \operatorname{Cos} \varphi}{R} \quad \text{and} \quad C(\lambda) = \frac{\operatorname{Sin} \delta(\lambda) \operatorname{Sin} \varphi}{R}$$

computed for specific latitudes are attested in the Islamic sources, the first function being more commonly tabulated than the second. Sibṭ al-Māridīnī's contemporary, the Egyptian astronomer Ibn Abī l-Fatḥ al-Ṣūfī,[42] actually tabulated the functions

$$B(\delta, \varphi) = \frac{\operatorname{Cos} \delta \operatorname{Cos} \varphi}{R} \quad \text{and} \quad C(\delta, \varphi) = \frac{\operatorname{Sin} \delta \operatorname{Sin} \varphi}{R}$$

for each degree of both arguments. It is not difficult to show that the hour-angle t is defined in terms of these auxiliary functions by

$$\frac{\operatorname{Cos} t}{R} = \frac{\operatorname{Sin} h - C}{B}.$$

With B and C tabulated, the calculation of time from solar or stellar altitude is reduced to very simple procedures, whether one is calculating by hand or using a Sine quadrant.

In chapters 4 and 5 of his treatise Sibṭ al-Māridīnī describes the following straightforward procedures for finding these auxiliary quantities:

To find the center-altitude, place the thread on the vertical scale and put the marker on the Sine of the latitude. Then move the thread to the declination measured upwards on the altitude scale. The marker will indicate the center-altitude on the vertical scale.

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[40] A Xerox copy of this manuscript was kindly provided by the Director of the Special Collections Library, Princeton University.

[41] On Ibn al-Majdī, see Suter, "Die Mathematiker und Astronomon der Araber," no. 432, and Brockelmann, *Geschichte der arabischen Litteratur*, 2:158–59, and ibid., Supp., 2:158–59. The treatise on the *qibla* is entitled *Tuḥfat al-aḥbāb fī naṣb al-bādahanj wa-l-miḥrāb*. On this work see Wiedemann, *Aufsätze der arabischen Wissenschaftsgeschichte*, 1:546–47 and 589, and King, "The Astronomical Works of Ibn Yūnus," pp. 262–63, and idem, "Ibn Yūnus' *Very Useful Tables*," p. 373.

[42] On Ibn Abī l-Fatḥ see Suter, "Die Mathematiker und Astronomen der Araber," no. 447, and Brockelmann, *Geschichte der arabischen Litteratur*, Supp., 2:159. His auxiliary tables and all other known auxiliary tables of B and C and related functions specifically intended for facilitating the computations of astronomical timekeeping are discussed in my paper on Islamic tables for timekeeping (see n. 7).

To find the absolute base, place the thread on the vertical scale and put the marker on the Cosine of the latitude. Then move the thread to the declination measured downwards on the altitude scale. The marker will indicate the absolute base on the vertical scale.

In figure 8, mark OC equal to Sin φ. With this radius draw the arc CD, where D is on OE and the arc AE measures δ. Draw the horizontal DF. Clearly OF measures the center-altitude. Similarly, in figure 9 mark OG equal to Cos φ. With this radius draw the arc GH, where H is on OI and the arc BI measures δ. Draw the horizontal HJ. Clearly OJ measures the base.

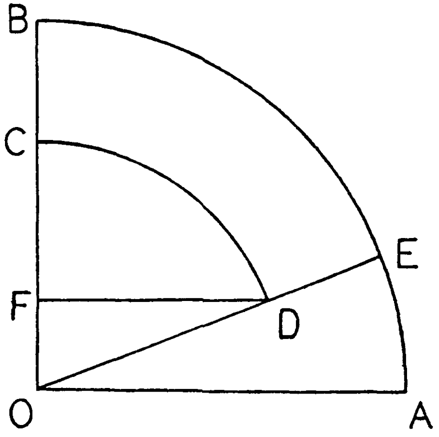

FIG. 8

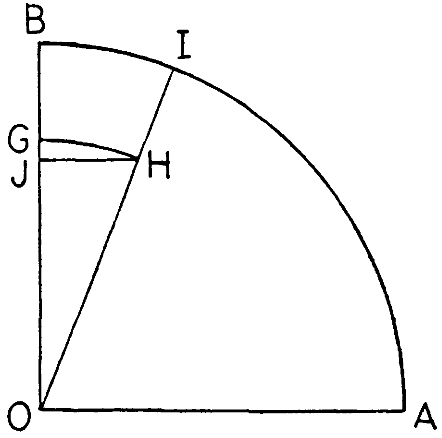

FIG. 9

We are now in a position to consider Sibṭ al-Māridīnī's method for finding the *qibla*, outlined in chapter 17 of his treatise. The instructions are terse and the terminology rather technical. I therefore present both the Arabic text and a free translation.

فى معرفة استخراج سمت القبلة استخرج الاصل وبعد القطر بالميل المساوى لعرض مكة وهو كا
درجة ثم ضع على الستينى وعلم على الاصل وانقل الخيط لفضل الطولين من معكوس القوس وهو فى
مصر يب درجة ثم زد على ما حازه المرى من الجيوب المبسوطة بعد القطر يحصل ارتفاع سمت
مكة وعلم على جيب فضل الطولين من الجيوب المبسوطة ثم انقل الخيط الى عرض مكة ونزل من
المرى فى الجيوب المنكوسة الى القوس تجد من اوله سمت مكة وهو شرقى إن كانت مكة
اعرض من بلدك و مساوية لها وإن كانت اقل عرضاً من بلدك فاستخرج الارتفاع الذى لا
سمت له بالميل المساوى لعرض مكة فإن كان اقل من ارتفاع سمت مكة فسمتها جنوبى وإن كان
اكثر من ارتفاع سمتها فهو شمالى والله اعلم .

Translation

On determining the azimuth of the *qibla*. First find the base and the center-altitude corresponding to a declination equal to the latitude of Mecca, which is 21°. Place the

> thread on the vertical axis and put the marker at the point corresponding to the magnitude of the base. Move the thread by the amount of the longitude difference, measured downwards on the altitude scale. This difference is 12° for Cairo. Next add the center-altitude to the reading on the vertical axis corresponding to the position of the marker. This will give you the altitude of the zenith of Mecca on the altitude scale. Now place the thread on the complement of this altitude and put the marker at a point on the thread corresponding to the Sine of the longitude difference on the vertical axis. Next move the thread to the value of the latitude of Mecca on the altitude scale. Read off the arc corresponding to the point on the horizontal axis directly below the marker. It will measure the azimuth of Mecca with respect to the east- or west-points. The azimuth will be eastern if Mecca has a greater longitude than your locality, otherwise it will be western. It will be northern if Mecca has a greater latitude than your locality or if Mecca has the same latitude. If Mecca is south of your locality, find the altitude in the prime vertical corresponding to a declination equal to the latitude of Mecca. If this altitude is less than the altitude of the zenith of Mecca, the azimuth will be southern. Otherwise it will be northern. But God knows better.

This construction is shown in figure 10 and may be justified as follows. Firstly, we mark

$$\mathrm{OC} = \frac{\mathrm{Cos}\,\varphi_{\mathrm{M}}\,\mathrm{Cos}\,\varphi}{R},$$

draw arc CD equal to ΔL, and draw the horizontal DE. Then

$$\mathrm{OE} = \frac{\mathrm{Cos}\,\Delta L\,\mathrm{Cos}\,\varphi_{\mathrm{M}}\,\mathrm{Cos}\,\varphi}{R^2}.$$

Now construct F″ on EB such that

$$\mathrm{EF}'' = \frac{\mathrm{Sin}\,\varphi_{\mathrm{M}}\,\mathrm{Sin}\,\varphi}{R}.$$

Then

$$\mathrm{OF}'' = \mathrm{OE} + \mathrm{EF}'' = \mathrm{Sin}\,h$$

by virtue of formula (22), so that arc FA measures h.

Now draw the arc AG equal to $\bar{h}$, and the arc AH equal to ΔL. The point I is the intersection of OG and HH″, and IJ is drawn perpendicular to OA. Then

$$\mathrm{OI} = \frac{\mathrm{OG}\cdot\mathrm{IJ}}{\mathrm{GG}'} = \frac{R\,\mathrm{Sin}\,\Delta L}{\mathrm{Cos}\,h}.$$

Note that if $\bar{h}$ *is less than* ΔL *the point* I *is outside the quadrant* AB. Now rotate OI to OK, where K lies on OL and arc AL measures φ_{M}. Draw QQ′ through K, perpendicular to OA. Then

$$\mathrm{OQ}' = \frac{\mathrm{Sin}\,\Delta L\,\mathrm{Cos}\,\varphi_{\mathrm{M}}}{\mathrm{Cos}\,h}$$

and AQ measures the azimuth of the *qibla*, measured from the east- or west-points, by virtue of formula (23).

Sibṭ al-Māridīnī notes the conditions on h which determine whether the azimuth of the *qibla* is north or south when the locality has latitude φ_{M}. These are expressed in terms of the altitude in the prime vertical of Z_{M}, here denoted by h_0. This is the altitude of the intersection of AB and ZE in figure 3, which is given by

$$h_0 = \mathrm{arc\ Sin}\left\{\frac{\mathrm{Sin}\,\varphi_{\mathrm{M}}}{\mathrm{Sin}\,\varphi}\right\}. \tag{24}$$

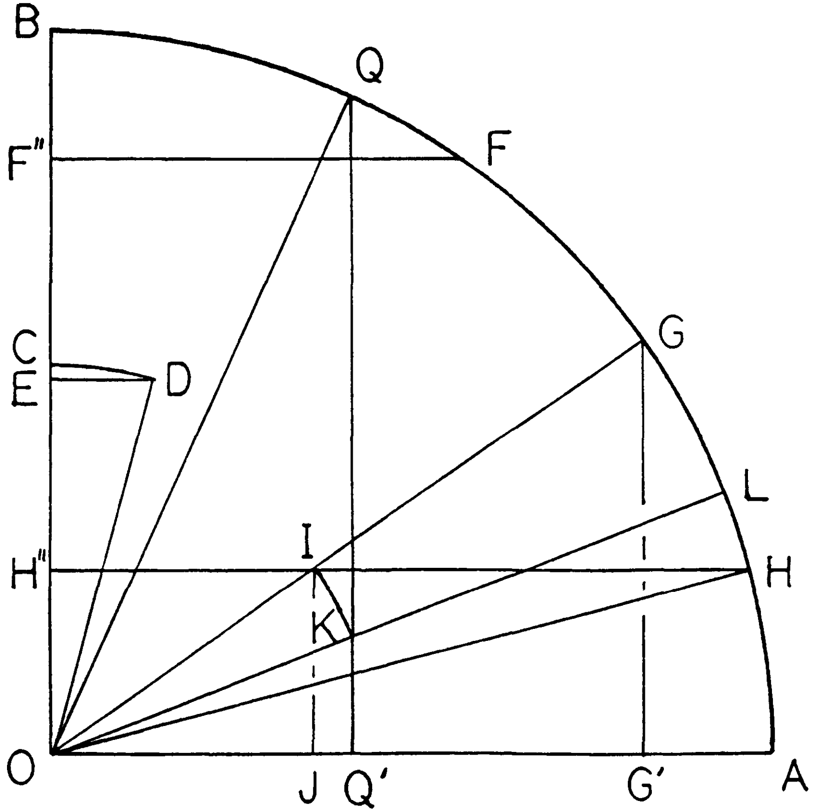

FIG. 10

Sibṭ al-Māridīnī correctly states that

$$q \text{ is} \begin{cases}\text{northerly}\\ \text{southerly}\end{cases} \quad \text{as } h \lessgtr h_0 \text{ for } \varphi = \varphi_{\mathrm{M}}.$$

Ibn Yūnus had incorrectly stated that the *qibla* is easterly or westerly for localities with the same latitude as Mecca.

APPENDIX A

On the Qibla *Construction of Ibn al-Haytham*

Although Ibn al-Haytham's construction has already been discussed in detail by K. Schoy,[43] I include it here because it is the simplest possible geometrical method for finding the *qibla*. There is no need to find the auxiliary quantity h. Ibn al-Haytham does not derive any trigonometric formula for q from the construction, but the modern formula follows from it immediately. Figure 11 is a slightly simplified version of the construction shown in the manuscript, and the broken lines are my own additions. The construction is described below only in outline, and the reader is referred to Schoy's translation of Ibn al-Haytham's entire treatise for the details.

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[43] See Schoy, "Abhandlung des al-Ḥasan ibn al-Ḥasan ibn al-Haiṯam (Alhazen)," summarized in his article in *Encyclopaedia of Islam*, 1st ed., s.v. "Ḳibla."

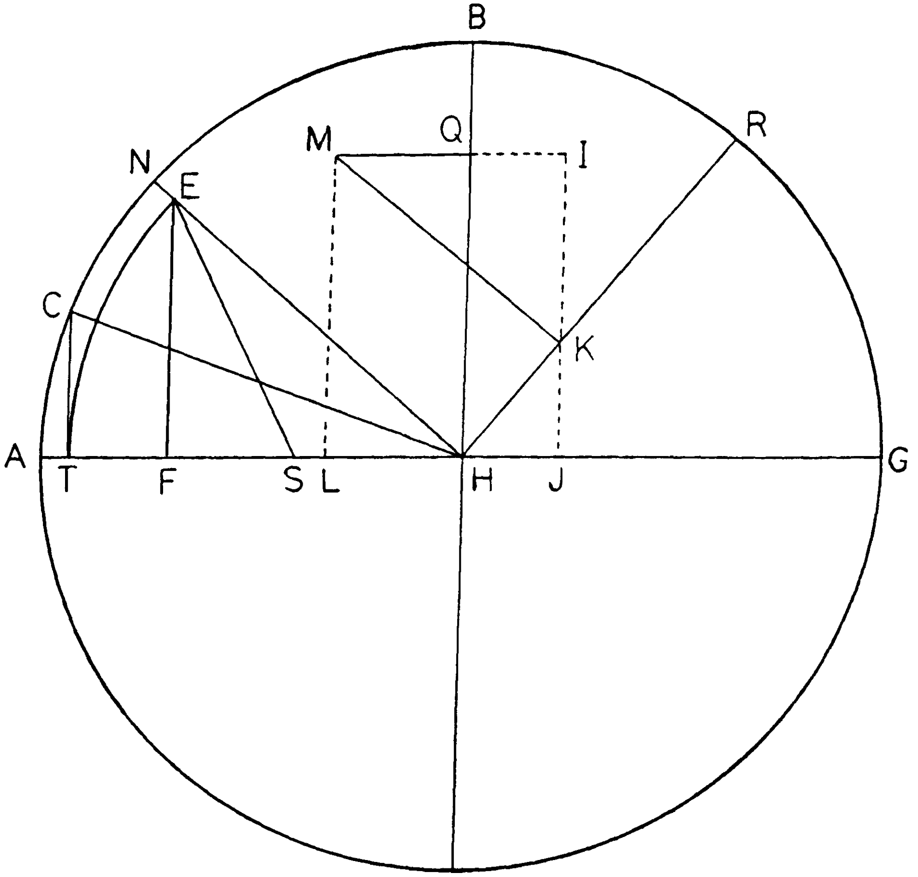

FIG. 11

The circle ABGD with center H is described on a horizontal plane, and AG and BD are perpendicular diameters. Mark an arc AC equal to the latitude of Mecca (φ_M) on the circumference of the circle, which represents the horizon of the locality for which the *qibla* is required. Similarly the arc GR is drawn equal to the local latitude (φ). Finally, the arc AN measures the longitude difference (ΔL). Now construct CT perpendicular to AH, and with center H and radius HT describe an arc TE to cut HN in E. Draw EF perpendicular to AH. Mark a point K on the radius HR such that HK equals CT. Then draw KM perpendicular to HR and such that KM equals FH. From M drop the perpendicular MQ onto HB, and construct a point S on FH such that FS equals MQ. Draw ES. Then the angle ESF measures the azimuth of the *qibla*, measured from the meridian.

Ibn al-Haytham's construction can be justified as follows. Consider ABGD as representing the plane of the celestial equator, so that AHG is in the meridian plane. Since arc AC measures φ_M, we have

$$EH = TH = \mathrm{Cos}\,\varphi_M.$$

Note that the arc TE represents part of the projection of the day-circle through Z_M onto the equatorial plane, and since the arc TE measures ΔL, the point E represents the projection of Z_M onto that plane. Further, the distance from the projection of Z_M on the meridian plane to the center of the day-circle of Z_M is

$$FH = \frac{EH\ \mathrm{Cos}\,\Delta L}{R} = \frac{\mathrm{Cos}\,\varphi_M\ \mathrm{Cos}\,\Delta L}{R}.$$

Now consider figure 11 as the projection of the celestial sphere in the meridian plane. MK is a segment of the projection of the day-circle of Z_M in that plane, since by the construction HR represents the celestial axis, inclined at an angle φ to the horizon, and also K is such that

$$HK = CT = \mathrm{Sin}\,\varphi_M,$$

and MK is perpendicular to HR. Further, since

$$MK = FH$$

it follows that M is the projection of Z_M in the meridian plane. Thus MQ measures the distance from M and also from Z_M to the plane of the prime vertical. But EF measures the distance from Z_M to the meridian plane. Therefore angle ESF as constructed by Ibn al-Haytham defines the azimuth of Mecca.

Formula (1) can easily be derived from this construction. First draw the rectangle MLJI. Then

$$\begin{aligned} FS = MQ &= MI - QI \\ &= MI - HJ. \end{aligned}$$

Also

$$MI = \frac{MK\ \mathrm{Sin}\,\varphi}{R} = \frac{\dfrac{\mathrm{Cos}\,\varphi_M\ \mathrm{Cos}\,\Delta L}{R}\ \mathrm{Sin}\,\varphi}{R}$$

$$HJ = \frac{\mathrm{Sin}\,\varphi_M\ \mathrm{Cos}\,\varphi}{R}$$

and

$$EF = \frac{\mathrm{Cos}\,\varphi_M\ \mathrm{Sin}\,\Delta L}{R}.$$

Thus

$$\frac{\mathrm{Cot}\, q}{R} = \frac{\mathrm{FS}}{\mathrm{EF}} = \frac{\mathrm{Cos}\,\Delta L\,\mathrm{Sin}\,\varphi - \mathrm{Cos}\,\varphi\,\mathrm{Tan}\,\varphi_{\mathrm{M}}}{R\,\mathrm{Sin}\,\Delta L}$$

which is equivalent to formula (1).

[*Note added in proof:* MS Istanbul Fatih 3439/12 (fols. 155r–157v) contains another treatise by Ibn al-Haytham which describes a quite different method for determining the *qibla*.]

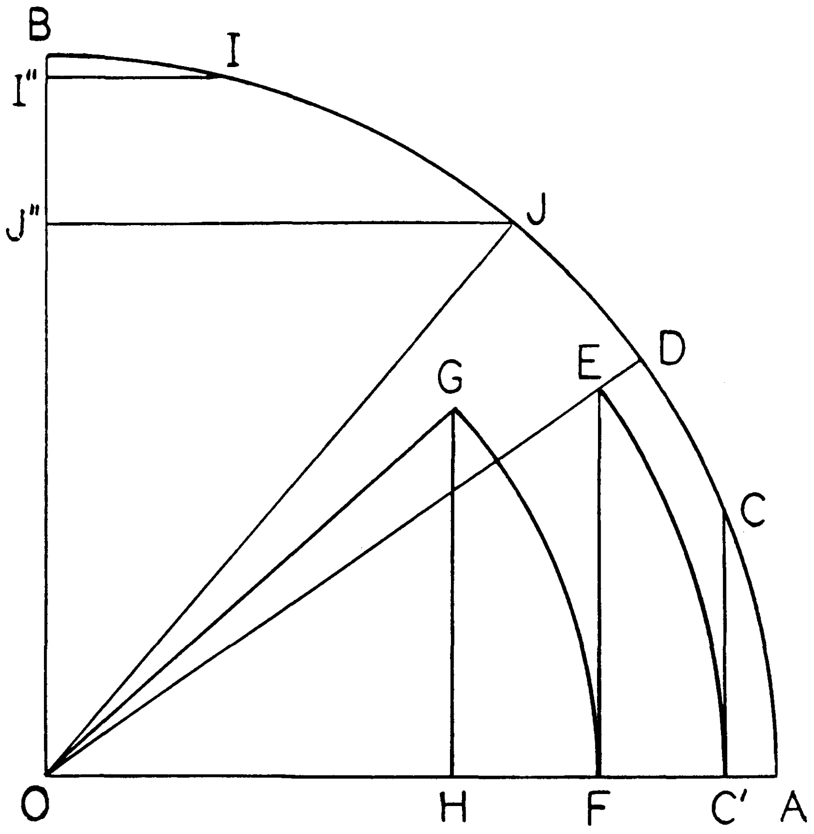

FIG. 12

APPENDIX B

A Theoretical Method for Determining the Qibla *Using a Sine Quadrant and the Formulae of al-Marrākushī*

Figure 12 shows a construction for determining $h(\varphi, L)$. On arc AB mark

$$\mathrm{AC} = \varphi_{\mathrm{M}} \quad \text{and} \quad \mathrm{AD} = \varphi.$$

Then draw the vertical CC′, and the arc C′E to meet OD in E. Draw the vertical EF and the arc FG such that

$$\mathrm{FG} = \Delta L.$$

Draw the vertical GH. Then we have

$$\mathrm{OC'} = \mathrm{Cos}\,\varphi_{\mathrm{M}}, \qquad \mathrm{OF} = \frac{\mathrm{Cos}\,\varphi_{\mathrm{M}}\,\mathrm{Cos}\,\varphi}{R},$$

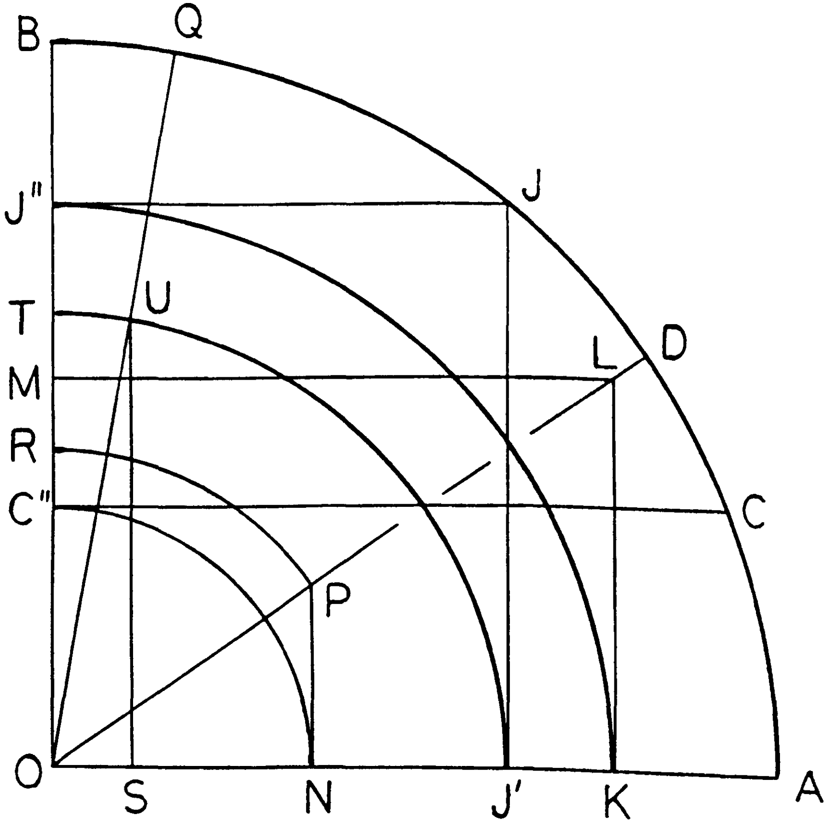

FIG. 13

and

$$\mathrm{FH} = \frac{\mathrm{Vers}\,\Delta L\,\mathrm{Cos}\,\varphi_{\mathrm{M}}\,\mathrm{Cos}\,\varphi}{R^2}.$$

Next mark the point I on arc AB such that

$$\mathrm{AI} = \bar{\varphi} + \varphi_{\mathrm{M}}$$

and draw the horizontal II″. Then

$$\mathrm{OI''} = \mathrm{Sin}\,(\bar{\varphi} + \varphi_{\mathrm{M}}).$$

On OI″ mark J″ such that I″J″ equals FH and draw the horizontal J″J to meet AB in J. Then, since

$$OJ'' = \text{Sin}\,(\varphi + \varphi_M) - \frac{\text{Vers}\,\Delta L\,\text{Cos}\,\varphi_M\,\text{Cos}\,\varphi}{R^2},$$

the arc AJ measures $h(\varphi, L)$ by virtue of formula (3).

Figure 13 shows a construction for determining $q(\varphi, L)$ given $h(\varphi, L)$. Points O, A, B, C, D, and J are as in figure 9. Firstly, draw the quadrant J″K and the vertical KL to cut OD in L. *Note that* L *may be outside the quadrant* OAB. Draw the horizontal LM. Then we have

$$OM = KL = \frac{\text{Sin}\,h\,\text{Tan}\,\varphi}{R}$$

Note that for large values of φ, M *may be on* OB *produced.* Now draw the horizontal CC″, the quadrant C″N, and the vertical NP to cut OD in P. Draw the arc PR. Then, since

$$ON = OC'' = \text{Sin}\,\varphi_M,$$

we have

$$OR = OP = \frac{R\,\text{Sin}\,\varphi_M}{\text{Cos}\,\varphi}$$

so that

$$MR = \frac{\text{Sin}\,h\,\text{Tan}\,\varphi}{R} - \frac{R\,\text{Sin}\,\varphi_M}{\text{Cos}\,\varphi}.$$

Now mark S on OA such that OS equals MR. Then draw the vertical JJ′ and the quadrant J′T. Draw SU vertically to cut the arc J′T in U. Produce OU to cut arc AB in Q. Then, since

$$OU = OJ' = \text{Cos}\,h,$$

arc AQ measures $q(\varphi, L)$, by virtue of formula (12).

APPENDIX C

Medieval Qibla *Tables Based on Approximate Formulae*

Since completing the analysis of al-Khalīlī's table I have come across four other *qibla* tables in the manuscript sources.[44] Each of the tables is based on an approximate formula, but only the fourth table listed below is based on the approximation (2) noted above. The function tabulated in all of the tables is $q(\Delta\varphi, \Delta L)$ rather than $q(\varphi, L)$ as in al-Khalīlī's table, and entries are given to two sexagesimal digits for various domains. A detailed investigation of these *qibla* tables would be worthwhile.

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[44] I am greatly indebted to the Directors of the Municipal Library in Alexandria, the Egyptian National Library in Cairo, the Ẓāhiriya Library in Damascus, the Forschungsbibliothek in Gotha, the Süleymaniye Library in Istanbul, and the Bibliothèque Nationale in Paris, for the generous facilities afforded me in each of these libraries. I am also grateful to Sayyid Aḥmad al-Ahdal of Zabid, Yemen Arab Republic, for allowing me to photograph a manuscript in his private collection.

The first *qibla* table can be attributed to an Abbasid source, as yet unidentified. Five copies of this table have come to light, namely:

A1: MS Istanbul Aya Sofia 4830, fols. 187r–188r
A2: MS Istanbul Hamidiye 1453, fols. 233v–234r
A3: MS Istanbul Esat Efendi 3769, fol. 62r
A4: MS Alexandria Municipal Library *ḥisāb* 5577C
A5: MS in the private collection of Sayyid Aḥmad ʿAbd al-Qādir al-Ahdal, Zabid, Yemen

Values of $q(\Delta\varphi, \Delta L)$ are given for each degree of both arguments from 1° to 20°. In none of the five sources is the compiler of the table mentioned. However, in MS A1, which was copied in Damascus in A.H. 626–27 (A.D. 1228–30), the table is accompanied by a treatise on the determination of the *qibla* which occurs amidst a number of minor treatises by the celebrated Muḥammad ibn Mūsā al-Khwārizmī (fl. Baghdad, ca. A.D. 825).[45] In MS A2 the *qibla* table is to be found between a set of instructions for al-Khalīlī's auxiliary tables and the tables themselves; this manuscript was copied in Edirne in A.H. 869 (A.D. 1464–65). In MS A3 the *qibla* table occurs in a collection of various Turkish treatises on astronomy. In both Ottoman manuscripts the last column of entries is garbled. MSS A4 and A5 are two medieval Yemeni sources: the first is an apparently unique copy of the *zīj* for Taiz entitled *Taysīr al-maṭālib* by the mid-thirteenth century Yemeni astronomer Muḥammad ibn Abī Bakr al-Kawāshī al-Fārisī, and the second is likewise a unique copy of an anonymous sixteenth century *zīj* for Sanaa entitled *Zād al-musāfir*.[46] In MS A5 it is stated that the *qibla* table is taken from the *Taysīr*, and this work in turn is based on an as yet unidentified Iraqi *zīj* and on a *zīj* by Ibn Yūnus. Since MS A1 was copied in Damascus over a century before al-Khalīlī's time, it seems reasonable to suppose that he would have been aware of the existence of this Abbasid *qibla* table.

A second *qibla* table is contained in

B1: MS Paris Bibliothèque Nationale supp. pers. 1488, fols. 130v–121r

of the Persian *Ashrafī Zīj*, compiled in Shiraz in the early fourteenth century by Muḥammad ibn Abī ʿAbd Allāh Sanjar al-Kamālī, known as Sayf-e-Munajjim.[47] Values of $q(\Delta\varphi, \Delta L)$ are given for each degree of both arguments from 1° to 30°, and in the text of the *Zīj* (fols. 104r–104v) the table is referred to as *jadvali-ye-morabbaʿ-e-lām-dar-lām*, "the square 30 × 30 table." This table may not be original to the *Ashrafī Zīj*.

A third *qibla* table is to be found in the early fourteenth century *Zīj* of the Persian astronomer Shams al-Munajjim al-Wābiknawī.[48] I have examined

C1: MS Istanbul Aya Sofya 2694, fols. 165r–166v of the table; other copies of this *Zīj* exist in Tehran and Bombay. The values of $q(\Delta\varphi, \Delta L)$ are given for each degree of $\Delta\varphi$ from 1° to 30° and each degree of ΔL from 1° to 60°.

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[45] On al-Khwārizmī see G. Toomer's article in the *Dictionary of Scientific Biography*, s.v.

[46] On al-Fārisī see Brockelmann, *Geschichte der arabischen Litteratur*, 1: 625, and ibid., Supp., 1: 866–67, and Kennedy, "A Survey of Islamic Astronomical Tables," no. 54. The *Taysīr al-maṭālib* is listed only by Brockelmann. Further information on this work, the anonymous Sanaa *Zīj*, and over a dozen other *zījes* compiled in medieval Yemen, is contained in a paper on mathematical astronomy in the Yemen currently in preparation.

[47] On this work see Kennedy, "A Survey of Islamic Astronomical Tables." no. 4, and C. A. Storey, *Persian Literature:* A Bio-Bibliographical Survey, vol. 2/1 (London, 1958), pp. 64–65.

[48] On this work see Kennedy, "A Survey of Islamic Astronomical Tables," no. 35, and Storey, *Persian Literature*, p. 65.

A fourth *qibla* table is located in at least five manuscripts:

D1: MS Cairo Dār al-Kutub *miqāt* 167M, fols. 205v–206r
D2: MS Cairo Dār al-Kutub *miqāt* 204M, fols. 83v–84r
D3: MS Damascus Ẓāhirīya 4804, fols. 30r–31r
D4: MS Damascus Ẓāhirīya 7564, fol. 1v
D5: MS Gotha Forschungsbibliothek A1411, fols. 7v–8r

The argument domains are

$$\Delta\varphi = 11°, 12°, \ldots, 40° \text{ and } \Delta L = 1°, 2°, \ldots, 40° \text{ (MS D3)}$$

and

$$\Delta\varphi = 1°, 2°, \ldots, 20° \text{ and } \Delta L = 1°, 2°, \ldots, 20° \text{ (MSS D1, D2, D4, D5)}$$

A characteristic of this fourth table is that the entries for $\Delta\varphi = \Delta L$ are 45;0°, and this suggests that the underlying formula is the standard approximation (2), a hypothesis borne out by inspection of the other entries. The table is referred to in MSS A2 and A5 as *al-jadwal al-ʿishrīnī*, "the 20 × 20 table," and in MS A5 it is attributed to Yūsuf ibn al-Damīrī, an individual unknown in the modern literature.

XIV

ON MEDIEVAL ISLAMIC MULTIPLICATION TABLES

SUMMARIES

Multiplication tables for sexagesimal numbers were used by the astronomers of the Islamic Middle Ages in their arithmetical calculations and in the preparation of more sophisticated tables for finding planetary positions and for reckoning time by the sun and stars. Most known examples of Islamic multiplication tables contain about 3,600 entries, although one has been found containing 216,000 entries.

ملخص: غرض هذا البحث هو وصف عدد من الجداول للضرب التى استخدمها الفلكيون المسلمون فى العصور الوسطى تسهيلا لاعمال الحساب الستينى كما استعملوها فى تحضير جداول اخرى لتحديد اماكن الكواكب ولحل مسائل علم الميقات وغيرها من مسائل علم الفلك

Las tablas de multiplicación en el sistema sexagesimal fueron empleadas por los astrónomos de la Edad Media Islámica en sus cálculos aritméticos y en la preparación de tablas más sofisticadas para determinar las posiciones de los planetas y calcular la hora del dia o de la noche. En general esas tablas de multiplicación contienen unas 3600 valores; no obstante, un ejemplo de las mismas contiene un total de 216000 valores.

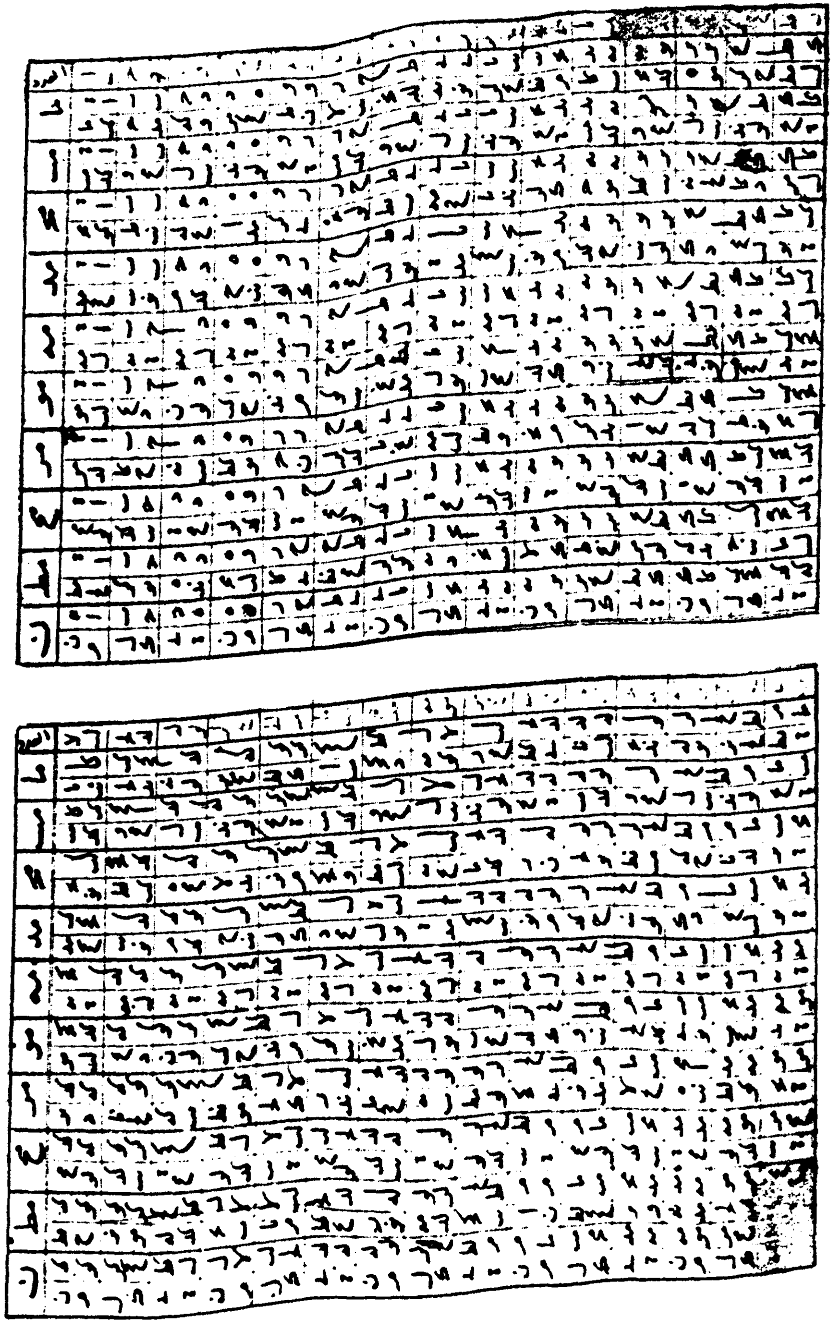

Plate I: Extract from the multiplication table in MS Paris Bibliothèque Nationale, ar. 2531

One of the major concerns of the astronomers of medieval Islam was the compilation of tables for planetary and spherical astronomy. [1] In virtually all of these the entries are expressed sexagesimally, that is, to base 60, using the standard Arabic alphabetical (*abjad*) notation. [2] In the modern notation used to represent sexagesimals, a number written 2,25;30,17 means 2 x 60 + 25 + 30/60 + 17/3600. As in decimal arithmetic, computation in the sexagesimal system is facilitated by having multiplication tables at hand, from which one can read, for example, the product 27 x 51 = 22,57 (=1377). Sexagesimal multiplication tables of a special kind are attested already in Babylonian sources [Neugebauer 1969, 18-29; Neugebauer-Sachs 1945, 19-33].

Most Islamic sexagesimal multiplication tables display products $m \times n$ to two sexagesimal digits for the domains $m, n = 1, 2 \ldots, 59$ or 60. These tables thus have about 3600 entries. Numerous examples of such tables are now known to exist in the vast manuscript sources available for the study of Islamic astronomy, and they are referred to in Arabic as *al-jadwal al-sittīnī* or *jadwal al-nisba al-sittīnīya*. [3] The earliest known mention of a table of this kind in an Islamic treatise is by Kūshyār ibn Labbān, writing about the year 1000 [Levey-Petruck 1965, 98; Luckey 1953, 169-170].

Plate I shows an extract from a multiplication table in MS Paris Bibliothèque Nationale ar. 2531, fols. 115v-127r, which may have been that used by the fifteenth-century Egyptian astronomer Ibn al-Majdī [de Slane 1883-1895, 542; King 1974, Sec. 7]. The argument m, in this case 41, 42 ..., 50, is read from right to left across the top of the page, and the argument n is read downwards, from 1 to 30 on the right hand page and from 31 to 60 on the left hand page. In this table the argument m actually runs to 120 (= 2,0).

In a rectangular table of products $m \times n$ the entries for $m \neq n$ are duplicated because of the commutativity of multiplication. Thus a triangular table giving products for $m \leqslant n$ will suffice. Only one such table is known to me in the Islamic sources [4], but doubtless this form was also commonly used. One small Islamic table giving products for the domains $m, n = 1, 2 \ldots, 10, 20 \ldots, 50$ has already been described in the literature. [5]

A rather unusual table is contained in MS Paris Bibliothèque Nationale ar. 2552. Miscataloged as a table for timekeeping [6], it is in fact a sexagesimal multiplication table displaying products $m \times n$ to three sexagesimal digits for the domains $m = 0;1, 0;2 \ldots, 59;59, 60;0$ and $n = 1, 2 \ldots, 60$. One can thus read directly from the table products of the form 4;27 x 37 = 24,56;39.

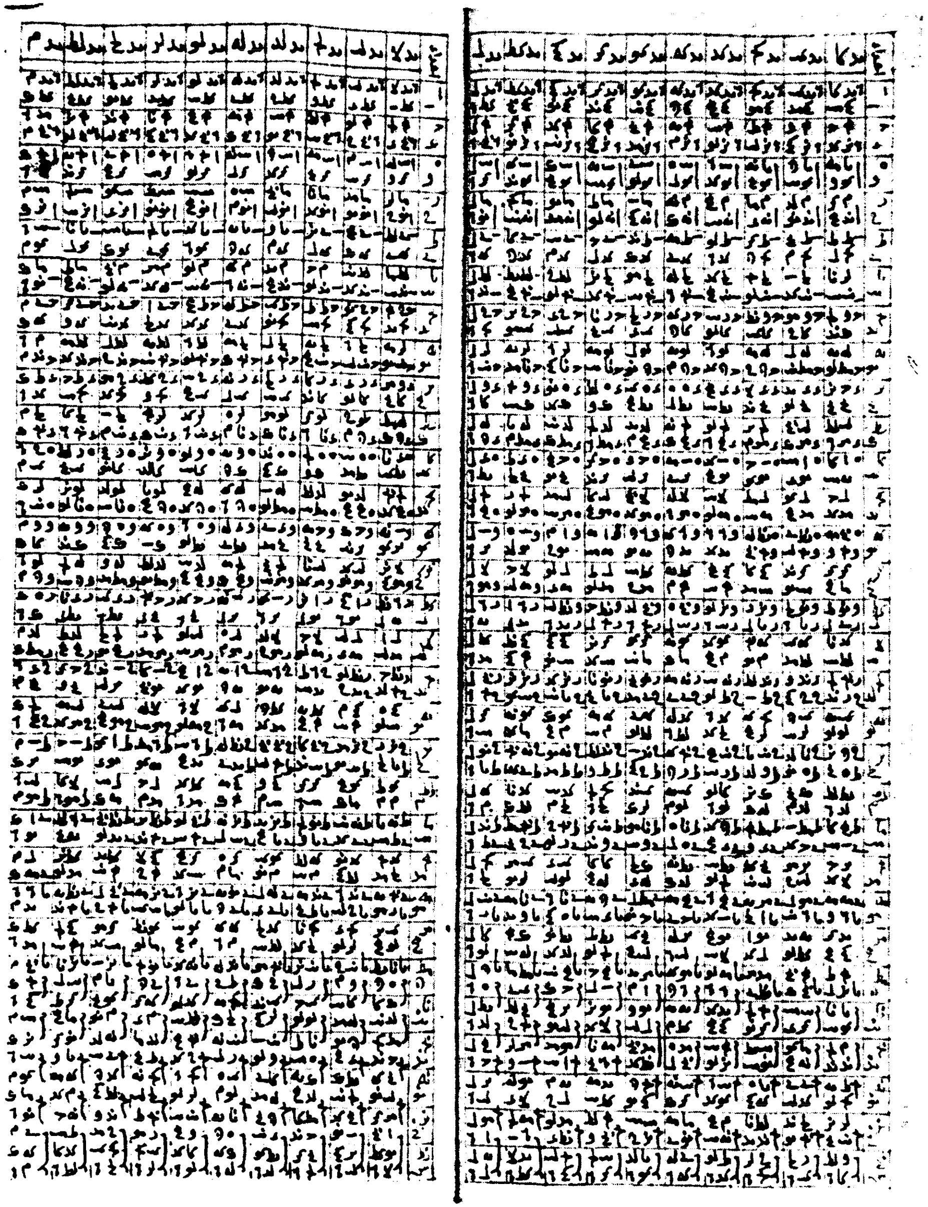

Plate II : Extract from the multiplication table in MS Paris Bibliothèque Nationale, ar. 2552

Plate II shows one pair of facing pages of the manuscript. The argument *m*, in this case 14;21, 14;22 ..., 14;40, is read from right to left at the head of each column, and the argument *n* is read downwards. There are 180 such pairs of facing pages in the entire set, and the entries number a grand total of 216,000. The manuscript is undated, and the flysheets have been defaced with some notes on timekeeping written in Turkish. I think that the table, which is one of the largest known tables from the medieval Islamic period [7], was probably compiled by an Ottoman astronomer about the year 1600.

NOTES

My research in Egypt during 1972-73 was sponsored by the Smithsonian Institution and the National Science Foundation, Washington, D.C., and outside Egypt was financed by a grant from the Penrose Fund of the American Philosophical Society. This support is gratefully acknowledged. The two photographs are published with kind permission of the Bibliothèque Nationale, Paris. I am also indebted to the Directors of the Libraries mentioned in notes 3 and 7 for permission to inspect the manuscripts in their collections.

1. See, for example [Kennedy 1956] for a survey of over 100 Islamic *zījes* or astronomical handbooks with planetary tables, and [King 1973a, 1973b, 1974] on extensive Islamic tables for planetary equations, timekeeping, and solving problems of spherical astronomy. I have prepared a survey and analysis of all known medieval Islamic tables for reckoning time by the sun and stars and regulating the astronomically defined times of prayer. Such tables represent a particularly Islamic contribution to the development of astronomy.
2. On the notation see [Irani 1955]. The most reliable modern works on Islamic sexagesimal arithmetic are [Luckey 1948; 1951; 1953].
3. Sources containing multiplication tables which have come to my attention are:
   MS Aleppo Aḥmadīya 1310
   MS Berlin Ahlwardt 5695, 3, fols. 1v-6r
   MS Berlin Ahlwardt 5721, fols. 5-35
   MS Berlin Ahlwardt 5722, fols. 31-38
   MS Berlin Ahlwardt 5757, fols. 136-137
   MS Berlin Ahlwardt 5758, fols. 59v-71r
   MS Cairo Azhar *falak* 4386, fols. 465-467
   MS Cairo Dār al-Kutub *mīqāt* 45
   MS Cairo Dār al-Kutub *mīqāt* 64, fols. 79v-94r
   MS Damascus Ẓāhirīya 3099, fols. 190v-192r
   MS Damascus Ẓāhirīya 5623, fols. 1v-7r

MS Damascus Ẓāhirīya 7388, fols. 18r-24r
MS Gotha Forschungsbibliothek A1381, fols. 1r-15v
MS Munich 865 (313) and 866 (318)
MS Oxford Bodleian Marsh 75 (Uri 1035), fols. 3v-37r
MS Paris Bibliothèque Nationale ar. 2520, fols. 167v-173r
MS Paris Bibliothèque Nationale ar. 2531, fols. 115v-127r
MS Princeton Yahuda 4072, fols. 23r-29r

P Luckey [1951,68] has drawn attention to a table of the same kind in a manuscript which in 1939 was in the Institut für Geschichte der Medizin und der Naturwissenschaften, Berlin (see further Hartner and Ruska [1939, 54/208]). Also, O. Neugebauer [1960, 36b] has noted a sexagesimal multiplication table in a medieval Byzantine astronomical manuscript based mainly on Islamic sources. For examples of sexagesimal multiplication tables in medieval Hebrew astronomical sources see MS Paris Bibliothèque Nationale heb. 1042, fols. 97r-102v and 105r-108v.

4. This table is contained in a compendium of medieval scientific works entitled *Jāmiʿ i Bahādur-Khānī* (P. 286 of the edition published in Mandi, India, in 1834), to which I was referred by Prof. E.S. Kennedy. On this work see [Storey 1958, 20].
5. This table, contained in MS Princeton Yahuda 373, is discussed in [Irani 1952].
6. See [de Slane 1883-1895, 459]. The manuscript measures 28.5 x 10.5 cms., and de Slane states that is dates from the seventeenth century. The handwriting is similar to that in MS Princeton Yahuda 373 (see footnote 5), which is dated 1650.
7. I use the term medieval here since various branches of traditional Islamic astronomy were practised in the Near East until the turn of the present century. Two other very large Islamic astronomical tables are the following: (a) The table of Najm al-Dīn al-Miṣrī (*fl. ca.* 1250?, Cairo) giving the time since rising of the sun or stars as a function of their altitude for any latitude, which contains over a quarter of a million entries. This is preserved in MSS Cairo Dār al-Kutub *mīqāt* 132M and Oxford Bodleian Marsh 672, which are two halves of a unique manuscript. (b) The double-argument planetary equation tables of ʿAbd al-Raḥmān al-Ṣāliḥī (*fl. ca.* 1500, Damascus), which contain over 170,000 entries. These tables display the lunar and planetary equations as functions of arguments which can be taken directly from the mean motion tables, and they are extant in numerous sources, including, for example, MSS Princeton Yahuda 4402 and Cairo Azhar *falak* 4386.

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XV

SUPPLEMENTARY NOTES ON MEDIEVAL ISLAMIC MULTIPLICATION TABLES

SUMMARIES

The astronomical tables compiled by Muslim astronomers in the medieval period are based on the sexagesimal system, and to facilitate their computations these astronomers used multiplication tables displaying products expressed sexagesimally. In a recent publication these tables were described for the first time. The present paper points to the existence of more of these tables, as well as to tables displaying sexagesimal quotients and others for facilitating linear interpolation.

ملخص: فى عام ١٩٧٤ نشر المؤلف بحثا فى هذه المجلة وصف فيه لاول مرة بعض الجداول لضرب الاعداد الستينية التى كان يستخدمها الفلكيون المسلمون فى العصور الوسطى . وها هنا ذيل لذلك البحث، يحتوى على وصف لبعض الجداول الاخرى للضرب الستينى وايضا للقسمة وللتعديل بين السطرين . وتم العثور على هذه الجداول فى عدة مكتبات وهى تحتوى على مادة علمية يرجع عهدها الى العصور الاسلامية الوسطى .

The following notes are intended to supplement an earlier paper [King 1974] describing the multiplication tables in the sexagesimal system used by the astronomers and mathematicians of medieval Islam. In my first paper I drew attention to various medieval Arabic manuscripts, preserved mainly in libraries in Europe, which contain tables displaying sexagesimal products

$$m \times n \text{ for } m, n = 1, 2, \ldots, 60.$$

These tables, called in Arabic *al-jadwal al-sittīnī* or *jadwal al-nisba al-sittīniya,* contain 3600 entries. I also pointed to the existence of a more extensive Arabic sexagesimal multiplication table preserved in MS Paris Bibliothèque Nationale ar. 2552. This table displays products

$$m \times n \text{ for } m = 0;1,\ 0;2,\ \ldots,\ 59;59,\ 1;0,0 \text{ and } n = 1, 2, \ldots, 60,$$

and hence contains 216,000 entries. The compilation of such an enormous table is as simple as it is tedious, and most medieval Muslim astronomers were content to use the smaller variety.

Since preparing the first paper I have come across a number of other multiplication tables in the vast manuscript sources available for the further study of medieval Islamic science. These are discussed below.

STANDARD SEXAGESIMAL MULTIPLICATION TABLES

The earliest dated copy of a standard sexagesimal multiplication table with both arguments running from 1 to 60 which has come to my attention is preserved in MS Cairo Dār al-Kutub *mīqāt* 64,7 (fols. 79v-94r) copied in 803 Hijra (= 1400-1401). The manuscript is of Syrian provenance, and contains a number of treatises on the use of different varieties of quadrants.

Another relatively early dated copy of a sexagesimal multiplication table was copied by the late 15th-century Damascus astronomer al-Tīzīnī [Suter 1900, No. 450]. This table, preserved in MS Oxford Bodleian Marsh 75, fols. 1r-37v, was copied in 890 Hijra (= 1490-1491). It is entitled *jadwal al-nisba al-sittīnīya fi l-a^{c}māl al-falakīya ilā mi'a wa-^{c}ishrīn*, "table of sexagesimal ratios for astronomical calculations [with argument] up to one hundred and twenty," and displays products

$$m \times n \text{ for } m = 1, 2, \ldots, 2{,}0 \ (= 120) \text{ and } n = 1, 2, \ldots, 1{,}0 \ (= 60).$$

It is followed in the manuscript by al-Tīzīnī's tables of the sine, cotangent, and versed sine functions (to base 60), as well as of the solar declination (using obliquity 23;35°). The purpose of tabulating sexagesimal products up to m = 2,0 was to facilitate operations such as the determination of time from solar or stellar altitude, which involve the use of the versed sine. This function (to base 60) assumes values between 0 and 2,0. In MS Istanbul Topkapi 7125-A3476,2 (fols. 33v-68r), copied in 912 Hijra (= 1506-1507), there is another multiplication table, and the argument m, for some reason, runs to 140 (= 2,20). In MSS Bombay K. R. Cama Oriental Institute 18, fols. 71v-81r and 19, fols. 14v-24r, both copied about 1600, there is a multiplication table in which the argument m runs to 100 (1,40).

Several other manuscripts preserved in Istanbul and Cairo, the largest repositories of medieval Arabic scientific manuscripts, contain sexagesimal multiplication tables of the standard kind, with both arguments running from 1 to 60. Some examples from Istanbul, all of which postdate the two mentioned above, are the following:

MS Bagdatli Vehbi Ef. 887, fols. 21r-36r,
MS Esat Ef. 3769, fols. 9v-13r,

MS Hamidiye 843, fols. 50v-56r,
MS Nurosmaniye 2904, fols. 47v-53r.

Other manuscripts containing standard sexagesimal multiplication tables preserved in the Egyptian National Library in Cairo are:

MS Zakīya 740 (15 pp., cop. *ca*. 1450),
MS Dār al-Kutub *mīqāt* 797 (26 fols., cop. *ca*. 1600)

and twenty others listed in my forthcoming catalogue of the Cairo scientific manuscripts.

None of these tables has any accompanying instructions of consequence. Simple treatises on the use of such tables and on sexagesimal arithmetic were compiled by a number of Egyptian astronomers in the 15th and 16th centuries. These include Ibn al-Majdī, al-Wafā'ī, Sibṭ al-Māridīnī, and Ibn Abi l-Fatḥ al-Ṣūfī [Suter 1900, Nos. 432, 437, 445 and 447].

Sexagesimal multiplication tables are attested, if rarely, in certain treatises on arithmetic and mathematics and also in *zīj*es. Some examples follow.

Firstly, MS Hyderabad Andra Pradesh State Central Library 298 (*ca*. 200 fols., copied *ca*. 1400) is the only known copy of the *Zīj* of Aḥmad ibn ^{c}Alī ibn Isḥāq al-Tūnisī [Suter 1900, No. 356], a work which was compiled in Tunis in the thirteenth century and which was widely used in North Africa for the next two or three centuries. Amongst the many tables in this precious manuscript is a sexagesimal multiplication table of the standard kind.

A Persian *zīj* prepared after the time of Ulugh Beg (early fifteenth century) by an individual named Malik Maḥmūd Karrāmī ibn Malik Aḥmad Khwānsārī [Storey 1958, 106, No. 27] and entitled *Kanz al-fawā'id fī dhikr al-qawā^{c}id* contains a sexagesimal multiplication table in the Introduction. I have examined MS Cairo Ṭal^{c}at *falak fārisī* 6 (375 fols., copied *ca*. 1700) of this work: the table in question occurs on fols. 31r-38v and is of the standard 60 × 60 type.

Some, but not all, copies of a *zīj* entitled *Ghāyat itqān al-ḥarakāt* by the seventeenth-century Yemeni astronomer named ^{c}Abd Allāh al-Sarḥī [Kennedy 1956, No. X212] contain a standard sexagesimal multiplication table. Copies which do contain such a table include MS Hyderabad Salar Jung *hay'a* 29 and an unnumbered manuscript in the Grand Mosque Library in Sanaa.

Another sexagesimal multiplication table is contained in an extensive treatise on arithmetic entitled *^{c}Uyūn al-ḥisāb* by Muḥammad Bāqir ibn Zayn al-^{c}Ābidīn Yazdī [Brockelmann 1943, SII, 591 and 1024, No. 72], who lived in Iran around the year 1640. I have examined MS Cairo Dar al-Kutub *riyāḍa* 822 of this work, copied *ca*. 1650; several other copies survive in other libraries. The table, which occurs on fol. 48v of the Cairo manuscript, displays sexagesimal products $m \times n$ for

$$m = 1, 2, \ldots, 10, 20, \ldots, 50$$

and

$$n = 1, 2, \ldots, 60.$$

Another table, on fol. 50v, displays the powers of sexagesimal products.

MS Cairo Dār al-Kutub K 7584,1 (fols. 1r-81r, copied 1841) is a fragment, without title or author's name, of a treatise on sexagesimal arithmetic and spherical astronomy. The work was compiled in Sfax in Tunisia, as far as can be judged from the spherical astronomical tables. Furthermore, it postdates Ulugh Beg because the author uses the latter's value of the obliquity. There is a sexagesimal multiplication table of the standard 60 × 60 kind on fols. 16v-20r of the Cairo manuscript.

Tables displaying only products for $m \leq n$ in a table of triangular form [see King 1974, 319 and note 4] are attested thus far only in Indian Islamic sources. A new example is MS Bombay K. R. Cama Oriental Institute 46 of a simplified version of the *Zīj-i-Muḥammad Shāhī* of Maharaja Jai Singh [Storey 1958, No. 144], where such a table is found on fols. 237v-238r. There is no such table in Jai Singh's original *zīj*.

As noted in [King 1974] the earliest reference to a sexagesimal multiplication table in the Arabic sources is in a treatise on arithmetic by Kūshyār ibn Labbān (*fl.* Iran, *ca.* 1000). Also, MS Florence Biblioteca Medicea Laurenziana 238 (Assemani CCCIX) contains a highly sophisticated treatment of sexagesimal arithmetic (Arabic, *ḥisāb al-munajjimīn*, "the arithmetic of the astronomers") compiled by the scholar al-Samaw'al ibn Yahūdā al-Maghribī (*fl.* Iraq and Persia, *ca.* 1150; see [Suter 1900, No. 302]). The treatise preserved in Florence deals with such topics as taking the square roots of sexagesimal numbers using a sexagesimal multiplication table. Thus, although most extant Islamic sexagesimal multiplication tables are of Egyptian, Syrian, or Turkish provenance and date from the 15th century or thereafter, I think it is reasonable to assume that such tables were also standard equipment of the Muslim astronomers in earlier centuries. For this reason alone, it is hardly surprising that no multiplication tables predating the 15th century have survived. Another reason is provided in an 11th century Persian astrological compendium, *Rawḍat al-munajjimīn*, by Mardānshāh ibn Abī l-Khayr Rāzī [Storey 1958, No. 81]. In the first *bāb* (chapter) of the eighth *maqāla* (treatise) of his work Mardānshāh writes (MS Cairo Ṭalᶜat *falak fārisī* 11, copied *ca.* 1300, fol. 58v):

> *In order to facilitate this operation, a table has been made, known as the sexagesimal table (*jadwal-i-sittīnī*), for performing multiplication and division. We put it on one separate piece (*juz'*) [of paper] in this work so that at any place where it is needed it may be placed before the eyes. It is not to be numbered among the pages.*

(This passage was located and translated by my colleague Dr. E. S. Kennedy.)

LARGE SEXAGESIMAL MULTIPLICATION TABLES

As noted in [King 1974], MS Paris Bibliothèque Nationale ar. 2552 (182 fols., copied *ca.* 1600) contains a sexagesimal multiplication table displaying products $m \times n$ to three sexagesimal digits for $m = 0;1, 0;2, \ldots, 0;59, 1;0$, and $n = 1, 2, \ldots, 60$. This manuscript is probably of Turkish origin, and was owned by Ahmad Muᶜayyid (?), and Sulaymān Durrī, a *muwaqqit* (timekeeper) at the Mosque of Sultan Muḥammad Khān (in Istanbul) about 1725. Two other tables of this kind have been located. They appear to be independently calculated, although I have not investigated the incidence of errors.

MS Cairo Taymūr *riyāḍa* 119 contains a multiplication table of the same kind as the one contained in the Paris manuscript. This Cairo manuscript, which dates from about 1500, comprises 600 pages of tables, each containing 180 products, representing only one-half of the whole set because the horizontal argument runs only to 29;59. The work is entitled *Kitāb al-nisba al-sittīniya al-kubrā*, which means "large sexagesimal table," and is attributed to Yūsuf ibn Qarqmās al-Ḥamzāwī, also known as Amīr al-Ḥājj al-Ḥalabī. al-Ḥamzāwī lived about 1475 and is known from his other treatises on sundials and astrology. (A work of his of particular interest and importance is an astrological history of political events in Aleppo about 1475, preserved in MS Cairo Dār al-Kutub Muṣṭafā Fāḍil *miqāt* 13,3.) At the end of the multiplication table there are some tables of proportional parts of the principal trigonometric ratios (to base $R = 60$; thus $\text{Sin}\, x = R \sin x$, etc; *cf.* Kennedy 1956, 139-140) of the obliquity of the ecliptic, ε, that is,

$$f(n) = \frac{n}{R} \cdot z \quad (n = 1, 2, \ldots, R)$$

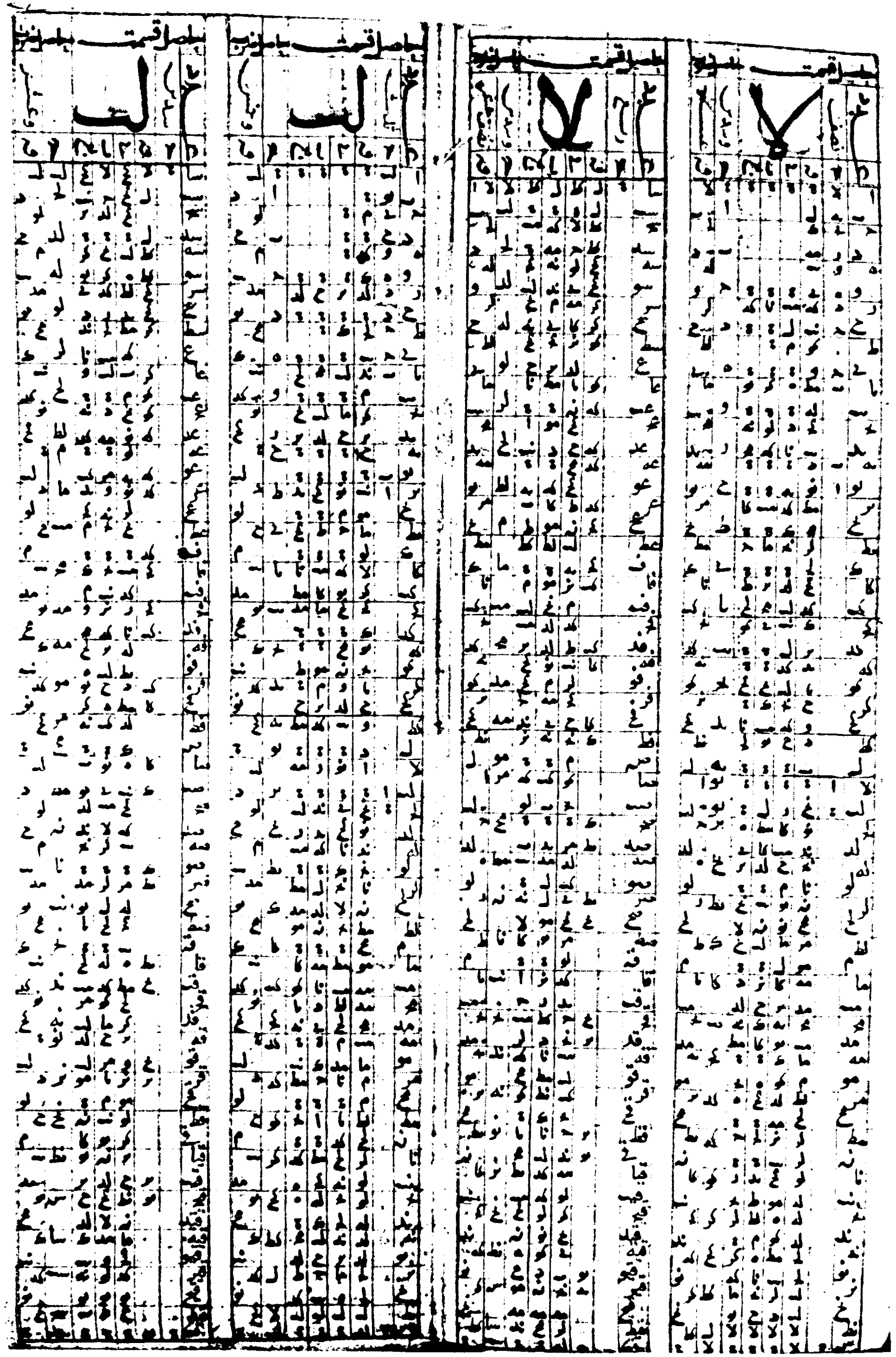
where

$$z = \text{Sin}\,\varepsilon, \quad \text{Cos}\,\varepsilon, \quad \text{Tan}\,\varepsilon, \quad \text{and Cot}\,\varepsilon\ .$$

The underlying values of Sin ε, etc., are related to those in the trigonometric tables of Ulugh Beg, compiled in 15th-century Samarqand [see Kennedy 1956, No. 12], but the parameter used for ε is the common Islamic one, 23;35°, rather than Ulugh Beg's value 23;30,17°. These values are

Sin 23;35° = 24;0,17,48,38,0,
Cos 23;35° = 54;59,19,29,45,0,
Tan 23;35° = 26;11,33,15,51,49,
Cot 23'35° = 2,17;26,36,44,30,37,51,5,12,46(!),

each of which is correct as far as the fourth sexagesimal



fractional digit. (Such tables of the trigonometric ratios of ε are attested already in 10th-century Islamic sources--see, for example, [Kennedy 1956, 145b]--and are of limited use in certain calculations of spherical astronomy.)

MS Cairo Dār al-Kutub K 3992 (120 fols., copied *ca.* 1750) contains another incomplete sexagesimal multiplication table of the larger variety with entries for m = 0;1, 0;2, ..., 48;0 and n = 1, 2, ..., 60. The table is entitled *Kitāb al-Nisba al-sittīnīya maḥlūla daqīqa daqīqa*, "Table of the sexagesimal ratio computed for each minute," and is attributed to Maḥmūd Efendī Jamliyān who lived in Cairo in the mid-eighteenth century and had a substantial library of astronomical manuscripts.

In MS Jaipur Maharaja Sawai Mansingh II Museum Library 14 of the *Zīj-i-Shāhjahānī* of Farīd al-Dīn al-Dihlawī, a work prepared in Delhi *ca.* 1630 [Storey 1958, No. 133], there is an incomplete sexagesimal multiplication table (fols. 299v-313v) in which the argument m runs up to 7;15 (n = 1, 2, ..., 60). Other manuscripts of this *zīj* which I have examined, such as MS Jaipur 12, do not contain such a sexagesimal multiplication table.

SEXAGESIMAL QUOTIENT TABLES

MSS Istanbul Aya Sofia 2698, Cairo Dār al-Kutub *mīqāt fārisī M* 8,1, and Istanbul Beyazit Umumi 4645 are three copies of an anonymous set of tables of sexagesimal quotients. The first manuscript is incorrectly entitled *Jadāwil fi l-miqāt*, "tables for timekeeping"; the second and third bear the more accurate title, *Kitāb al-mi'a wa-l-ᶜishrīn ᶜalā ṭarīq jadwal al-sittīnī*, "one hundred and twenty tables after the model of the sexagesimal multiplication table." The main tables in both sources display the quotients m/n (called *ḥāṣil-i-qisma*) to five significant sexagesimal digits for each integral value of both arguments from 1 to 120. The corresponding products $m \times n$ (called *ḥāṣil-i-ḍarb*) are also given. The Aya Sofia and Cairo manuscripts have a brief introduction in Persian and are beautifully copied; the tables are framed in gold ink. The Beyazit manuscript is less ornate and contains no introduction. All three sources are anonymous, but the Cairo manuscript dates from about the year 1300.

TABLES FOR LINEAR INTERPOLATION

Two relatively late Syrian sources contain multiplication tables specifically intended to facilitate linear interpolation

Plate 1: An extract from the sexagesimal quotient tables in MS Cairo Dār al-Kutub *mīqat fārisī* 8,1, fols. 23v-24r. The extract shows quotients m/n for m = 31 and 32, and n = 1,2, ..., 120. Reproduced by courtesy of the Egyptian National Library.

(Arabic, *al-taᶜdīl bayn al-saṭrayn*: literally, "correction between the two lines"). The first of these is contained in MS Damascus Ẓāhirīya 7387 of the *Zīj* (astronomical handbook) of ᶜAbd al-Raḥīm ibn ᶜAbd al-Karīm al-Qazwīnī al-ᶜAjamī, an astronomer associated with the Umayyad Mosque in Damascus about the year 1600, who made various minor modifications to the *Zij* of his illustrious predecessor Ibn al-Shāṭir. The table in al-Qazwīnī's *Zīj* which concerns the present study is found on fols. 69r-70r of the Ẓāhirīya manuscript and consists of six subtables entitled *jadāwil taᶜdīl mā bayn al-saṭrayn yu'khadh bi-l-tafāḍul wa-bi-l-kasr,* "tables for linear interpolation, found from the first difference (between the two consecutive entries) and the fraction (of the argument interval for which the value is required)." The function displayed in the subtables is

$$f(m,n) = n \times \frac{m}{60}$$

for arguments

$$m = 1,\ 2,\ \ldots,\ 5,\ 10,\ 20,\ \ldots,\ 50$$

and

$$\begin{aligned} n = {} & 0;0,\ 0;1,\ 0;2,\ \ldots,\ 0;5,\ 0;10,\ 0;15,\ \ldots, \\ & 0;55,\ 1;0,\ 1;1,\ 1;2,\ \ldots,\ 1;5,\ 1;10,\ 1;15, \\ & \ldots\ \ldots,\ 11;55. \end{aligned}$$

The argument n is labelled *al-tafāḍul*, "first difference," and the values $f(m,n)$ are labelled *qāf yā'*, which is an abbreviation for *daqā'iq thawānī*, "minutes and seconds." All the numbers are expressed sexagesimally in *abjad* notation.

The second Syrian interpolation table is contained in MS Damascus Ẓāhirīya 7976 of the *Zīj* of Muḥammad ibn ᶜAbd al-Raḥīm al-Mukhallalātī, who died in 1792. His *Zīj* is likewise based on that of Ibn al-Shāṭir and includes extensive double-argument planetary equation tables. The tables which concern the present study are found on pp. 48-68 of the Ẓāhirīya manuscript and are entitled *jadwal taᶜdīl faḍl mā bayn al-saṭrayn wa-taᶜdīl mā bayn al-tafāḍulayn,* literally, "table for correcting the difference between the two lines and the difference between the two first differences." The function tabulated is

$$f(m,n) = n \times m$$

for the argument domains

$$m = 1,\ 2,\ \ldots,\ 59 \text{ and } n = 0{,}1,\ 0{,}2,\ \ldots,\ 12{,}0.$$

All numbers are expressed in *abjad* notation.

MISCELLANEOUS

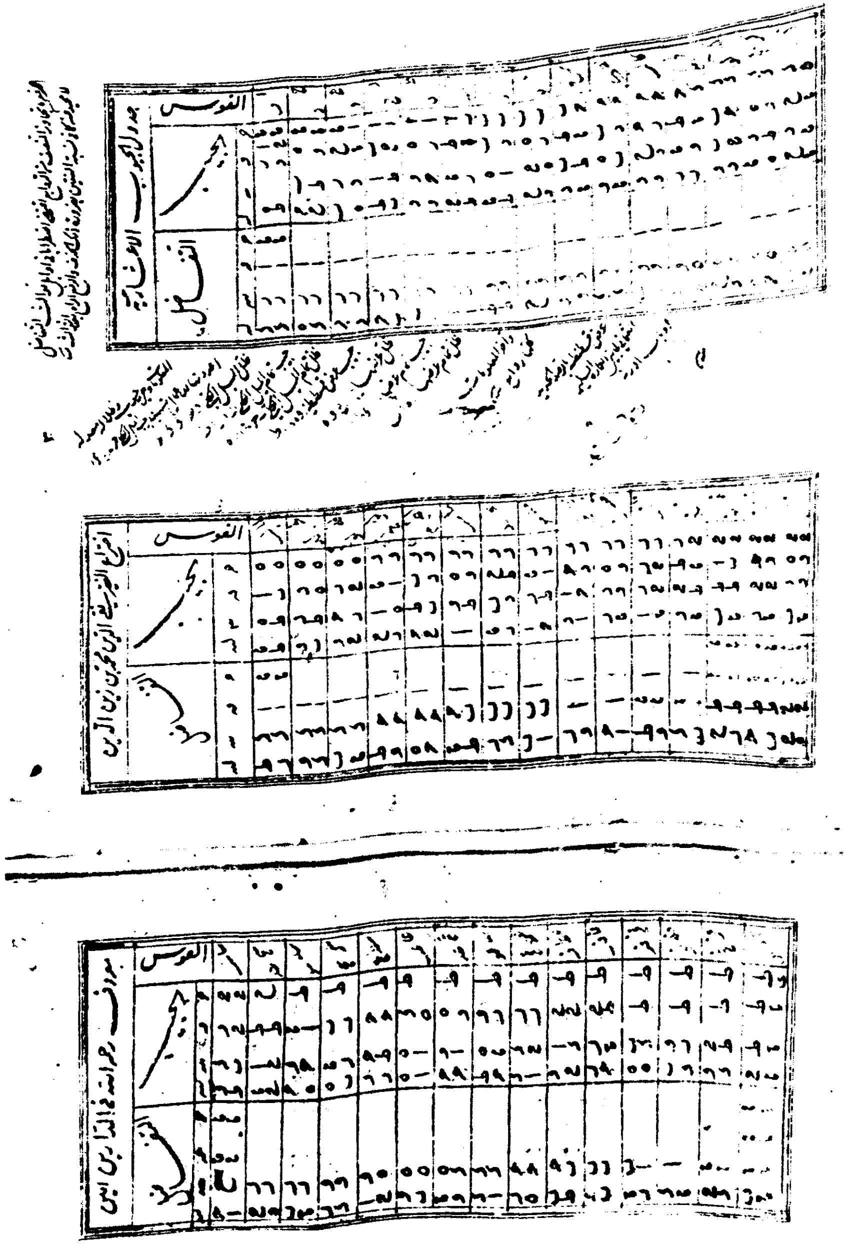
MS Istanbul Hafid Ef. 181, fol. 104v, contains a table entitled *Jadwal sittīnī li-basṭ al-kawākib laṭīf,* which means "nice sexagesimal multiplication table for finding the mean motions of the planets." Sexagesimal products $m \times n$ are given for the domains

$$m = 1,\ 2,\ \ldots,\ 120 \text{ and } n = 5,\ 9,\ 10.$$

The table occurs at the end of a redaction of the mid-13th-century Persian astronomical handbook *al-Zīj al-Īlkhānī,* prepared by the 16th century Syrian astronomer Aḥmad ibn Ibrāhīm ibn Khalīl al-Ḥalabī [see Kennedy 1956, No. 6; Suter 1900, No. 434]. I fail to see the advantage to be gained from using a table of this kind to calculate planetary mean motions.

MS Milan Ambrosiana C49 inf., fol. 104r, contains a small multiplication table displaying products $m \times n$ for each integral value of both arguments from 1 to 10. The table occurs at the end of a simple treatise on arithmetic by Ibn Fallūs, who worked in Mecca in the early 13th century [see Suter 1900, No. 359]. The products are expressed decimally in the Arabic *abjad* notation: thus, for example, the entry for 9 × 9 is *fā' alif*, representing 80 and 1, i.e., 81. In the arithmetic of the early 14th-century scholar Ibn al-Bannā' of Marrakesh [Suter 1900, No. 399; *cf.* Souissi 1969, 56-62] there are some notes on the use of a simple multiplication table of this kind; no table is presented in the text, and the products quoted as examples are written out in words.

MS Istanbul Esat Ef. 1976, fol. 19r, contains a similar multiplication table in which the products are expressed decimally in the Arabic *abjad* notation: thus, for example, the entry for 9 × 9 is *ḥā' alif* representing 8 and 1, i.e., 81. There is also a table of reciprocals $1/n$ for each integral value of n from 1 to 60, with values expressed as four-figure decimals in *abjad* notation. These two tables occur amidst various other tables due to the 16th-century Ottoman astonomer Taqī al-Dīn [Suter 1900, No. 471], preceding a complete copy of his astronomical handbook for Istanbul, entitled *Jarīdat al-durar wa-kharīdat al-fikar*. In this work the entries in the trigonometric and certain spherical astronomical tables are expressed decimally: these are the earliest examples of Islamic trigonometric tables in the decimal system. The base for Taqi l-Dīn's trigonometric functions is 10 rather than 1, and the names of the units are those of Islamic sexagesimal arithmetic. Thus for Sin 30° (the capital notation indicates that the base is 10, i.e., $\mathrm{Sin}\ x = 10 \sin x$), the entry is 5000, and the abbreviations at the head of the columns of entries are *j* (for *daraj*, "degrees"), *q* (for *daqā'iq*, "minutes"), *yt* (for *thāniya(t)*, "one second"), *lth*



(for *thawālith*, "thirds"). Likewise, the entry for Tan 45° is 10000, and the columns are labelled ᶜ (for *marfūᶜ*, "first power," literally "raised"), *j* (for *daraj*), *etc.*

Plate 2 shows the sine table from Taqi l-Dīn's *Zij*, taken from a manuscript of the work preserved in Cairo.

In Ms Cairo Dār al-Kutub *mīqāt* 900,1 (fols. 3v-5r, copied *ca.* 1785), preceding a copy of Taqi l-Dīn's *Zīj*, there is a set of decimal multiplication tables written in *abjad* notation displaying products,

$$x \times n \quad (n = 1, 2, \ldots, 10),$$

for x = sin ϕ_M, cos ϕ_M, sin ϕ_I, cos ϕ_I, sin ε , cos ε , tan ε , cot ε, tan ϕ_I, cot ϕ_I, sin ϕ_M, cos ϕ_M, and four other functions relating to timekeeping; ϕ_M and ϕ_I are the latitudes of Mecca and Istanbul and ε is the obliquity. Such tables are of (very) limited use in timekeeping calculations and in determining the *qibla* (direction of Mecca) at Istanbul. They are preceded by a decimal multiplication table displaying products $m \times n$ for $m,n = 1, 2, \ldots, 10$. The products are written in *abjad* notation so that, for example, 81 is represented by *ḥā' alif*, that is, 8 followed by 1.

MS Aleppo Awqāf 965, which was copied in Aleppo about 1800, contains a table entitled *jadwal ḥall al-darajāt,* literally "table for solving degrees," and displays the function

$$f(m,n) = 60 \times m + n, \qquad m = 1, 2, \ldots, 60 \text{ and } n = 1, 2, \ldots, 60.$$

The entries are written decimally in modern Arabic numerical notation, and the table can be used to find, for example, the number of minutes corresponding to a sexagesimal number $m;n$.

MS Princeton Yahuda 353 of the corpus of tables for timekeeping prepared for the latitude of Istanbul in the mid-18th century by Ṣāliḥ Efendī contains an additional table that is not found in any of the other known copies of the corpus. The table displays values to three sexagesimal digits of the function

$$f(x,y) = \frac{8y}{x}$$

for the domains

$$x = 10,\ 10;30,\ \ldots,\ 25;30, \quad y = 1, 2, 3, 4, 5, \text{ and } 10.$$

The purpose of tabulating this function is not stated. However, the same table occurs in MS Istanbul Kandilli Observatory 370

Plate 2: The sine table in the *Zīj* of Taqi l-Dīn, in which values are given to base 10 decimally for each degree of argument. This extract is taken from MS Cairo Talᶜat *mīqāt* 76, fols. 3r-4r, by courtesy of the Egyptian National Library.

with a note that it was prepared by the astronomer Shukrī Zāde to supplement Ṣāliḥ Efendī's tables of coordinates for constructing sundials.

A treatise in three *maqālas* on arithmetic, algebra, and surveying by ᶜAbd al-Majīd al-Sāmūlī, apparently compiled in Mecca about 1600, contains a small decimal multiplication table displaying products $m \times n$ for $m,n = 1, 2, \ldots, 9$, written in Arabic numerical notation and expressed decimally. I have examined MS Cairo Taymur *riyāḍa* 314 (117 pp., copied *ca.* 1800) of the first half of this treatise, in which the table occurs on p. 7.

MS Cairo Taymūr *riyāḍa* 321,2 (fols. 2v-6r, copied *ca.* 1800) contains a table, with neither title nor accompanying text, which displays the decimal products

$$44 \times n \quad (n = 1, 2, \ldots, 1000),$$

written in Arabic numerical notation. I have no idea what the purpose of this table might be.

MS Cairo Dār al-Kutub *mīqāt* 770,2 (fol. 7r, cop. *ca.* 1750) contains a table entitled *jadwal al-taᶜdīl al-aṣghar*, "table of the smallest equation," displaying the sexagesimal products

$$0;17,24 \times n \quad (n = 1, 2, \ldots, 60)$$

to three digits. The table is unrelated to the other works in the manuscript, and its purpose eludes me.

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XVI

A Handlist of the Arabic and Persian Astronomical Manuscripts in the Maharaja Mansingh II Library in Jaipur

THE MAHARAJA JAI SINGH (d. 1743) is well known to the history of science as the founder of the stone observatories of northern India, of which the most spectacular is in the "pink city" of Jaipur.[1] Having convinced his patron, the Emperor Muḥammad Shāh, of the inaccuracy of the current ephemerides, computed with the *zījes* of Ulugh Beg and al-Kāshī (*ca.* 1425) of Samarqand and with the Indian recensions of the *zīj* of Ulugh Beg made by Mullā Chānd (*ca.* 1600) in the reign of Akbar, and by Mullā Farīd al-Dīn (*ca.* 1630) in the reign of Shāhjahān, Jai Singh was ordered to undertake new observations with the help of Muslim, Brahman, and European astronomers. Besides constructing the observatories, Jai Singh collected manuscripts of Sanskrit, Persian, and Arabic astronomical works, as well as printed books from Europe. Some of these, surely only a fraction of his original collection, are still preserved in the library adjacent to the observatory in Jaipur, although not all of them date from the time of Jai Singh, notably the two manuscripts of his own *zīj*.

The purpose of this note is simply to identify the Arabic and Persian astronomical manuscripts preserved in the Library.[2] The manuscripts mentioned below add little to the corpus of material available for the further study of the history of Islamic astronomy in general, but are of interest in that they illustrate the kind of works that were being studied in Turkey, Iran, and India in the seventeenth, eighteenth, and nineteenth centuries. For each work represented I give only the most basic information, such as title and author, together with the accession number, number of folios, and date of copying (Hijra/Christian calendar), as well as the date of acquisition where this is available.[3] All of the authors and their works are well known to the history of Islamic science. The references given below, particularly those to the surveys of Arabic literature by C. Brockelmann and F. Sezgin and the survey of Persian literature by C. A. Storey, will guide the reader to other manuscripts of the same works preserved in other libraries.[4]

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* Department of Near Eastern Languages and Literature, Faculty of Arts and Sciences, New York University, 50 Washington Square South, New York NY 10003, U.S.A.

[1] On Jai Singh's astronomical activities see in the appended bibliography, for example, *Kaye*; *Sayili*, pp. 359-361; *Blanpied*; and *Price*. For an overview of Mogul astronomy see *Ansari*. On the translation of Islamic works into Sanskrit see *Pingree*.

[2] A list of the holdings of the Maharaja's Museum and Library, including most but not all of various Sanskrit, Islamic, and European astronomical works, is contained in *Das*. I have not been able to consult *Bahura*, which apparently lists only Sanskrit manuscripts.

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[3] Since these are given in Sanskrit, I have relied on *Das* for this information.

[4] The standard reference works on the sources for Islamic science are *Sezgin* (covering the period up till the mid-eleventh century); and *Suter* and *Brockelmann* (still the main sources for the later period); and *Storey* (for Persian works). Additional information on scientific manuscripts in Istanbul and Cairo is given in *Krause* and *King*, respectively. A survey of the Islamic astronomical handbooks known as *zījes* is in *Kennedy*.

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It is a pleasure to thank Dr. Asok Kumar Das, Director of the Maharaja Sawai Mansingh II Museum in the City Palace of Jaipur, for affording me every possible assistance in the Library of the Museum, and also Mr. Yaduendra Sahai, conservationist at the Museum, for ensuring that not a minute of my short visit to the Library was wasted.

List of manuscripts

1. The Arabic version by Thābit ibn Qurra of Ptolemy's *Almagest*: 20 (*ca.* 150 fols., copied *ca.* 1600, breaks off after the beginning of the sixth *maqāla*), and the Arabic recension of Ptolemy's *Almagest* by Naṣīr al-Dīn al-Ṭūsī: 19 (97 pp., copied *ca.* 1500, acquired 1725).

References: *Sezgin*, VI, pp. 89 and 93.

2. An anonymous Arabic commentary on the *Kitāb al-Manāẓir* (*Optics*) of Ibn al-Haytham, actually the *Tanqīḥ al-manāẓir* by Kamāl al-Dīn al-Fārisī: 17,1 (*ca.* 150 fols., copied 1070H = 1659-60, checked 1079H = 1668-69, acquired 1725, clear *naskhi* script with carefully-drawn diagrams). The manuscript appears to have been copied by Abū Muḥammad Samānī (?) for al-Shāh Qiyād ibn ᶜAbd al-Jalīl al-Ḥārithī al-Badakhshī known as Diyānthiḥān(?).

References: On Ibn al-Haytham and his *Optics* see the article by A. I. Sabra in *DSB* and the references there cited. Prof. Sabra is currently completing an edition of this work. On Kamāl al-Dīn al-Fārisī see the article by R. Rashed in *DSB*, and on the available manuscripts of the *Tanqīḥ* see *Brockelmann*, I, p. 619 and SI, p. 853, and *Krause*, no. 389.

3. The Arabic treatise on the rainbow and lunar halo by Ibn al-Haytham: 17,2 (8 fols. in the same hand as 17,1 – see no. 2 above).

References: On Ibn al-Haytham see 2 above. On this treatise see *Krause*, no. 204(19) and *Brockelmann*, SI, p. 853. The only other known copy of this work appears to be the Istanbul copy listed by Krause.

4. The Persian version of *al-Tafhīm li-awā'il ṣināᶜat al-tanjīm* by Abu'l-Rayḥān al-Bīrūnī: 7 (*ca.* 150 fols., copied *ca.* 1300, acquired 1725, fine copy).

References: *Storey*, no. 80; *Boilot*, no. 73. On Bīrūnī see also the article by E. S. Kennedy in *DSB*.

5. The Arabic commentary by Qāḍīzāda al-Rūmī on the treatise on theoretical astronomy entitled *al-Mulakhkhaṣ fi'l-hay'a* by Maḥmūd ibn ᶜUmar al-Jaghmīnī: 18 (106 fols., copied *ca.* 1600, acquired 1725).

References: On al-Jaghmīnī see *Suter*, no. 403; *Krause*, no. 403; *Brockelmann*, I, pp. 624-625, and SI, p. 865, and *Storey*, no. 88. The *Mulakhkhas* was compiled in 618H = 1221 (*contra Sezgin*, V, p. 115). On Qāḍīzāda see *Suter*, no. 430; *Brockelmann*, II, p. 275, *etc.*

6. The Arabic commentary by al-Nīsāpūrī on the treatise on theoretical astronomy entitled *al-Tadhkira* by Naṣīr al-Dīn al-Ṭūsī: 21 (250 fols., copied *ca.* 1600), and 22 (*ca.* 120 fols., copied *ca.* 1600, acquired 1725).

References: On al-Ṭusī see *Suter*, no. 368; *Krause*, no. 368; *Brockelmann*, I, pp. 670-676 and SI, pp. 924-933; *Storey*, nos. 10 and 91; and the article in *DSB* by S.H. Nasr. The *Tadhkira* is currently being investigated in detail by J. Rajeb of Harvard University. On al-Nīsāpūrī see *Suter*, no. 395, *Brockelmann*, II, p. 256 and SII, p. 273.

7. The Arabic commentary by ᶜAlī al-Birjandī on the treatise on arithmetic called *al-Shamsīya* by al-Nīsāpūrī: 10 (197 fols., copied 924H = 1518, acquired in 1725).

References: On al-Nīsāpūrī see 6 above. On ᶜAlī Birjandī see *Suter*, no. 456; and *Storey*, no. 121. Other copies of this commentary are listed in *Brockelmann*, SII, p. 273 (to which add MS Princeton Mach 4800).

8. The Persian astrological treatise *Lawā'iḥ al-qamar* by Ḥusayn ibn ᶜAlī al-Bayhaqī al-Kāshifī: 91 (*ca.* 100 fols., copied *ca.* 1600, acquired 1725).

References: *Storey*, no. 116.

9. An unidentified anonymous Persian work on astrology: 2 (*ca.* 150 fols., copied *ca.* 1700). The author quotes Dorotheos frequently. Incipit: ...

دليلها هفت ستاره بر مولودها

References: This manuscript is not listed in *Das*. No Persian astrological works based on Dorotheos are listed in *Storey*.

10. The Persian *Zīj-i Khāqānī* of Ghiyāth al-Dīn al-Kāshī: 9 (184 pp., copied *ca.* 1600, acquired 1728, fair copy, diagrams unlabelled).

References: *Storey*, nos. 104 and 105; *Kennedy*, no. 20. On al-Kāshī see also the article in *DSB* by A. P. Youschkevitch and B. A. Rosenfeld. An edition and translation of the *Khāqānī Zīj* is currently being prepared by E. S. Kennedy.

11. The Persian *Zīj-i Sulṭānī* of Ulugh Beg: 11 (*ca.* 195 fols., copied *ca.* 1500, fair copy), plus Persian commentaries by ᶜAlī Birjandī: 5 (*ca.* 200 fols., 1015H), and Mollā Chānd: 6 (*ca.* 250 fols., copied *ca.* 1600, acquired 1725).

References: *Storey*, no. 104; *Kennedy*, no. 12. On the Arabic versions of this *zīj* see also *Brockelmann*, II, pp. 275-276 and SII, p. 298. Only the Persian introduction of the *Zīj* of Ulugh Beg and the star catalogue have been published; for a brief survey of the remaining tables, which merit detailed study, see *Kennedy*, pp. 166-167.

12. The Persian *Zīj-i Shāhjahānī* compiled in Delhi by Farīd al-Dīn Masᶜūd ibn Ibrāhīm al-Dihlawī: 12 (438 fols., copied *ca.* 1800), and 14 (328 fols., copied *ca.* 1700, acquired 1725). In the second copy, the *zīj* is followed by an incomplete sexagesimal multiplication table (on which see *Historia Mathematica*, 1 (1974), 317-323, and 6 (1979), 405-417), and an incomplete table for computing the *mizāj* of the moon.

References: *Storey*, no. 133; and *Kennedy*, no. X204. The *Shāhjahānī Zīj* has, as far as I know, never been studied, and merits investigation.

13. The Persian *Zīj-i Muḥammad Shāhī* of Jai Singh: 4 (*ca.* 150 fols., copied *ca.* 1800??), and 8 (222 fols., copied *ca.* 1800).

References: *Storey*, no. 144; *Kennedy*, no. X203. Another copy which I have come across that is not listed in *Storey* is MS Aligarh University Library 30. Kaye, writing in 1918, implies that he was unable to locate a Persian copy of this *zīj* in Jaipur (*Kaye*, p. 7). The *zīj* of Jai Singh is unpublished, although much has been written on Jai Singh's astronomical activity (see *Storey* for references).

Sanskrit Translations of Islamic Works

For the sake of completeness I list the following manuscripts of Sanskrit versions of Islamic astronomical works, for which I have relied mainly on the handlist of the collection prepared by Dr. Asok Das and on Dr. David Pingree's survey of Islamic astronomical works in Sanskrit translation (see *Das* and *Pingree* in the bibliography). Other Sanskrit astronomical manuscripts are preserved in the Library, and also some European books on astronomy: see further *Das*. The Sanskrit manuscripts are listed in *Bahura* and are classified in D. Pingree's *Census of the Exact Sciences in Sanskrit* (see *CESS* for a full reference). Dr. Pingree kindly provided me with the information on the works numbered 18, 19, and 20 below.

14. *Zīj* of Nityānanda: 23 (443 pp.).

References: *Das*, p. 7, no. 127; *Pingree*, pp. 323-326.

15. *Hayatagrantha*: 24 (*ca.* 50 fols.)

References: *Das*, p. 6, no. 112; *Pingree*, pp. 326-328.

16. An extract from the tables in what is apparently a Sanskrit version of the *Zīj-i Ulugh Beg*: 45 (*ca.* 100 fols.).

References: *Das*, p. 7, no. 115; *Pingree*, p. 326.

17. Nayanasukhopādhyāya's translation of what purports to be Naṣīr al-Dīn al-Ṭūsī's *Tadhkira* in the commentary of al-Birjandī: 46 (*ca.* 60 fols.) (unique?).

References: *Das*, p. 6, no. 114; *Pingree*, p. 328. This work remains to be studied. For al-Birjandī on the *Tadhkira* see *Brockelmann*, SI, p. 931. This Sanskrit work is more probably a translation of al-Birjandī's Persian *Risāla-i-hay'a* (see *Storey*, no. 121), the relation of which to al-Ṭūsī's *Tadhkira* remains to be established.

18. Nayanasukhopādhyāya's translation of Naṣīr al-Dīn al-Ṭūsī's recension of the *Sphaerica* of Theodosius: 44.

References: *CESS*, A3, 132a, and A4.

19. *Yantrarājarisala bisa bāba*, a translation of Naṣīr al-Dīn al-Ṭūsī's treatise on the use of the astrolabe: 42.

References: *CESS*, A3, 145a, and A4.

20. *Virodhamardanagrantha*, a work in Maraṭhī composed by Yajnesvara Punakara Jyotirvit in 1837 and based on the *Zīj-i-Khāqānī* (see no. 10 above): 205 (16 fols.), unique?

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DSB: *Dictionary of Scientific Biography*, 15 vols., (New York: Charles Scribne's Sons, 1970-78).

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XVII

ISLAMIC MATHEMATICS AND ASTRONOMY

The chapters on mathematics and astronomy in *Islamic Science: An Illustrated Study*. S. H. Nasr (World of Islam Festival Publishing Co. Ltd, London, 1976). Pp. xiv+273. £12·50.

This lavishly-illustrated volume was prepared for the occasion of the Exhibition "Science and Technology in Islam" held in the Science Museum in London in 1976 as part of the so-called Festival of the World of Islam. The author is Professor S. H. Nasr, a professional historian of science and specialist on Islamic science, who is currently Chancellor of the Arya-Mehr University of Technology in Tehran. The Exhibition in London and also this new book by Professor Nasr attempted to cover the whole spectrum of Muslim activity in the sciences, defined in their widest sense, and to illustrate the depth and importance of the Muslim achievements in the sciences. The book claims to be (p. xiii b) "the first ever written on Islamic science in which the study and analysis of the texts is combined with illustrations from sources throughout the Islamic world".

In view of the nature of this journal and the limitations of my own experience in working on Islamic scientific material, I shall restrict my attention entirely to Nasr's chapters on mathematics and astronomy, which take up about one quarter of the book. In so doing I regrettably ignore what Nasr has written on physics, medicine, natural history, and "Man in the Universe". Likewise I shall not discuss Nasr's philosophy of Islamic science which enables him to see "the integration of the sciences of the Cosmos [in Islam] into the total scheme of knowledge drawn from the Quranic relevation" (front jacket). This philosophy, coupled with his contempt for Western rationalism, and disdain for Western science and civilization in general, which are in evidence in every chapter, make the work very much a personal interpretation rather than an historical survey. Typical of Nasr's personal interpretation of Islamic science is a remark found on p. 133a: "... Muslims had all the technical knowledge necessary to overthrow the Ptolemaic system [*sic*], including knowledge of the heliocentric system [*sic*], but they did not do so because they had not as yet become forgetful of the symbolic content of traditional astronomy nor of the fact that the best way to remind most men of the presence of God is to remind them of the limited character of the created world [!]."

However, Nasr's survey, which contains no "study and analysis" of any texts, is further marred by an inordinately large number of inadequately-documented claims and outright errors. Most of Nasr's statements referring to an individual, or a work, or a discovery, contain some kind of distortion or exaggeration resulting from the author's lack of familiarity with the original sources, or his innocence of the mathematics or astronomy involved. Not all the errors are Nasr's; many are found already in his sources. But this is not a book by an amateur; the author is a professional historian of science. Notwithstanding the fact that he is currently compiling an *Annotated bibliography of Islamic science* (of which the first volume appeared in 1975), he seems to be quite out of touch with much recent scholarship in the field of Islamic science.

Nasr is of course at his best when quoting directly such authorities on Islamic science as D. Pingree and E. S. Kennedy. But the general picture of Islamic science which he portrays indicates that he has not used some of the most important references which he cites, such as Kennedy's *Survey of Islamic astronomical tables*, Sayili's *Observatory in Islam*, Wiedemann's *Aufsätze zur Geschichte der Naturwissenschaften*, and Juschkewitsch's *Mathematik im Mittelalter*. Nor has he used any of the relevant and important articles in the *Encyclopaedia of Islam* (hereafter *EI_1* and *EI_2* for the first and second editions) or the *Dictionary of scientific biography* (hereafter *DSB*). Nasr's book is at first sight carefully documented with footnotes and references to numerous scholarly articles, but a superficial investigation of his choice of an authority here, or selection of a reference there, gives the impression that the book was written in some haste.

A curious omission from Nasr's chapters on mathematics and astronomy is any serious discussion of the peculiarly Islamic aspects of Islamic mathematics and astronomy. The arithmetic of inheritance is a rather important aspect of Islamic mathematics which Nasr does not even mention. The three Islamic aspects of Islamic astronomy, which are the sole surviving elements of a millenium of Islamic astronomy, are firstly, the determination of the visibility of the lunar crescent at the beginning of each Muslim month, secondly, the determination of the astronomically-defined times of Muslim prayer, and thirdly, the determination of the qibla or direction of Mecca. Nasr ignores the first of these aspects completely, and devotes three or four noncommittal sentences of his own to the second and third, which he calls the "cosmic dimension of the Islamic rites". Muslim astronomers concerned themselves with the determination of crescent visibility and of the prayer-times and qibla for over a millenium. This activity and the vast corpus of Islamic literature dealing with it are worth more than a few sentences in a book bearing the title *Islamic science*. Why are the prayer-times astronomically defined? How were they regulated in medieval times? How are they regulated today? Why do medieval mosques in general not face the true qibla? Why must the crescent be sighted before the new month begins? Why is there so much confusion about the beginning of the Muslim months in the modern Islamic world? These are fair questions, and their answers add a new dimension to our knowledge of Islamic science.

The following examples, taken from a few pages of Nasr's text, are presented to illustrate the major problems of at least the sections of the book dealing with mathematics and astronomy.

Let us begin with some of Nasr's remarks about the achievements of various Muslim scholars in number theory and computation (pp. 79–81). On p. 80a–b he states that "Kāshānī, the outstanding Persian mathematician, . . . invented the decimal fraction . . ." and contradicts this claim in a footnote saying "al-Uqlīdusī [*sic*] seems to have invented them . . .". In fact, as far as we know, al-Uqlīdisī of Damascus wrote on decimal fractions five centuries before al-Kāshānī (or al-Kāshī), and they were further developed in the intervening centuries. Further on in this section on number theory and computation we read that: "[al-Kāshānī] must also be considered the first person to have invented a calculating machine", but al-Kāshī's *ṭabaq al-manāṭiq* is a planetary equatorium, not a calculating machine.

On p. 81a Nasr writes of the prolific seventeenth-century writer Bahā' al-Dīn al-ᶜĀmilī that "[he] was particularly influential because he was a truly universal genius . . . it is not accidental that Suter concluded his now classical work on Islamic mathematics with him . . .". Suter (1900) indeed concluded his study with al-ᶜĀmilī, but he said of him: "Seine Werke zeigen keinen Fortschritt mehr, sondern im Gegenteil einen Rückschritt, von Selbständigkeit oder Originalität ist bei ihm und seinen Nachfolgern keine Spur mehr zu finden." The fact that al-ᶜĀmilī was indeed influential is more a reflection of the stagnation of intellectual life in Islam in the late medieval period than of the universality of his genius.

In the section on trigonometry (p. 4) Nasr states: "It was Muslim mathematicians who for the first time formulated the trigonometric functions explicitly. In fact the word 'sine' is the direct [*sic*] translation of the Arabic word *jayb*." Curiously, though, the Arabic word *jayb* is derived from Sanskrit *jiva*, where in the astronomical context it also means "sine". Nasr's discussion of algebra (pp. 84–86) begins: "As in trigonometry so in algebra Muslims must be considered as the founders of this science whose very name reflects its origin. . . . al-Khwārizmī . . . firmly established this branch of mathematics. . . ." In fact, both Greek and Hindu algebra had advanced well beyond the elementary stage of al-Khwārizmī's work. Nasr continues: "[Algebra] is closely related to certain metaphysical principles so central to Islamic doctrines", but unfortunately gives no examples. When he writes on p. 85a that "after Khayyām the study of algebra gradually declined among Muslims . . .", he ignores the results of much recent research on Islamic algebra which has radically improved our knowledge of the subject, notably the work of Rushdi Rashed.

In a remark about the Muslim contribution to mathematical geography on p. 98b we meet the statement that: "Altogether the astronomers of al-Ma'mūn made many original observations, one of the most outstanding being the measurement of the meridian near Mosul, which they found to be 111,814 metres", and the footnote "the actual value is 110,938 metres", which display a complete misunderstanding of the purpose of the observations and the results obtained. On p. 48 of Nasr's chapter on geography we read that "already during the 3rd/9th century they had measured a degree of latitude at about 36° N to be 2877 feet [!] . . .", which is somewhat at variance with the other result cited.

In a section on the sources of Islamic astronomy (pp. 95–98) Nasr misinterprets the meaning of *anwā'* and *nasī'*, concepts basic to the primitive folk astronomy of the Arabian peninsular in pre-Islamic times (*cf.* the articles *Anwā'* and *Nasī'* in EI_2 and EI_1 respectively). When discussing the non-Islamic sources of Islamic astronomy, Nasr quotes D. Pingree and then on p. 97b ventures the following remark of his own: ". . . by the 3rd/9th century Muslims became thoroughly acquainted with the astronomical traditions of the Indians, Persians and Greeks and of course, through them, with those of the ancient Babylonians and Egyptians." In fact the Muslims knew nothing of the astronomical traditions of either ancient Babylon or ancient Egypt as such: when isolated examples of Babylonian and Egyptian techniques do occur in Islamic sources their origins are not usually identified. Nasr claims on p. 106a: "Until the 4th/10th century Muslims followed Indian and Persian planetary models, while from the time of al-Battānī the Ptolemaic theory became completely dominant", but three of the four extant *zīj*es from ninth-century Iraq are based on Ptolemaic planetary theory.

The whole of Nasr's brief categorization of Islamic astronomical literature on p. 98a betrays a lack of familiarity with the most basic works of Islamic astronomy. For example, the *Nihāyat al-idrāk* of al-Shīrāzī belongs to precisely the same category of Islamic astronomical literature as al-Ṭūsī's *Tadhkira*. Also, al-Bīrūnī's *al-Qānūn al-Masᶜūdī* is a *zīj*. Nasr states that ". . . *zīj*es or tables . . . usually include results of observations in tabular form with extensive mathematical analysis and even occasionally discussion of mathematics itself". But *zīj*es are not tables. Nor do they include results of observations in tabular form. Furthermore, no example of a *zīj* containing any discussion of mathematics other than sexagesimal arithmetic comes to mind. In fact, as one can learn from E. S. Kennedy's survey of Islamic *zīj*es cited in Nasr's footnote, *zīj*es are astronomical handbooks sometimes containing several hundred pages of text and tables, and usually covering a variety of topics of concern to the medieval astronomer from chronology, to solar, lunar, and planetary motions, to astrology.

Nasr's conclusions on the achievements of individual Mulsim astronomers (pp. 98–105) include the following. On p. 105a he states that "al-Biṭrūjī . . . is responsible for . . . the most developed criticism against Ptolemaic astronomy to appear in the Western land of Islam" and on p. 106b that "Al-Biṭrūjī marks the peak of this movement . . . to create astronomically meaningful models based on the Aristotelian system . . .". Does the author not know that it is not possible to make meaningful models based on the Aristotelian system? On p. 105a Nasr perpetuates an old misconception that "al-Zarqālī . . . edited the *Toledan tables*". G. Toomer has shown in his penetrating study of the *Toledan tables* that the only part of the corpus which may be due to al-Zarqāllu is the planetary mean motion tables; the rest was lifted from the *Zīj*es of al-Khwārizmī and al-Battānī. On p. 105a we read also that "al-Zarqālī . . . proved the motion of the apogee of the sun with respect to the fixed stars . . .", but on p. 132a we read as one of the achievements of the Muslim astronomers that ". . . the motion of the solar apogee was observed and tied to the movement of the fixed stars [*i.e.*, the gradual precession of the equinox] . . .". Is the solar apogee sidereally fixed or not? On p. 99a Nasr states "al-Battānī was a careful observer who discovered the amount by which the Sun's apogee had increased since Ptolemy". But the Islamic sources inform us of various solar apogee measurements from the early ninth century onwards. (On such measurements prior to al-Battānī see, for example, W. Hartner's article "al-Battānī" in *DSB*.)

Nasr's section on number theory and computation concludes with a remark about the achievements of the Persian astronomers at Maragha. On p. 81a he writes: "Finally the Muslims developed techniques of computation far beyond what had existed before. This is to be seen especially in the circle of Naṣīr al-Dīn al-Ṭūsī at Maraghah, where a precision of one in ten million was attained for the table of tangents. To have a large number of mathematicians work on problems together, to coordinate their computation and finally to develop a means of checking error as one progresses is no easy task. But this is precisely what was achieved in Persia in the 7th/13th century, although the means whereby it was done are not as yet fully known. Be it as it may, it represents a major achievement of Islamic mathematics." On the contrary, we know precisely how the Maragha astronomers compiled their extensive tables of the tangent (and also the sine) function. They simply plagiarized them from the *Ḥākimī Zīj* of the tenth century Cairo astronomer Ibn Yūnus. The most superficial investigation of the tables themselves forbids making such extravagant claims for the achievements of the Maragha astronomers. However, it is well established that the trigonometric tables in the *Sulṭānī Zīj* of Ulugh Beg are in general accurate to five sexagesimal places: these tables Nasr does not mention.

In assessing the achievements of the astronomers of the observatories at Maragha and Samarqand on pp. 105, 109a, and 111a, Nasr might have pointed out that no analytic or comparative study has been made of the *Īlkhānī Zīj* of al-Ṭūsī and the *Sulṭānī Zīj* of Ulugh Beg, or of the other *zīj*es compiled at Maragha and Samarqand. However, we do know that the *Īlkhānī* trigonometric and planetary tables were lifted from the *Zīj*es of Ibn Yūnus and Ibn al-Aᶜlam, and we also know that the *Zīj* of Ulugh Beg is based on strictly Ptolemaic planetary theory. Thus the only innovations of the Maragha astronomers, namely, modifications to Ptolemaic planetary theory, were completely ignored by the astronomers of Ulugh Beg at Samarqand. On the other hand, in his section on the development of planetary theory Nasr might have mentioned that Ibn al-Shāṭir of Damascus is the only medieval Muslim astronomer who is known to have applied the new planetary theory in his astronomical tables (see the bibliography in my article "Ibn al-Shāṭir" in *DSB*). It is also important to stress that the Islamic modifications to the Ptolemaic system (p. 111a) were based entirely on philosophical speculation, not on any kind of observations.

Let us turn to Nasr's section on astronomical instruments. The section on the astrolabe (pp. 118–123) reads like an essay by a first-year student of Islamic art. On p. 118a we read that "this multi-functional instrument can determine the altitude of the stars, the Sun, the Moon and other planets [!] . . . It can also be used to tell time, and to measure the height of mountains and the depth of wells." A meagre description of an analogue computer devised to solve all of the standard problems of spherical astronomy! The markings on the plates of the astrolabe are then described as "the circle of declination and the azimuthal coordinates"—read instead "altitude circles and azimuth circles". On p. 120 Nasr writes that "the earliest treatise in Arabic on the astrolabe dates from the 3rd/9th century and is by Māshā'allāh", but al-Fazārī wrote on the astrolabe in the 2nd/8th century (neither treatise is extant in Arabic). Nasr also states that ". . . the earliest [astrolabe] extant is from 4th/10th century Isfahan", but six earlier Islamic astrolabes from Iraq were on display at the Exhibition in London.

Nasr's discussion of planetary equatoria (pp. 123a–b) is also confused. al-Bīrūnī did not, as far as we know, possess a "mechanical astrolabe by which the positions of planets and stars were determined with the help of a gear mechanism". He did, however, describe an eclipse computer. Nasr continues: "[al-Bīrūnī's instrument] was later perfected by the Andalusian astronomer Ibn al-Samḥ with his 'plates of the seven planets'." But al-Bīrūnī's eclipse computer has nothing to do with Ibn al-Samḥ's planetary equatorium. The diagrams of "the universal instrument of Jābir ibn Aflaḥ" presented on p. 122 (Figures 57 and 58) are meaningless without explanation; also this instrument should hardly be mentioned in a discussion of astrolabes (p. 122a).

Two instruments which the Muslim astronomers really developed were sundials and quadrants. Nasr devotes one sentence to the former (p. 114) and ignores the latter altogether, although there are some nice photographs of both sundials and quadrants. One of the photos (pl. 72 on p. 119) bears the caption "a sundial from Persia" but there is no sundial visible.

Nasr rightly cites the works of S. Tekeli on the relationship between early European and late Islamic observational instruments (p. 126a). But he pushes this relationship too far as an "achievement" of Taqi al-Dīn or, of course, of his Persian predecessors at Maragha. He writes: "The mural quadrant in which Tycho took great pride and which he called Tichonicus was also built by Taqī al-Dīn and called by him *libnah*. The same is true of many of the other instruments described by Taqī al-Dīn and going back ultimately to the Maraghah observatory." It so happens that in medieval Arabic *libna* is the term used for the mural quadrant of Ptolemy described in the *Almagest*. But even in this paragraph there occur more serious distortions of historical fact (p. 126a, note 31): "S. Tekeli is of the view that the torquetum so popular in the West did not come from Turkey but from the equatorial armillae (*dā'irat al-muᶜaddil*) so common in the eastern lands of Islam up to the 9th/15th century." Tekeli is here misquoted, and the footnote to one of her publications is a blind reference. In fact the instrument called *dā'irat al-muᶜaddil* was invented in Egypt in the 9th/15th century. Plate 20a on p. 44, labelled "an Ottoman compass", is actually a *dā'irat al-muᶜaddil*!

Nasr mentions the illustrious Egyptian astronomer Ibn Yūnus. Consider, for example, the statement on p. 112a: "Occasionally also observatories were built for individual astronomers, such as that of Ibn Yūnus on the Muqattam hill on the outskirts of Cairo." A. Sayili, in his definitive work *The observatory in Islam*, which is actually cited by Nasr, has shown conclusively that Ibn Yūnus did not have an observatory at all. Again on p. 101: "Ibn Yūnus was also the first person to make a serious study of the oscillatory motion of a pendulum, which finally led to the invention of the mechanical clock." It is significant that no source is cited for this statement, because there is not a shred of evidence that Ibn Yūnus knew of the pendulum. Again on p. 84: "[Ibn Yūnus] discovered that $\cos a \cos b = \frac{1}{2}[\cos(a+b)+\cos(a-b)]$." In fact this identity underlies two equivalent methods for determining the time from solar or stellar altitude in Ibn Yūnus's *zīj*. He did not "discover" the prosthaphaeresis formula: it was Delambre (*ca* 1820) who discovered that Ibn Yūnus's two methods were related by this formula. Again on p. 101a: "Ibn Yūnus's *Ḥākimī zīj* is a masterpiece of observational astronomy in which many constants have been measured anew. . . ." To be sure, Ibn Yūnus in the *Ḥākimī zīj* recorded over one-hundred observations made by his predecessors in the ninth and tenth centuries and also by himself. But it is remarkable that he did not bother to record in his *zīj* a single observation or calculation that he used to derive his own planetary parameters. The real achievements of Ibn Yūnus are not lessened by the fact that he did not have an observatory, that he did not know the principle of the pendulum, and that he did not discover the prosthaphaeresis formula of trigonometry.

Nasr's concluding chapter on the achievements of Islamic astronomy is perhaps the weakest tribute to the Islamic astronomical tradition. One of his final comments on p. 133a must suffice as an example: "But perhaps the most enduring contribution of Muslims to the history of astronomy was their transforming the Ptolemaic spheres from merely mathematical models to physical realities." It happens that we now know that Ptolemy's *Planetary hypotheses*, which was the starting point for Muslim investigations of theoretical astronomy, is a description of the arrangement of the heavens based on the assumption that the models of planetary motions have a physical existence. Also, since modern astronomers no longer believe in the physical reality of the planetary spheres, this "enduring contribution" of the Muslims (and the Greeks) is not generally recognized today.

The above quotations by no means exhaust the factual errors and misinterpretations even in the few pages singled out for discussion in this review. By these inaccuracies Nasr has unintentionally failed to do justice to a scientific tradition which knew no rival from the eighth to perhaps the fifteenth century. Nevertheless the book has the advantages of being very finely illustrated and having been published in the full radiance of the limelight of the Exhibition, and so it stands a good chance of being considered an authoritative work on the subject. Fortunately, a valuable and richly-documented catalogue of the London Exhibition has been prepared by F. Maddison and A. Turner. This catalogue provides in its introductory sections a much more carefully documented and much more accurate—and no less laudatory!—overview of Islamic science. However, the catalogue was not ready at the time of the Exhibition and is not yet published.

Nowhere is this writer more in agreement with Nasr than when he states on p. 98b "unfortunately today even some of the most basic works . . . have been only partially studied, not to speak of many which remain forgotten in manuscript form in various libraries", so that "almost every day there are still important and major discoveries made in this field". There is certainly also a need for a synthesis of the available secondary literature in a reliable form. It is doubtful that any modern scholar could hope to do justice to the entire

spectrum of Islamic science, and persons better qualified than myself must judge whether Nasr has done justice to Muslim activity in branches of science other than mathematics and astronomy. However, the major achievements of the Muslim scholars at least in these two fields were of a highly technical nature and do not lend themselves to a popular survey, unless the author is fully in control of his subject.

Smithsonian Institution Project in Medieval Islamic
Astronomy,
American Research Center in Cairo

XVIII

ISLAMIC MATHEMATICS

The Muslim Contribution to Mathematics. ALI ABDULLAH AL-DAFFA' (Croom Helm, London, and Humanities Press, Atlantic Highlands, N.J., 1977). Pp. 121.

There is an obvious need for a popular but reliable account in English of Muslim contributions to each branch of medieval science. No such accounts exist even for astronomy, mathematics, and medicine, major sciences in which the Muslims worked for centuries without rivals. S. H. Nasr's recent publication *Islamic science* is unfortunately riddled with errors of fact and interpretation (see the review in the *Journal for the history of astronomy,* ix (1978), 212-19). Now, however, another new book has appeared with a title which might encourage the reader to believe that the gap has been filled at least for mathematics. The book is nicely produced and promises to "present the Muslim contribution to mathematics during the golden age of Muslim learning".

The book consists mainly of a collection of citations from the tertiary literature, strung together like a high-school term paper with a nice reference for every few sentences. There are a few quotations from the secondary literature, with the usual over-emphasis on al-Khwarizmi. Errors of fact, interpretation, and orthography occur even more frequently than the footnotes. The index contains about one hundred and fifty items, but only twelve of the entries are names of Muslim mathematicians (including the greatest scientist of Islam, al-Biruni, listed under the A's as Abdul Rihan!). The bibliography contains over two hundred items, but the vast majority of these have nothing to contribute to the history of Islamic mathematics (including Dwight Eisenhower's Address to the U.N. General Assembly!). There are no works listed in French or German, or even Russian, which is particularly unfortunate because most of the basic secondary literature on Islamic mathematics is in French (by Woepcke), or German (by Suter and Luckey), or Russian (by Rosenfeld and Youschkevitch). None of the more recent important studies of A. Saidan on Islamic arithmetic (in Arabic and English) or A. Anbouba and R. Rashed on Islamic algebra (Arabic and French) has been consulted. The author is apparently unaware of the existence of H. Suter's *Arab mathematicians and astronomers and their works* (in German), F. Sezgin's *History of Arabic literature,* vol. v: *Mathematics* (in German), and A. P. Youschkevitch's *History of mathematics in the middle ages* (available in Russian and German), or even of the various reliable articles in English in *The encyclopaedia of Islam* and the *Dictionary of scientific biography.* Youschkevitch's book is still the best survey of the secondary scholarly literature on Islamic mathematics, and the chapter dealing with Islamic mathematics is now available in a new French translation with additions by the author and translators, and an updated bibliography (see the review in *Journal of the history of Arabic science,* i (1977), 111).

In view of the fact that the author has not used any of this standard secondary material on Islamic mathematics it is not surprising that his book hardly touches on the impressive achievements of the mathematicians of medieval Islam. For these there is still no adequate survey in English, but there is a wealth of reliable secondary literature from which such a survey could be made.

New York University

ADDENDA AND CORRIGENDA

Bibliographical abbreviations:
* *Cairo Survey* = D.A. King, *A Survey of the Scientific Manuscripts in the Egyptian National Library,* (Malibu, Ca.: Undena, 1985).
* *IAI* = D.A. King, *Islamic Astronomical Instruments,* to be published by Variorum Reprints in 1986.
* *IY* = *Ibn Yūnus* = D.A. King, *Spherical Astronomy in Medieval Islam: The Ḥākimī Zīj of Ibn Yūnus,* to appear.
* *JAOS* = *Journal of the American Oriental Society.*
* *JHAS* = *Journal for the History of Arabic Science.*
* *JNES* = *Journal of Near Eastern Studies.*
* *Kennedy Festschrift* = D.A. King and G. Saliba, eds., *From Deferent to Equant: A Volume of Studies in the History of Science in the Near East in Honor of E.S. Kennedy, Annals of the New York Academy of Sciences,* (1985).
* Kennedy, *Studies* = E.S. Kennedy, Colleagues, and Former Students, *Studies in the Islamic Exact Sciences,* (Beirut: AUB Press, 1983).
* *MAY* = D.A. King, *Mathematical Astronomy in Medieval Yemen,* (Malibu, Ca.: Undena, 1983).
* *SATMI* = D.A. King, *Studies in Astronomical Timekeeping in Medieval Islam,* to appear.
* *ZGAIW* = *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften.*

General remarks: The numerous papers of E.S. Kennedy cited throughout my own are now reprinted in Kennedy, *Studies.* A bibliography of studies either by Kennedy or inspired by him is contained in *Kennedy Festschrift.*

II: It should be stressed that the tables described here are generally not found in *zījes* and so were not mentioned by E.S. Kennedy in his *zīj* survey – see p. 40. * Pp. 40-41: For a survey of multiplication tables, see now XIV and and XV. * Pp. 41-44: For a more detailed survey of Islamic trigonometric tables, see *IY,* Part II, Section C. * Pp. 44-46: For a more detailed survey of Islamic tables of spherical astronomical functions, see *IY,* Part II, Section E, and for a survey of all known tables for timekeeping by the sun and stars, see *SATMI,* Part I. * Pp. 46-48: See *SATMI,* Part II for a survey of all known prayer-tables. * Pp. 48-49: See now XIII, 120-122 for a survey of qibla-tables. * Pp. 49-51: See *SATMI,* Part I, Section 9 on auxiliary tables. * Pp. 51-55: See further *IAI.* * P. 55: On planetary equation tables, see now V and the references there cited, to which add G. Saliba's studies in *JHAS,* 1 (1977), pp. 24-32, and 2 (1978), pp. 53-65. * P. 56: On this Abbasid material, see now my "al-Khwārizmī and New Trends in Mathematical Astronomy in the Ninth Century," *Occasional Papers on the Near East* (New York University, Hagop Kevorkian Center for Near Eastern Studies), 2 (1983).

III: An abridged version of this paper, with different illustrations and a discussion of the orientation of Cairo, was published in *Muqarnas*, 2 (1984), pp. 73-84.

IV: See now *MAY* for a more detailed survey of Yemeni astronomy.

V: Pp. 132-133. On the meaning of the curious term *ḥabṭaq*, see now *Cairo Survey*, no. C10.

VIII: P. 4: The photos of the Escorial manuscript were so poorly reproduced in *JHAS* as to be virtually illegible. They are here suppressed altogether, so that the pagination jumps from 21 to 42. * P. 5: An astronomer who worked in Qayrawan in the tenth century was the Jew Dūnas ibn Tamīm. His activities are discussed in S.M. Stern, "A Treatise on the Armillary Sphere by Dūnas ibn Tamīm" (first publ. 1956), repr. as XII in *idem, Medieval Arabic and Hebrew Thought*, (London: Variorum Reprints, 1983). * Pp. 19-20: On this table, see now my paper in the *Kennedy Festschrift*.

IX: This study was based mainly on MS G of the Cairo corpus, in which all of the tables are attributed to Ibn Yūnus on the title folio (see *Cairo Survey*, Pl. XXVI). The attribution of the tables is actually much more complicated: see III 540-542 and *SATMI*, Part II for more information. * Pp. 362 and 364: For al-Maqdisī, read al-Maqsī (from a suburb of Cairo known to this day as al-Maqs). The use of the word "plagiarized" was ill-advised - see above. See further *Cairo Survey*, no. C15. * Pp. 368 and 371-373: On the qibla and the astronomically aligned ventilators of medieval Cairo, see my paper "Architecture and Astronomy: The Ventilators of Medieval Cairo and Their Secrets," *JAOS*, 104:1 (1984), pp. 97-133. * P. 373, n.68: For al-Manūfī, read al-Minūfī. This individual (on whom, see *Cairo Survey*, no. C120) is not to be identified with Ibn Abi 'l-Fath al-Sufi. * P. 376: These instructions were certainly not written by Ibn Yūnus, and are probably due to al-Bakhāniqī (*MAY*, no. 13 and *Cairo Survey*, no. C28).

XI: P. 109: On tables of the Secant function, see *SATMI*, Part I, Section 6.9. On multiplication tables, see now XIV and XV.

XII: P. 255, n.28: See now F.E. Barmore, "Turkish Mosque Orientation ...," to appear in *JNES*.

XIII: P. 84, n.10: MS Paris B.N. ar. 2560,11 is indeed a third copy of al-Khalīlī's table. * Pp. 120-122: On these tables, see my paper "Some Early Mathematical Methods and Tables for Finding the Direction of Mecca," to appear in *ZGAIW*.

XVI: See now D. Pingree, "Indian and Islamic Astronomy at Jayasiṃha's Court" in *Kennedy Festschrift*.

XVII: P. 216: The tangent tables in the *Zīj-i-Īlkhānī* are lifted from al-Bīrūnī, not Ibn Yūnus. It is the *Īlkhānī* sine tables which are taken from Ibn Yūnus.

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